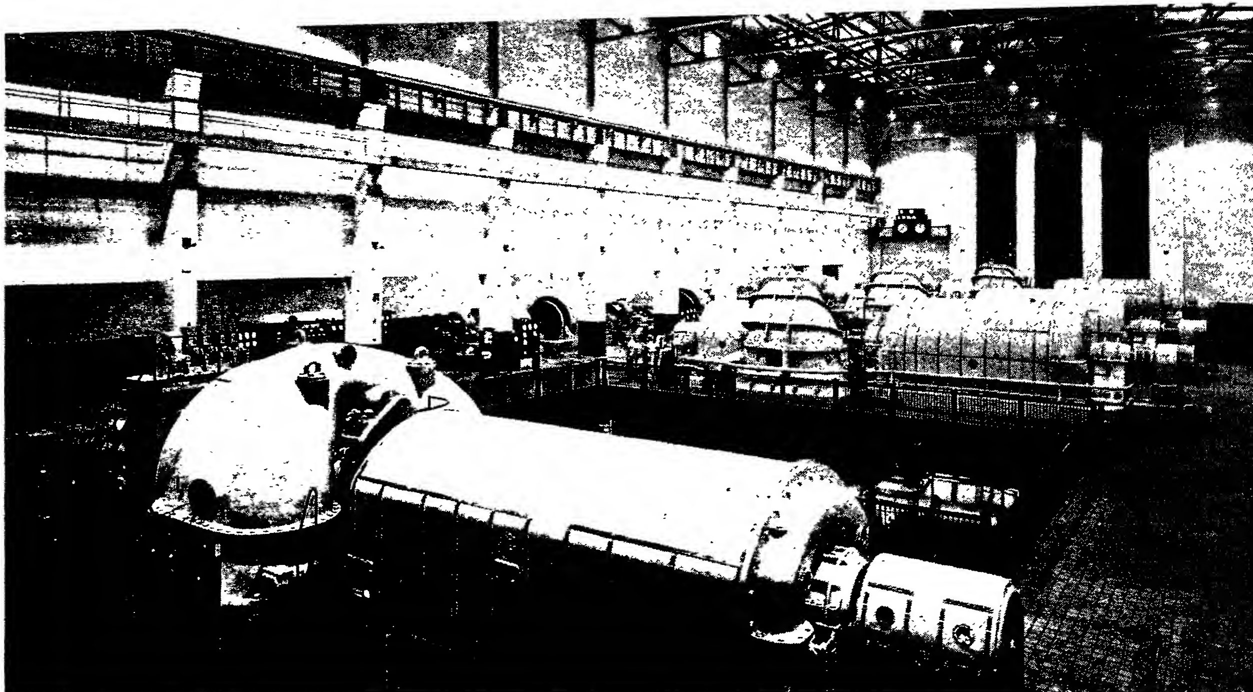
A. V. LEPPANEN

GENERATING STATIONS



Delray station No. 3. New 75,000-kw. turboalternator in foreground, first of three units, 14.4 kv., 1,800 r.p.m., operating on straight regenerative cycle at 815 p.s.i., 900°F. Older units in rear are 50,000 kw., operating at 375 p.s.i. and 700°F. The 4,000-kw., 240-volt, direct-current, house-service turbogenerators are shown in the left-hand bay. (*Courtesy of Detroit Edison Co.*)

GENERATING STATIONS

ECONOMIC ELEMENTS
OF ELECTRICAL DESIGN

BY

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GENERATING STATIONS

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To

JOHN CASTLEREAGH PARKER

Teacher and Friend

PREFACE

Since the second edition in 1935 there have been great changes in generating stations. Within the past year there has been a drive for rapid increase in the total power capacity available for service.

In the hydroelectric field the expansion has been phenomenal, more than 1,000,000 hp. in waterwheel-driven generators being under construction by one manufacturer alone at the end of 1939. Both the Federal government and the privately owned utilities have constructed, or have under way, many new plants as well as planned additions to the generators already installed.

In the steam plants the development of better metals has permitted thermal cycles to advance decisively to new high pressures and temperatures with pioneering installations planned at 1,800 and 2,400 lb. Generator developments of a 3,600 r.p.m. machine with hydrogen cooling have greatly increased kilovolt-ampere capacities and improved the already high efficiency. In 1937 superposition was introduced to increase the capacity and improve the economy of existing low-pressure stations. For the first time standardization has been suggested. In order to expedite production and installation which should be adequate for both peacetime industry and possible war expansion, the National Defense Power Committee has recommended preferred standards for steam-turbine generators.

The large hydro plants with their associated high-voltage lines have placed additional emphasis upon the transmission of bulk power with the related problems of maintaining stability and rapid clearing of faults. In this connection there have been developments of high-speed relays and new faster opening circuit breakers. As a result of research in recent years several types of oilless circuit breakers have been presented for service at 15 kv.

Special thanks are due to the engineers and officers of many power companies, the Tennessee Valley Authority, the Hydro-Electric Power Commission of Ontario, the U.S. Bureau of

Reclamation, and the electrical manufacturing companies for data furnished and courtesies extended. Many engineers and economists have generously allowed the use of their work, for which acknowledgment is made in the text. Also numerous references have been made to developments and problems in the standard engineering publications in order to encourage the student to use these sources.

ALFRED H. LOVELL.

UNIVERSITY OF MICHIGAN,
ANN ARBOR, MICH.,
January, 1941.

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GENERATING STATIONS

CHAPTER I

ELEMENTARY PRINCIPLES OF CORPORATE FINANCE

1. The Economic Motive.—With the exception of the construction of works of art or memorial buildings, all engineering projects are undertaken from an economic motive, either direct or indirect. In particular, the design and construction of a power system are undertaken for the purpose of producing power to be sold at a profit, either to other persons or, virtually, to the owner himself for use in his principal business. In either case, the object is to produce the power as cheaply as possible, taking into account the quality of the power produced. Indeed, this quality will in itself affect the cheapness or expensiveness of the product, since a bad power service is likely to be very expensive in the long run.

The cheapest power is not necessarily produced by the most efficient plant, since in general it will involve a great deal more investment to secure such a plant than would be required to secure a less efficient plant. The difference between real engineering and the mere technical selection of the academically best lies in the proper proportioning of investment and operating cost.

We are not unfamiliar with a very inexact form of expression of this general idea of evaluating efficiency in machines. A man, on being shown a superlatively fine automobile, may admit its excellencies while deciding that for him they are not worth the higher price charged for such a car. The engineer must be in a position to determine with reasonable certainty whether a higher degree of efficiency justifies the greater investment involved, whether more assured continuity of service, greater safety, further ease of operation, or additional automatic features are worth their extra cost, and in order to accomplish this must have some fundamental understanding of the economics back of his design. Usually, other things being equal, the more expensive the first cost of the design, the cheaper it will be to operate the plant, and

vice versa. Note that here we have to compare original construction costs, incurred within a comparatively brief time, with those recurring costs which are repeated year by year and day by day. Therefore in all plants an itemized record of plant performance and cost of operation must be kept in order to obtain economic results. These and the capital accounts are almost universally kept in accordance with the Uniform System of Accounts prescribed on Jan. 1, 1938, by the state Public Utility Commissions and the Federal Power Commission. The value of such records lies in the fact that they will inform the engineer as to the exact costs at all times and enable him to proceed intelligently in reducing the losses. The intrinsic value of adequate operating data was demonstrated abundantly in the recent depression, when with major construction programs halted they enabled engineering staffs to effect real operating economies and to cut the capital costs on such minor extensions as had to be made. These economies more than justified the cost of such engineering.[1]

2. The Preliminary Report and Estimate.—In reporting upon the feasibility of any power project, the engineer, as has been customary, must consider all its economic phases: the cost of the system, the amount and uniformity of the power that can be developed, the available market that might justify the power development, the price at which the power can be sold, and all the charges and expenses involved in the construction and operation of the plant. In recent years, public opinion has come to recognize that electric power supply is a vital and necessary modern service, that it is a fundamental element in our industrial life which must be adequate and assured even for war-time expansion, and that it is a dominating factor in the recent Federal programs for social and economic betterment in the South and Northwest. Hence the report should also consider the general social aspects of the problem. That is, if the engineering project is to be valuable to humanity, the value of the *benefits* to be derived from the construction must be greater than the cost of doing without it.[2]

[1] See Sporn, Philip, Progressive Engineering Pays, *Elec. World*, June 9, 1934.

[2] See discussion of "Humanities in Engineering Education," by Prof. J. K. Finch, *Soc. Promotion Eng. Educ. Jour.*, April, 1934.

In the case of fuel-burning plants, the costs are generally fairly well known, although plans may have to be modified during construction, extras may be allowed, and severe weather conditions and storms often delay the progress and add greatly to the cost of the work. On the other hand, each hydraulic project presents many special problems. In developing a water power, the hazards are much greater not only in the construction of the property but in the operation and maintenance.[1] The structures have to be built in and across a river where flood and ice conditions may raise the ultimate cost of construction by a large contingency item.

In addition to the cost of the hydro plant itself, large investments have to be made for auxiliary steam power, for navigation and flood-control works, for storage reservoirs, for dams, canals, fish ladders, and log chutes, for land and flowage rights, for new roads and embankments, and for long high-voltage transmission lines with their switching and substations; all of which result in very high fixed charges to be carried by the development throughout its life. In spite of the many hydroelectric plants built in the last 10 years and the wide publicity given to their economic significance, the general public, and even some technical writers, still concentrate all their attention on the low operating costs of producing hydro kilowatt-hours at the plant switchboard some hundred miles away from the market. They forget or ignore entirely that the maximum power will not be available in the dry-season flows and must be replaced by other power and that the fixed charges on the plant and all the fixed and operating charges on such transmission lines as will be necessary for the hydro development but not for a steam plant must be added to the operating-plant cost to determine the total cost as delivered at the load. When the real total cost is thus considered, the excessive profits which have been thought to exist in every water project will be found to be largely nonexistent. As an alternative, the engineer should consider whether a fuel-burning plant at the load point would not furnish the power at a lesser total cost, having as it would the advantages of a saving of the transmission cost, the smaller investment per kilovolt-ampere, and the reliabil-

[1] See MEAD, DANIEL W., "Water Power Engineering," McGraw-Hill Book Company, Inc.

ity of a full available power capacity unaffected by drought or flood.

3. Initial and Annual Costs.—The construction and operating costs mentioned above, the one being lump sums paid once and for all during construction, the other occurring each year of the plant's operation, are incommensurate. To render them commensurate, we shall express the lump sums in terms of the annual

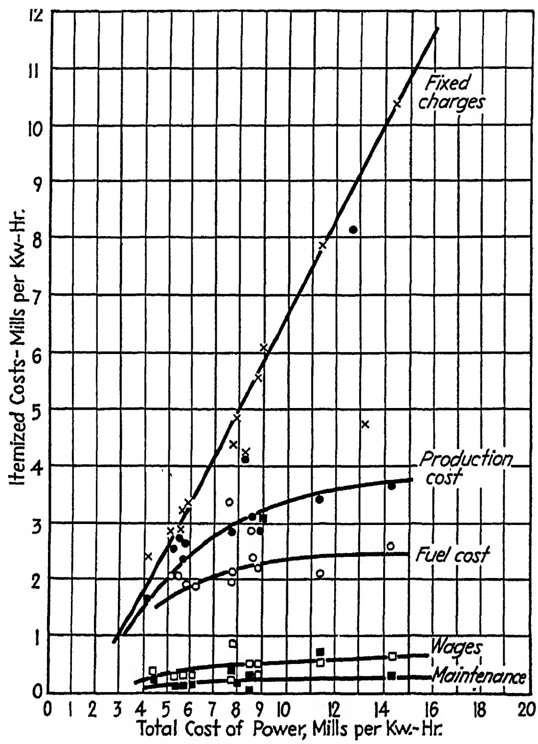


FIG. 1.—Fixed charges and operating costs of 16 steam stations. (*Electrical World, Nov.* 23, 1935.)

costs necessitated by the consignment of such capital to the project. There will then be annually two recurring costs:

1. Fixed or overhead costs—which do not vary with the operation of the plant.

2. Operating or direct costs—dependent on the manner and extent of the operation of the plant.

Figure 1 shows the relation between these two sets of costs for 16 representative plants, 15 to 200 M kw., base load and stand-by

stations when built (since 1927), and their operating costs for the year 1934, from the Steam Station Cost Survey of the *Electrical World*, Nov. 23, 1935.

4. Fixed Costs.—These will consist of

1. Taxes.
2. Insurance.
3. Depreciation (wearing out of the depreciable part of the plant, augmented by obsolescence).
4. Money use (interest and bond discount, dividends).
5. Risk insurance (profit).

5. Operating Costs.—These will be made up of

1. Fuel.
2. Wages.
3. Supervision.
4. Water, oil, waste, and supplies.
5. Repairs and maintenance.

6. Taxes.—A corporation engaged in generating and selling electric power or gas, like an ordinary householder, is subject to the general property tax, *i.e.*, the tax on real estate and personal property. Generating and substations, transmission and distribution systems, gas, water, and steam mains are considered as personal property and assessed where they are located. For example, the tax rate for the city of Ann Arbor for the year 1939 was $29.39 per thousand. The Michigan State Tax Commission imposes a tax for the primary school fund on the utilities based on the average property tax rate of Michigan communities. The 1939 rate was $27.4909 per $1,000 valuation.

At the time of organization of the corporation, in addition to the trustee's fees, fees for engraving, etc., the state of Michigan will charge an organization or incorporating fee of 50 cts. for each $1,000 of capitalization. This usually will be capitalized.

There is also the franchise privilege fee. The state of Michigan charges 2.5 mills on the paid-up capital and surplus of the preceding year from a minimum tax, set at $10, up to a maximum tax of $50,000. In addition, the state of Michigan charges a general sales tax of 3 per cent on sales of electricity. Many of the other states levy on gross receipts or on a kilowatt-hour basis as listed in the *Electrical World* of April 15, 1933, and later issues.

As of 1940, the Federal government taxes on corporations in general were as follows:

TAX ON CORPORATIONS IN GENERAL

	Excluding defense tax	Including defense tax
Income:		
Normal-tax net income over $25,000 (general rule)	19%	24%
Normal-tax net income of $25,000 or less:		
First $5,000	13½	14.85
$5,000 to $20,000	15	16.5
$20,000 to $25,000	17	18.7
Normal-tax net income slightly over $25,000: (alternative)		
First $25,000	$3,775.00	$4,152.50
Over $25,000	33%	36.3%
Capital stock:		
Per $1,000 of adjusted declared value of capital stock	$1.00	$1.10
Excess Profits Tax:		
Net income in excess of 10% and not in excess of 15% of adjusted declared value	6%	6.6%
Net income in excess of 15% of adjusted declared value	12%	13.2%

There is also the Federal excise tax of 3⅓ per cent of the amount paid for electrical energy for domestic or commercial consumption as furnished by either privately or publicly owned operating companies.

For Social Security taxes based upon the pay roll for 1940–1942 the rate for Old Age Retirement is 1½ per cent paid by employer and 1½ per cent withheld and paid for employee. For 1943–1945 these rates increase to 2 per cent. The rate for Unemployment stays at 3 per cent, total of Federal and state payments, paid by the employer.

Closely allied to taxes is municipal compensation. In distributing the electric power, a public utility necessarily has to make use of the streets and alleys of the city, according to the permission given to it in the franchise. The city thus becomes vitally interested in the construction work of the utility on its

thoroughfares and takes steps to protect its interests and safeguard the public. The utility must obtain civic permission to open up a pavement for underground construction, and generally such permission is accompanied by a definite time limit imposed, by which time the work must be completed and the roadway again be ready for traffic. The city then places its inspector on the work, his time being charged to the utility company, at a certain cost per day, to see that city piping is not interfered with, that backfill is thoroughly made, the pavement relaid, etc. Where overhead lines are constructed, these have to pass inspection by the city lighting commission for which inspection the utility is charged, say at the rate of $0.25 per pole. Installations of interior wiring are inspected to see if they comply with the prevailing electrical code. In the power plant itself, the boilers are periodically inspected by the city or state department and a charge made against the utility for such inspection. In the large cities, the cost of public-utility work is constantly increasing, owing to the many complications encountered and the numerous restrictions which are placed upon such operations. For example, under this heading is the 3 per cent of its total receipts from the sale of electric energy which the Commonwealth Edison Company pays to the city of Chicago under its franchise, and the 1 per cent of gross monthly revenues charged to its public utilities by the city of New York, over and above the 2 per cent charged by the state, for unemployment relief.

For the year 1939, the taxes paid by the Detroit Edison Company were as shown in Table 1.

These total taxes considered as a ratio of the gross earnings for 1939 of $59,534,612 amount to 14.6 per cent. Such a ratio is more intelligible than the ratio of total taxes to book value of taxable property since the large items of Federal income and excise tax, which are included in the total figure, bear no relation to book value. The designing engineer, however, in order to determine the total fixed charges against the estimated cost of his development, would prefer the ratio of the total taxes to the value of the tangible property in the fixed capital account of $317,348,186, which is 2.74 per cent.

With the tremendous increase in the various activities of government and the resulting expenditures, particularly those of state and local governments, taxation has increased at a very

TABLE 1.—TAXES PAID BY DETROIT EDISON COMPANY, 1939

Ad valorem property taxes	$5,233,415.56
State corporate privilege (franchise) fees	50,958.13
State unemployment compensation tax	440,931.67
State chain-store tax	14,820.00
Total local taxes	$5,740,125.36
Federal income tax provision	$1,548,000.00
Federal excise tax on sales of electricity for commercial and residential uses	1,063,209.15
Federal unemployment compensation tax	48,094.98
Federal old-age benefit tax	147,202.60
Federal capital stock tax	124,365.00
Total Federal taxes	$2,930,871.73
Total taxes charged to operation	$8,670,997.09
Taxes charged against incomplete construction, as part of its cost	122,095.54
State sales tax paid by customers as an increment to their bills	1,131,112.15

rapid rate and the electric light and power industry has been called upon to absorb an increasingly larger amount of the payments. Table 2 shows the growth of taxes paid by the power companies and its relation to their revenue.

TABLE 2.—OPERATING REVENUE, EXPENDITURES, TAXES*
Privately Owned Establishments
(In Thousands of Dollars)

	Revenue for electric service	Operating expenditures	Taxes	Taxes, per cent of revenue
1912	$ 287,139	$152,630	$ 13,147	4.56
1917	502,060	319,912	30,063	5.98
1922	948,905	498,731	73,128	7.70
1927	1,702,020	756,850	150,253	8.82
1932	1,873,364	750,597	203,858	10.88
1937	2,207,109	947,022	312,271	14.13
1938†	2,050,000	789,000	325,000	15.82
1939†	2,165,000	810,000	345,000	15.90

* *Elec. World*, Jan. 13, 1940.
† Edison Electric Institute. Preliminary figures.

It will be noticed that the taxes charged to a power corporation include much more than the general property tax. The exception to this generality is that municipal power plants are almost exempted from taxation, but may make contribution of service

either free or at a nominal charge. For a detailed study of taxes as applied to private and publicly owned utilities, the reader should consult the report Rate Series 5 of the Federal Power Commission, 1936.

7. Insurance.—In addition to the foregoing charge against the investment, there is the evident charge for fire risk on those portions of the property insurable, but not including such things as hydraulic conduits, coal supply tracks, condenser intake and discharge tunnels, all of which are outside any building subject to fire risk, and are not capable of damage from fires occurring within the insured property. On the other hand, investment in steam turbines and boilers, waterwheels, and governors must carry an insurance burden since fires in the buildings in which they are housed probably would be disastrous to them; hence they increase the value of the property insured. Electrical insurance covers accidental breakdown or burnout of a machine from any accidental cause while in operation or connected up ready for operation. By breakdown is meant only a sudden, substantial, and accidental burning out of a machine which stops the functions of the machine and necessitates repair. The engineer should familiarize himself with the nature of the insurance policy to be carried and with those things covered under the risk in order to know what fixed charge lies against any individual piece of equipment. Based upon 80 per cent coinsurance, the rate for hydro buildings may range from 0.096 to 0.192 and for hydroelectric equipment from 0.146 to 0.242. For steam-electric plants, 0.056 to 0.070 on building and contents may be typical. Insurance may be carried with a regular insurance company at the established rates, or the corporation may create its own fund to take care of losses of this nature. Insurance in a modern plant may seem unnecessary, but the regular visits of insurance inspectors have a beneficial influence on the operation of the plant.

If a corporation or contractor undertakes the construction and operation of a plant, then insurance under the Workmen's Compensation Act must be carried for protection against damage suits resulting from injuries to workmen. This may be carried with the state or with an insurance company, and the cost represents a certain percentage of the estimated annual pay roll. In Michigan as of July, 1940, for power-house work the rates

per $100 of annual pay roll were $4.20 for excavation, $5.80 for carpentry, $3.59 for masonry, $4.35 for steelwork, etc. In case of injury under such a policy, the workman is provided with medical service and after the first week gets a payment of $17 to $28 per week. This is furnished, if necessary, up to 500 weeks.

Contractors are also commonly required to furnish two bonds when bidding on a construction job, one to insure that if their bid is accepted they will execute a contract to do the work and the other to insure that they will perform all the work in the time specified. As an example, bidders on the construction of the Grand Coulee Dam for the Columbia River were required by the

TYPICAL RATES AS OF JULY, 1939, FOR WATER-TUBE BOILERS BASED ON THE LOSS EXPERIENCE AND INCLUDING THE VALUE OF THE OBJECT FOR A TERM OF THREE YEARS*

Size, heating surface, square feet	Object charges without piping		Add for piping	
	Without water walls	With water walls	Main steam piping	Pressure piping
Class 1. Steam Pressure 15 lb. or less				
10,000	$29	$34	..	$10
20,000	45	50	..	11
Class 2. Steam Pressure 16 to 300 lb.				
20,000	$170	$240	$7	$11
40,000	320	460	7	11
Class 3. Steam Pressure 301 to 600 lb.				
20,000	$195	$279	$7	$11
40,000	365	533	7	11
Class 4. Steam Pressure 601 to 1,000 lb.				
20,000	$220	$318	$7	$11
40,000	410	606	7	11
Class 5. Steam Pressure over 1,000 lb.				
20,000	$245	$357	$7	$11
40,000	455	679	7	11

* Courtesy Hartford Steam Boiler Inspection and Insurance Co.

U. S. Bureau of Reclamation to post a $2,000,000 bid bond and a $5,000,000 performance bond.

Boilers are also insured against accidental explosion with a boiler insurance company whose experts will make periodic inspections of the equipment.

An industrial plant may also carry use and occupancy insurance as a safeguard against loss in business because of failure of power supply. Under such a policy, the insurance company may insist upon a stand-by power service being provided.

8. Depreciation.—It must be realized that from the very day the construction of a power-machine unit or plant is completed deterioration begins, and by the wear and tear from use and the age and physical decay from lapse of time, there results a reduction of value, a loss of some part of the original investment in the perishable property. The rate of the wear and disintegration will, of course, depend upon the conditions under which the apparatus is operated, how it is protected from the elements, and how promptly required repairs are made. It is therefore necessary, as the property decreases from its original cost completely installed, to its final scrap or salvage value as merely so much metal at the end of its useful life, that the owner have in hand nearly as much money at any given date as represents the shrinkage in value; and at the time of retirement of the plant, he must surely have in hand the full sum of the depreciable part of the property. This amount added to the net salvage value will enable the owner to rebuild the same type of property that he built in the first instance, build some other property of equivalent earning power, or merely invest the amount so that it will earn the cost of money use on the original plant.

The Supreme Court has clearly recognized this principle in the case of *City of Knoxville v. Knoxville Water Company*, Jan. 4, 1909, in a consideration of value for rate purposes. The Court says:[1]

> The cost of reproduction is one way of ascertaining the present value of a plant like that of a water company, but that test would lead to obviously incorrect results if . . . not diminished by the depreciation which has come from age and use. . . . It is not easy to fix at any given time the amount of depreciation of a plant whose component parts are of different ages with different expectation of life. But it is clear that

[1] See Riggs, H. E., "Depreciation of Public Utility Properties," p. 163, McGraw-Hill Book Company, Inc.

some substantial allowance for depreciation ought to have been made in this case. . . . Before coming to the question of profit at all the Company is entitled to earn a sufficient sum annually to provide not only for current repairs, but for making good the depreciation and replacing the parts of the property when they come to the end of their life. The Company is not bound to see its property gradually waste, without making provision out of earnings for its replacement. . . . It is not only the right of the Company to make such a provision, but it is its duty to its bond- and stock-holders, and, in the case of a public service corporation at least, its plain duty to the public. . . . If, however, a company fails to perform this plain duty and to exact sufficient returns to keep the investment unimpaired, whether this is the result of unwarranted dividends upon over issues of securities, or of omissions to exact proper prices for the output, the fault is its own. . . .

In general, physical decay is not complete at the end of the useful life, and indeed the whole matter of physical decay is more or less completely mixed up with operating costs incurred through current repairs. A boiler may pit as used, but if the tubes are cut out one by one and replaced as they pit, those portions of the boiler subject to rapid deterioration will be always in a state somewhat approximating the "as new" condition, though the boiler as a whole can never be 100 per cent new from the time the fire is first built under it. The furnace also is in a state of partial depreciation but, through the replacement of grate bars and the rebricking of burned-out portions at its regular inspection periods, never falls into a state of absolute worthlessness. The insulation on an electric machine may break down in spots, or the commutator or brushes wear out, but current repairs bring these parts individually back to as good a state as when new, while other parts are approaching their individual time of replacement. Withholding repairs may hasten the deterioration of a structure to a point where the owner cannot afford to use it any further. On the other hand, even with most careful attention to repairs, the time will come when they will be so frequent and so expensive that it will be cheaper to retire the individual piece of apparatus and replace it with a new one, even though of the same type as the replaced apparatus was when it was first installed.

The net physical life for the apparatus, then, is that period within which it will be necessary to replace the apparatus,

with as skillful and frequent attention to repairs as the character of the labor at hand and the exigencies of service will justify. In arriving at his determination of such life, the engineer will have to be guided by sound judgment based on observation of similar apparatus in the past. If he is calculating the useful life of a piece of refined apparatus installed in a crude industry with no skilled workmen, or with the responsibility of plant maintenance necessarily concentrated in the hands of the engine runner, he will not assume so long a useful life as might be expected in a large central power station with a number of skilled operators, adequate machine shop, and highly technical supervision. No general rules can be formulated for such factors of judgment, as they must depend on local conditions and on the best "guess" of the engineer guided by his previous experience.

As the engineer, in arriving at his depreciation costs, anticipates the scrapping date as a result of physical decay, he must take cognizance of the condition of the apparatus when scrapped. The apparatus, though of no use in its initial location, may be of some use to the owner somewhere else, or may have a value for sale to someone else. Such value constitutes salvage and will be high for materials having little fabrication value as compared with their raw constituents, such as the lead sheathing and copper of conductors, the babbitt metal in bearings; will be relatively lower for materials such as bricks, still lower for structural steel which has experienced some fabrication for its specialized use, and will be lowest for individual elaborate machines and other pieces of station apparatus. The reason is entirely evident since cable when pulled out can, if not sold as cable, be resolved into some of its constituent metals with the loss of the purchase price of only the insulating material and the initial cost of fabrication. The bricks will have to be sold as bricks and not as brickwork, unlike copper and lead, and even a simple length of I beam will be salable only to a purchaser who happens to be able to use that particular length and section of structural material, in which case its salvage value probably will be determined by the junkman.

The salvage value of some equipment at the end of its useful life may be even less than nothing, as in the case of concrete foundations which, because of the cost of breaking up and removal, are a liability rather than an asset. The depreciation

cost of such an investment as a concrete foundation should be computed for the physical life of the apparatus to be installed on it, and should be computed not on the cost of the foundation when first installed, but on the cost of the foundation plus the probable cost of getting rid of it. Conversely, the depreciation rate of materials having a positive salvage value should be computed, not on their original first cost, but on the depreciable portion of that first cost, *i.e.*, on the first cost in place, including all the labor necessary to get the equipment installed, plus the cost of removing, preparing for sale and selling, minus the presumptive gross proceeds from such sale.

In addition to the depreciation of wear and tear mentioned above, there is also depreciation from inadequacy, from obsolescence, both sentimental and economic, from changes in the demand of the public for service, from requirements of regulating authority, and from accidents. These forms of depreciation will be more fully discussed in Chap. III.

If any of these factors become operative, they will tend to force the actual retirement of the unit or plant from service before the end of its useful life and hence shorten the years in which its depreciation expense can be collected. These special factors will require such an increase in the depreciation rate, over and above that estimated for physical life, that the depreciation reserve shall be built up so as to be adequate for the actual retirements. For normal performance, the amount of this adequate reserve may be estimated from the past history of the utility itself. If the reserve is not so maintained, the deficiencies may be made up by gradually increasing depreciation rates in the future, or vice versa.

The Consolidated Income Statement of the Detroit Edison Company for 1939 gives charges of Retirement Reserve (depreciation) as $8,000,000, which on the Fixed Capital, Tangible Properties (including real estate and grounds not subject to depreciation) of $317,348,186 is 2.52 per cent.

9. Methods of Providing for Depreciation.—The following methods are available:[1]

1. Straight-line method.
2. Diminishing-value method.

[1] See CURTISS, W. B., Depreciation of Property, *Gen. Elec. Rev.*, December, 1915.

3. Retirement-expense method.
4. Sinking-fund method.

The straight-line method provides for setting aside each year an equal proportional part of the depreciable cost based upon the life of the property. Thus, if a machine costs $10,000 and its life is estimated as 10 years with a scrap value of $1,000, the annual depreciation will be 10 per cent of $9,000, or $900. This method has the advantage of extreme simplicity and ease of application when the only causes for retirement are wear and tear or the gradual action of the elements. It is extremely difficult to estimate when obsolescence may occur or when overhead construction may be destroyed by hurricane, earthquake, or ice storms or may be ordered moved or put underground by municipal or highway authority.[1]

The straight-line method of depreciation accounting received general approval over the sinking-fund and retirement-expense methods in the report of the special committee on depreciation of the National Association of Railroad and Utilities Commissioners in 1939. In Table 3 are given the estimated years of life for power-plant equipment from the "Standard Handbook for Electrical Engineers," sixth edition, page 1305, which represent the judgment and experience of the authorities listed.

Table 4 lists representative rates of straight-line depreciation on a composite basis, as determined by the engineers of the Tennessee Valley Authority and listed in the annual report for June 30, 1939.

The diminishing-value method provides for the setting aside each year of a fixed rate, first applied to the cost and then to the diminishing value, such rate being based upon the estimated life of the property. Our example of the $10,000 machine would thus carry a rate of 20.57 per cent of $10,000, or $2,057, for the first year and decreasing amounts for the following years. This gives the heaviest charges for depreciation in the early years when the maintenance charges are lightest and so evens out the total expense of the unit for depreciation plus maintenance over its life. It would, however, impose an extremely heavy burden on the early years of a new plant which had to develop its load and build up its earnings as it went along.

[1] See FERGUSON, SAMUEL, "Depreciation Accounting," *Elec. World*, June 15, 1940.

TABLE 3.—LIFE EXPECTATION TABLE FOR POWER-PLANT EQUIPMENT[1]

Class of equipment	Years of life	Authority	Controlling cause
Belting	20 to 25	A	D
Boilers, fire tube	10 to 15	A	D
Boilers, water tube	20 to 30	A	D
Buildings, masonry	75	A	O
Buildings, wood frame or second class	50	A	D and O
Chimneys and stacks, masonry	30	B	O
Chimneys and stacks, steel	10	B	D
Condensers	20 to 30	C	D
Conveyors, coal or ash	10	A	D
Engines, gas	10 to 15	A	D and O
Engines, steam, high speed	15 to 20	A	D
Engines, steam, slow speed	25 to 30	A	D and O
Feed-water heaters	20 to 30	A	D
Fuel-oil equipment	25	C	D
Generators, motors and converters:			
High speed	15	B	D
Slow speed	20	B	O
New types	20	A	D
Old types	15	A	O
Turbine driven	20	A	D
Lightning arresters	15 to 20	A	D
Pipes and pipe covering	20 to 30	A	O and D
Pumps:			
Boiler feed	15 to 20	A	D
Small centrifugal	20 to 30	A	D
Shafting	20 to 40	A	O
Station wiring	30	A	O and D
Stokers	20	C	D
Storage batteries	15	A	D
Switchboard instruments and wiring	25 to 30	A	O and D
Switchboards:			
Old types	20 to 30	A	O
New types	15 to 20	A	O
Turbines:			
Hydraulic, old types	25 to 40	A	O
Hydraulic, new types	30 to 50	A	O
Steam, large units	20	A	D and O
Auxiliary units	10 to 20	B	D and O
Transformers, station	20	A	O and D
Wire:			
Insulated copper line	10 to 15	A	D
Lead-covered aerial cable	10 to 15	A	D and O
Lead-covered underground cable	20 to 25	A	O

[1] From "Standard Handbook for Electrical Engineers," 6th ed., McGraw-Hill Book Company, Inc.

A—Wisconsin Railroad Commission.

B—Barker.

C—Chicago Traction Valuation Commission.

D denotes deterioration.

O denotes obsolescence.

TABLE 4

Item of plant	Per cent			
	Pickwick	Wilson	Wheeler	Norris
Dam and spillway	1.44	1.40	1.16	1.13
Power house and intake	1.22	1.26	1.19	1.37
Intake gates	2.58	2.86	2.70	2.06
Roads	4.00	2.86	1.40	2.16
Lock and lock machinery	1.34	1.44	1.40	
Turbines and generators	2.51	2.81	2.50	2.51
Accessory electrical equipment	3.66	2.87	3.80	3.75
Transmission lines	2.59			
Substations	4.00			
Rural distribution lines	3.39			

The retirement-expense method, formerly adopted in the uniform classification of accounts for electrical utilities, is no longer in favor with regulatory commissions. The Federal Power Commission and state regulatory bodies have prescribed a change from retirement to depreciation accounting in the new uniform system as of Jan. 1, 1939. The retirement-expense method was not based on the estimated life of the property but aimed to create an adequate reserve, to take care of retirements before such replacements actually occurred. The arbitrary charges made to operating expense plus the amounts appropriated from surplus "should in all cases be sufficient to provide during a period of years a reserve against which can be written off all losses sustained upon the retirement of property for any cause whatsoever."[1] In accumulating such a reserve then, the charges for a year would exceed the cost of the actual retirements in that period, but after the accumulation the reserve could be maintained by annual appropriations very similar to depreciation charges. This was a very practical method for an operating utility working on its actual experience, but might be a very difficult method for the engineer to apply in estimating on a new design. It has been objected that meeting the cost of replacement out of the operating income of the period of retirement

[1] National Association of Railway and Utilities Commissioners, State Law Reporting Company, New York.

loads the period with costs not properly assignable to that period.[1]

The sinking-fund method provides for setting aside each year such a sum as, invested at a certain interest rate compounded annually or semiannually, will equal the amount of the depreciable property at the end of its life. This is the method that the author will use throughout this text. It requires smaller annual amounts than does the straight-line method, and the amounts for the annuity are uniform.

After an investigation of actual experience with 14 important hydro plants, R. L. Thomas in the *Electrical World* for Mar. 14, 1936, prefers the sinking-fund method for modern hydro developments.

Let P = principal sum to be retired.
S_n = net salvage value available at end of life.
n = number of years estimated for the life.
a_n = annual rate necessary to effect the accumulation of the depreciable investment.
e = annual rate earned on the accumulation or depreciation reserve.

If the interest is paid semiannually and the allowance for depreciation expense is taken from earnings semiannually, we shall at the end of the first half year set aside a sum represented by $\frac{a_n}{2}(P - S_n)$. This will have increased by its interest earnings, at the end of the first year, to $\frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right)$. To this will be added, out of the second half year's earnings, a depreciation accumulation equal to that laid aside out of the first half year's earnings, $\frac{a_n}{2}(P - S_n)$, so that at the end of the second half year there is in hand $\frac{a_n}{2}(P - S_n) + \frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right)$. During the third half year, these two accumulations will increase to $\frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right) + \frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right)^2$, to which will now be added out of the third half year's earnings an equal

[1] See THOMPSON, C. W., Depreciation Accounting Favored, *Elec. World*, July 14, 1934.

accumulation $\frac{a_n}{2}(P - S_n)$. At the end of r years, the total accumulations will be represented by a geometric series $\frac{a_n}{2}(P - S_n) + \frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right) + \frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right)^2 + \cdots \frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right)^{2r-1}$, and at the end of the whole period of n years this must have accumulated to $2n$ terms, so that the series will terminate in an expression where $r = n$. At such time, the sum of the series must be equal to the depreciable investment $(P - S_n)$; thus:

$$(P - S_n) = \frac{a_n}{2}(P - S_n) + \frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right) + \frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right)^2 + \cdots \frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right)^{2n-1}. \qquad (1)$$

Multiply through by $\left(1 + \frac{e}{2}\right)$, then

$$\left(1 + \frac{e}{2}\right)(P - S_n) = \frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right) + \frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right)^2 + \cdots \frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right)^{2n}. \qquad (2)$$

Subtract (1) from (2).

$$(P - S_n)\frac{e}{2} = \frac{a_n}{2}(P - S_n)\left[\left(1 + \frac{e}{2}\right)^{2n} - 1\right], \qquad (3)$$

or

$$\frac{e}{2} = \frac{a_n}{2}\left[\left(1 + \frac{e}{2}\right)^{2n} - 1\right]$$

and

$$a_n = \frac{e}{\left[\left(1 + \frac{e}{2}\right)^{2n} - 1\right]}. \qquad (4)$$

Figures 2 and 3 show the variation of the annual depreciation expense rate with the years of life, for various earning rates on the depreciation reserve, for semiannual compounding.

As the depreciation reserve should be at least as secure as the original investment, the best place in which to invest it will be in the extension of the property, provided it is capable of extension. This may ensure an earning rate of 6 per cent, whereas if the funds were on deposit in the bank the earning rate would probably be only 1 per cent. We may assume then as a proper earning rate on the depreciation reserve the same rate that the

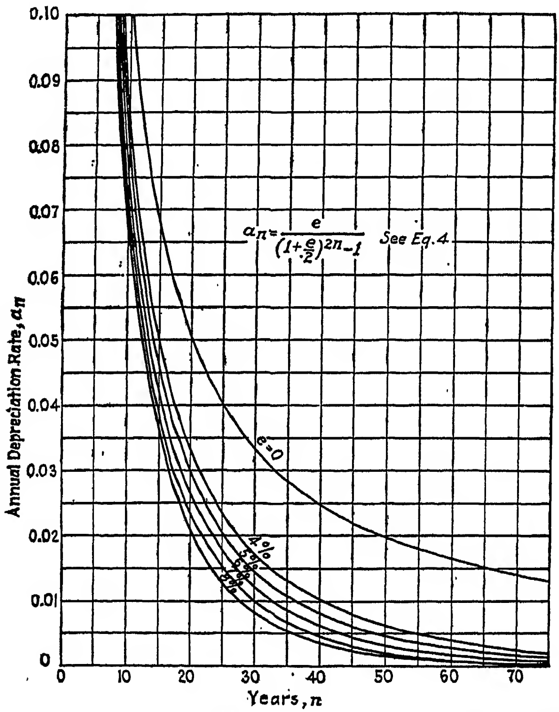


FIG. 2.—Annual depreciation rates, up to 75 years, for different earning rate e and life n years, compounding semiannually.

company had to pay on the proceeds of its initial sale of the bonds to establish the plant. Thus, if the earning rate were 6.5 per cent, our example of the \$10,000 machine with a 10-year life would carry a rate

$$a_n = \frac{0.065}{(1.0325)^{20} - 1} \qquad \log 1.0325 = 0.0138901$$

$$20 \log 1.0325 = 0.2778020, \text{ num.} = 1.8959$$

$$\text{denominator} = 1.8959 - 1 = 0.8959$$

$$= 0.0726.$$

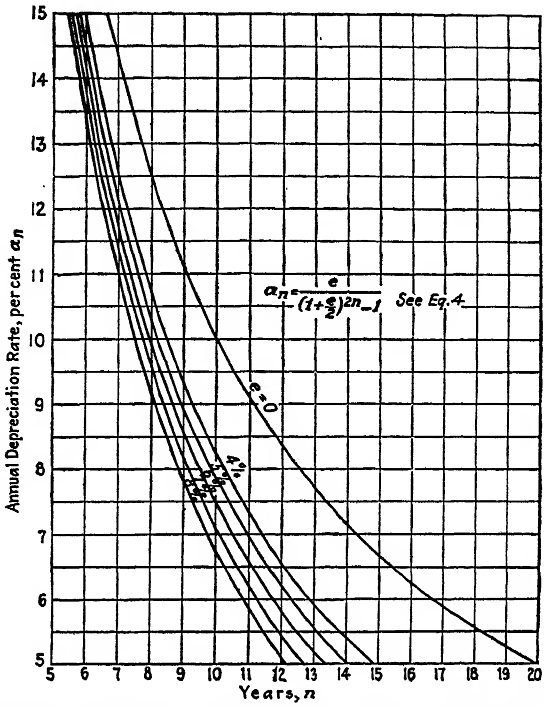


Fig. 3.—Annual depreciation rates, up to 15 years, for different earning rates e and life n years, compounding semiannually.

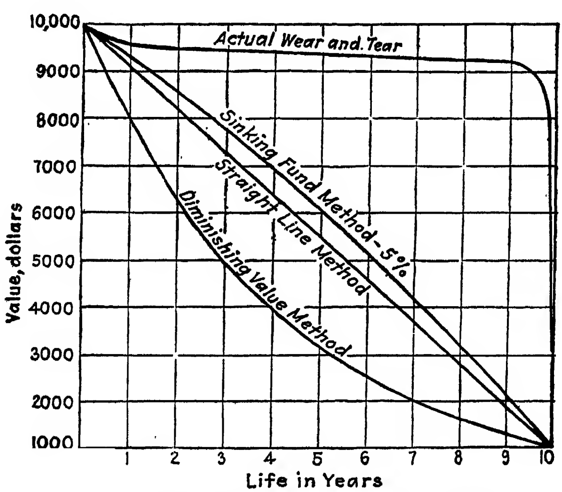


Fig. 4.—Methods of providing for depreciation.

The curves of Fig. 4 illustrate three of the methods described. The curve of actual wear and tear assumes that the property is kept in first-class repair and therefore will be useful throughout its life. This depreciation does not proceed at an even rate during the life of the property, and its amount at any time would be determined by a thorough inspection of the plant by an experienced engineer. For accounting purposes, however, it is not necessary to set up the actual depreciation occurring in each and every year. It is sufficient to the theory that the utility recover the full amount of the depreciation on an item by the time it has reached the end of its useful life.[1] It is advisable, therefore, to adopt a method that will impose a fairly uniform rate upon earnings. The curves of the straight-line, sinking-fund, and diminishing-value methods illustrate what assumptions are involved as to the uniformity of the depreciation.

10. The Depreciation Reserve.—In setting up the depreciation reserve on the sinking-fund basis, as previously explained, the annual allowances taken from earnings are credited to this account and reinvested in the plant extensions. Credit may or may not be given for the interest they earn while they remain there. In the case of the Detroit Edison Company, Accumulated Surplus and Retirement Reserve are reinvested in the business. No dividends or bond interest are paid out on this reinvested money. Against these sums is charged the excess of the original cost over the net salvage value of property retired from service. In this manner then, the utility reimburses itself for the losses due to the retirement of important items such as buildings, continuous sections of electric line, or identifiable units of plant. The replacement of the thousands of small items in the plant is taken care of by ordinary maintenance. This practice is similar to that of the railways in the retirement of ties, rails, ballast, etc., which are taken care of through maintenance owing to the large number of the units involved.

The annual depreciation rate a_n, Eq. (4), has been determined on the basis of having completely collected the depreciable value of the equipment $(P - S_n)$ at the end of the n years' life of the apparatus. If it is desired to know how much has been accumulated at some intermediate number of years, as r years, it is neces-

[1] See Cross, W. G., Fundamentals of Public Utility Depreciation, *Elec. World*, Oct. 22, 1927, p. 841.

sary to take only $2r$ terms in the series, instead of $2n$ terms, and the amount of the reserve will be

$$R_r = \frac{a_n}{2}(P - S_n) + \frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right) + \frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right)^2 + \cdots \frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right)^{2r-1}. \quad (5)$$

Multiply through by $\left(1 + \frac{e}{2}\right)$, then

$$R_r\left(1 + \frac{e}{2}\right) = \frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right) + \frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right)^2 + \cdots \frac{a_n}{2}(P - S_n)\left(1 + \frac{e}{2}\right)^{2r}. \quad (6)$$

Subtract (5) from (6)

$$R_r \cdot \frac{e}{2} = \frac{a_n}{2}(P - S_n)\left[\left(1 + \frac{e}{2}\right)^{2r} - 1\right], \quad (7)$$

or

$$R_r = \frac{a_n(P - S_n)\left[\left(1 + \frac{e}{2}\right)^{2r} - 1\right]}{e}. \quad (8)$$

But from Eq. (3)

$$(P - S_n) = \frac{a_n(P - S_n)\left[\left(1 + \frac{e}{2}\right)^{2n} - 1\right]}{e}. \quad (9)$$

Then

$$\frac{R_r}{(P - S_n)} = \frac{a_n(P - S_n)}{a_n(P - S_n)}\left[\frac{\left(1 + \frac{e}{2}\right)^{2r} - 1}{\left(1 + \frac{e}{2}\right)^{2n} - 1}\right] = \frac{a_n}{a_r}. \quad (10)$$

That is, the ratio of the amount accumulated in the depreciation reserve at any intermediate time, r years, to the total depreciable sum is equal to the inverse ratio of the annual depreciation rates. Hence

$$R_r = \frac{a_n}{a_r}(P - S_n). \quad (11)$$

Figure 5 shows the ratio of the depreciation reserve to the depreciable amount for various values of r/n years.

The balance sheet of the Commonwealth Edison Company as of Dec. 31, 1938, shows a Depreciation Reserve of $117,512,393. Similarly on the consolidated balance sheet of the Detroit Edison Company as of Dec. 31, 1939, the Retirement Reserve (depreciation) is given as $43,141,326. This is 13.6 per cent of the book figure for Fixed Capital (tangible property) at the same date.

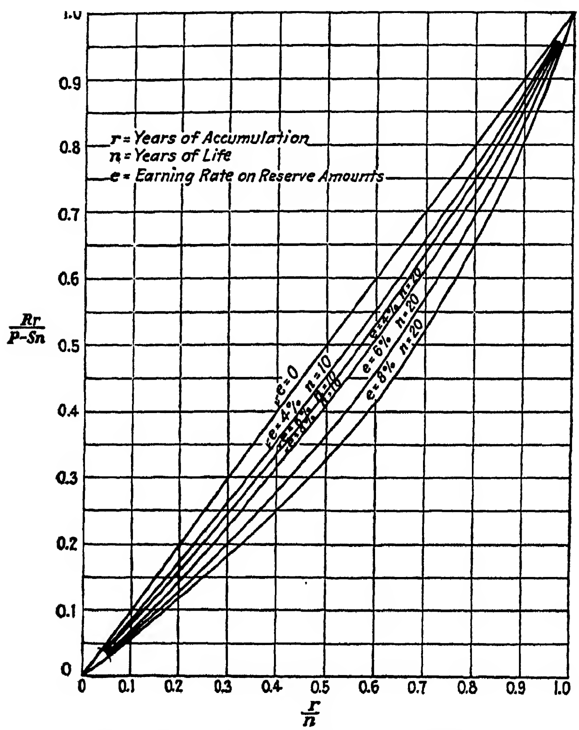


FIG. 5.—Ratio of depreciation reserve (R_r) to depreciable amount ($P - S_n$).

11. Cost of Money Use.—When a piece of property is purchased or constructed, money has to be expended therefor. This money either comes out of the owner's pocket or is borrowed from other people. It is entirely evident that if the money is borrowed from other people, interest will have to be paid on it until the loan is paid off, and in perpetuity if the loan is never repaid. It will cost just the same to keep one's own money invested in a property, since the owner tying his money up in a

property loses the interest that he might have got on the money had he kept it as such for lending to others.

In modern business, money for permanent investment is seldom the owner's individual capital in more than a minor degree. Most of the money for modern business undertakings is borrowed, and there are two general methods through which this borrowing is effected

1. By the sale of stock.
2. By the sale of bonds.

There are many subdivisions and varieties of these two general classes of security, but for our present purposes it will suffice to consider the fundamental ideas involved.

"These shares of stock are shares of interest in the fortunes of the business and these shares of interest involve the responsibility of management as well as participation in the profits or the losses."[1]

The bond, usually of $1,000 nominal value, is a first lien upon the property of the corporation and is a right to share in a certain contract, the indenture, made between the corporation and the trustee. "Bonds are promises to pay a certain precise sum of money and a definite rate of interest."[1]

A share of common stock, of no-par value, or of $100, $50, $25, or $10 nominal value, is therefore merely a statement of indebtedness, and unlike the bond carries no promise of repayment at any date, nor does it carry any promise of a definite interest rate. The cash dividends on the stock are paid annually out of the earnings of the property after all proper bills and taxes have been paid, and the interest paid on the bonds.

The holder of stock, then, has no assurance that his original investment will remain intact in case the business falls on bad days, and has every prospect that he will receive no dividends whatever during the early days of the enterprise. He has only the speculative probability that when the enterprise is well on its feet he may receive dividends considerably in excess of the interest rate on the bonds. It is this latter speculative probability that induces an investor to take the risk of loss of principal sum or interest.

[1] See DEWING, A. S., "Financial Policy of Corporations," The Ronald Press Company, New York. See also BOWERS and ROWNTREE, "Economics for Engineers," Chap. V, McGraw-Hill Book Company, Inc.

In case the corporation fails,

> . . . the single bondholder is usually powerless to enforce the payment of either the interest or the principal of his bond, and in many cases his hands are effectually tied by a provision which requires the approval of a majority of the bondholders before the trustee may take the necessary protective steps. . . . So many legal and business obstacles can be thrown in his way by the corporation officials, and so little value have the assets of a bankrupt corporation in liquidation, that experience has shown that most bondholders, like stockholders, are bound to bear some losses of a corporation failure.[1]

In many cases, the bond mortgage imposes on the owner, and the trustee enforces, the accumulation annually out of the earnings, before the stock dividends are paid, of a sum that shall, when deposited where it will earn interest, pay off the amount borrowed under the bond mortgage when due. The aggregate of such accumulation is known as an "amortization" fund, and the process of its accumulation would be entirely similar to the collection of the depreciation reserve discussed above. Good business policy in general requires the annual accumulation of a sum of money known as "corporate surplus" to be used in case of emergency, before cash dividends on the stock are paid.

In deciding to what extent an enterprise shall be financed through the sale of stock, and to what extent through the sale of bonds, the promoter and the bondholder have diametrically opposed interests. From the bondholder's point of view, his investment is much more stable and secure with a relatively large stock issue. For example, a $500,000 corporation having an issue of $200,000 worth of bonds and $300,000 of common stock would not be likely to fall into such a bad condition as to be unable to earn the interest on the bonds, and even if sold on the courthouse steps would probably bring a price well above the face of the bonds; but were the bond issue to be $400,000 and the stock issue only $100,000, it would take a shrinkage of only 20 per cent in the value of the property to bring the bondholders to the verge of losing some of their investment. A similar shrinkage in the earning capacity of the concern would embarrass it in meeting the interest on the bonds.

[1] See Dewing, A. S., "Financial Policy of Corporations," Ronald Press Company, New York.

On the other hand, the stockholder is correspondingly anxious to have a small stock issue in proportion to the bond issue. If the total net earnings of our $500,000 corporation, after paying for materials used, labor, taxes, etc., are $30,000, or 6 per cent, and if the stipulated bond interest is 5 per cent, there will be required $10,000 and $20,000, respectively, to pay the interest in the case of the $200,000 bond issue and in the case of the $400,000 bond issue. This will leave, in the respective cases, $20,000 and $10,000 to go to the stockholders. With the larger stock holding, this will pay only 6⅔ per cent, whereas the $10,000 aggregate dividend on stock would pay 10 per cent on the smaller $100,000 stock issue. It is noteworthy that the large bond issue

TABLE 5.—CAPITALIZATIONS—PUBLIC UTILITIES, AS OF DEC. 31, 1939*

Company	Capacity	Capitalization	
Shawinigan Water & Power Co.	1,046,039 hp.	Bonds	$ 90,947,000
		No-par com.	2,178,250sh.
Montana Power Co.	342,000 kw.	Bonds	$ 60,136,900
		No-par cum. $6 pfd.	159,588sh.
		No-par com.	2,481,665sh.
Pacific Gas & Electric Company	877,391 kw. hydro; 436,625 kw. steam	Bonds	$287,345,000
		$25 pfd. cum.	5,370,825sh.
		$25 par com.	6,261,357sh.
Niagara Falls Power Company.	373,200 kw. hydro	Bonds	$ 32,177,000
		No-par com.	742,241sh.
Mississippi River Power Company.	135,000 kw. hydro	Bonds	$ 18,722,000
		$100 par cum. 6% pfd.	$ 8,234,475
		$100 par com.	$ 16,000,000
Southern California Edison	490,120 kw. hydro; 415,000 kw. steam; 24,250 kw. inter. comb.	Bonds	$142,273,000
		$25 par cum. pfd.	3,466,857sh.
		$25 par com.	3,182,805sh.
Consolidated Edison Company of New York.	1,600,300 kw.	Bonds	$297,821,000
		No-par cum. $5 pfd.	2,184,590sh.
		No-par com.	11,476,527sh.
Commonwealth Edison Company of Chicago.	1,229,000 kw.	Bonds	$297,042,900
		$25 par com.	10,471,516sh.
Detroit Edison Company	1,011,375 kw. steam; 9,300 kw. hydro	Bonds	$134,320,000
		$100 par com.	$127,226,000

* Courtesy of Moody's "Manual of Public Utilities."

serves to increase the amplitude of fluctuations of earnings in their effect on the dividends on the stock. Evidently then it is to the interest of the promoter, who in general has either purchased common stock or taken it in compensation for the work of promotion, to use as much of the bondholder's money as possible; although it is to the interest of the bondholder to compel the promoter to issue as few bonds as possible against the given property. The prospective bond purchaser holds the whip hand in this matter since, unless the protection offered to his investment is high enough, he will either demand that the bond interest be made enough higher to give him some return for the risk he has taken, or else insist that a $1,000 bond be sold to him for less than $1,000.

For examples of the financial structure of modern public-utility corporations, the reader is referred to Table 5, Capitalizations —Public Utilities.

The reader should be warned against making any superficial comparisons among central station companies on the basis of the foregoing figures. These are materially affected by how much plant has been built from accumulations in depreciation reserve and surplus, by the character of the load supplied, whether it is large-block primary power or is converted secondary power at a large investment per kilowatt, and whether it is mainly dense urban load or widely distributed in suburban territory. Also it is important to consider how much of the plant has been built during the past years when construction prices were high. Thus it can readily be appreciated that an attempt to make accurate comparisons would involve almost endless ramifications and assumptions.

In explaining that public utilities find it impossible to finance a large construction program from their surplus earnings, because the proportion of invested capital required to produce a given volume of earnings is greater than in any other industry, Mr. Beckett, assistant treasurer, Pacific Gas and Electric Company stated[1] that "even the most conservatively financed hydro-electrical companies are compelled to invest $500 in plants and transmission and distribution systems, etc., to yield a gross revenue of about $100 per annum." This is confirmed by Table

[1] *Gen. Elec. Rev.*, March, 1922.

6, Relation between Capital and Revenue, for the entire light and power industry.

The engineer as such is little concerned with the relative sizes of the bond and stock issues, but in order to appreciate the proper rate that must be paid for money spent in carrying out his designs, he will have to take cognizance of the interest rate and the price at which bonds are sold.

The bonds are not usually sold directly by the owner, or by the trustee of the bond mortgage to the individual purchaser, but are purchased by one or a few firms of bankers, who dispose of these at a somewhat advanced price to their clientele. Such a banking concern is known as the "underwriter" of the bonds, and the price paid through the trustee to the mortgagor is said to be the price at which the bonds are "underwritten." If an issue of 5 per cent bonds running for a period of, let us say, 40 years is underwritten at 90, the sale of each one of these bonds will bring to the owner of the property $900 for its construction, for the use of which he will have to pay $50 a year, and for which he will have to repay $1,000 at the end of 40 years. It is evident that he is paying $50 on $900 which he received so that the interest rate is apparently $5\frac{5}{9}$ per cent. As a matter of fact, the interest rate is somewhat higher than this, since during the 40 years between the issue of the bond and its stipulated date of repayment, the $900 worth of property built through its sale must have accumulated an additional $100, in other words must have "amortized" the bond discount. The method of computation of the amortization rate necessary to accumulate this bond discount from the earnings of the property within the life of the bonds is exactly similar to that for the accumulation of the depreciation reserve developed above.

During a considerable period before the first World War (1889 to 1914) the interest rates on the same type of security remained remarkably constant. Four per cent predominated for railroads, 5 per cent was consistently used by public utilities, and 5 and 6 per cent by industrials.[1]

To determine the real cost of borrowing money by bonds,
let F = face value of the bonds.
i = annual interest rate carried by the bonds.

[1] See WILLCOX, O. B., Cost of Money and Credit of Utilities, *Elec. World*, June 5, 1920.

a_n' = annual accumulation rate for the bond discount.
d = discount rate.
n' = number of years in which to amortize the bond discount.
e = rate for cost of money use.

Then the real "market" rate of interest or the "cost of money use" is the total annual expense incurred due to the borrowing divided by the amount of money actually borrowed; *i.e.*,

$$e = \frac{iF + a_n'Fd}{F(1 - d)}. \tag{12}$$

But from Eq. (4),

$$a_n' = \frac{e}{\left(1 + \frac{e}{2}\right)^{2n'} - 1}.$$

Therefore

$$e = \frac{i + d\left[\frac{e}{\left(1 + \frac{e}{2}\right)^{2n'} - 1}\right]}{(1 - d)}. \tag{13}$$

Since this equation for the annual rate for money use is difficult of solution, e may be more readily obtained by successive approximations. For the first approximation, let the bond discount be accumulated on the straight-line method instead of the sinking-fund plan. Thus in the example cited, one-fortieth of the discount would be accumulated each year. Then

$$e_1 = \frac{i + \frac{d}{n'}}{1 - d}. \tag{14}$$

We know that this rate will be slightly larger than that using a sinking-fund method, because d/n' will be greater than $a_n'd$. Now use e_1 as the cost of money use in setting up the amortization of the bond discount on the sinking-fund plan, and

$$e_2 = \frac{i + d\left[\frac{e_1}{\left(1 + \frac{e_1}{2}\right)^{2n'} - 1}\right]}{1 - d}. \tag{15}$$

This will give a closer value of e, and in general the second approximation is sufficiently accurate. However, a third approximation may be taken by using e_2 for the sinking-fund plan, thus

$$e_3 = \frac{i + d\left[\dfrac{e_2}{\left(1 + \dfrac{e_2}{2}\right)^{2n'} - 1}\right]}{1 - d}. \tag{16}$$

For the example suggested above, these would evaluate as follows:

$$e_1 = \frac{0.05 + \dfrac{0.10}{40}}{1.00 - 0.1} = \frac{0.0525}{0.9} = 0.0583.$$

$$e_2 = \frac{0.05 + 0.10\left[\dfrac{0.0583}{(1.02915)^{80} - 1}\right]}{0.9} = 0.0563.$$

$$e_3 = \frac{0.05 + 0.10\left[\dfrac{0.0563}{(1.02815)^{80} - 1}\right]}{0.9} = 0.0563.$$

For practical purposes, it is more convenient to obtain a_n' from the curves of Fig. 2. For this method, write Eq. (15) in the form

$$e = \frac{i}{1 - d} + \frac{da_n'}{1 - d}. \tag{17a}$$

Then in the foregoing problem, the values would be

$$e = \frac{0.05}{0.9} + \frac{0.1a_n'}{0.9} = 0.05556 + 0.11a_n'..$$

Thus it is evident that the rate will be very close to 0.055, since this is increased only by the product of 0.11 and a_n'. Therefore an approximate a_n' may be read from the curve for earning rate of 5.5 per cent at a life of 40 years. This gives $a_n' = 0.0070$. Then $e = 0.05556 + (0.11)(0.0070) = 0.05556 + 0.00077 = 0.0563$, or 5.63 per cent.

In the rather rare event that the bonds sell above par, *i.e.*, at a premium (p), during each year of the life of the bond a proportional amount of the premium must be written off since only the face value of the bond will be paid at maturity. Then

the cost of money use will be determined as is developed in Eq. (21).

If p, the premium, is counted as lost *at the purchase date* of the bonds, then p is the present value of an annuity at e per cent for n years. Thus the net income rate e will be the annual rate corresponding to the interest rate i, minus the annuity corresponding to p.

That is,

$$e = i - p(e + a_n).$$

Then

$$e = i - pe - a_n p$$

and

$$e(1 + p) = i - a_n p.$$

Therefore

$$e = \frac{i}{1 + p} - \frac{pa_n}{1 + p}. \tag{17b}$$

To obtain the cost of money when funds are obtained by the use of both bonds and stocks, the public-utility company ordinarily computes the cost by dividing the bond interest and annual amortization of bond discount, plus the dividends on the stock, by the par values of the outstanding securities.

12. Risk Insurance (Profit).—One additional fixed cost must be computed for every detail occasioning investment. This may be expressed by the two general terms "risk insurance" and "profit." No financier would undertake the construction of a property with all the liability of error in judgment as to the possibility of sale, the cost of operation, the continuity of business, instability of cost of supplies and labor, and changes in governmental attitude in such matters as taxes and regulation were his return to be merely that which the bondholders receive on their investment, protected as they are by the stock holdings, which take the brunt of any financial disaster. It is therefore proper, and only the part of wisdom, to add to the other fixed charges a percentage of the investment which may form a sort of risk insurance. The necessity for this becomes apparent immediately when the original promoter attempts to induce other capital to come in through the sale of stock. Not all the excess of stock-dividend rate above bond-interest rate is true profit, but indeed a very large portion of it is risk insurance. Although the risk insurance must

in general be carried by the business as a whole, it has yet to be recognized that the inclusion of a power plant as an auxiliary to a business enterprise increases the amount of capital that must be risked, and that, in case of disaster overtaking the enterprise, the power plant cannot advantageously be removed and taken away intact for use in some other place or sold separately from the rest of the property. It therefore suffers the same burden of risk insurance as does the rest of the property, excepting only very highly specialized manufacturing machines, which might in case of complete wreckage of the business have no sales value whatsoever. Just how much risk insurance the engineer should compute for any given property is not very easy of determination.

Suppose the property is already in existence and the engineer's work is to build or extend a power plant for the property. If the property is highly competitive with similar properties in the immediate vicinity, *i.e.*, is not monopolistic in character, and the stock is not watered, then all stock dividends paid in excess of the rate of nominal bond interest may be said to be "risk insurance," with the probability of very slight "clear profit" being included therein. This is simply a result of the fact that free competition will probably have reduced the stock earnings to a point where there is not anything in them other than enough to make it a matter of indifference to the purchasing public whether they take a somewhat risky stock at a high income rate or more stable bonds at a low interest rate. For example, a $2,000,000 concern has $1,200,000 of bonds outstanding, which net the holders an average earning of 5.5 per cent. It also has $800,000 worth of common stock, which over the period of years since the concern came into existence, has netted stockholders an average return of 7 per cent on stock purchased at par. Then the actual cost of money for the enterprise is 5.5 per cent of $2,000,000, or $110,000, per year, and although the stockholders have averaged $56,000 per year, only $44,000 of this, *i.e.*, 5.5 per cent, represents the value of money use. The remaining $12,000 is risk insurance, which protects the stockholders against the risk distributed over the whole $2,000,000 investment. In this case then, we have $12,000 annual risk insurance on an investment of $2,000,000, or 0.6 per cent.

In the rare event that the power plant is to be built in conjunction with a monopolistic and unregulated enterprise, whose

dividends on stock are well above the ordinary rates of investment return, the engineer will have to form his own judgment as to what return would content him as a possible investor in the stock of this enterprise, in comparison with other and non-monopolistic enterprises of the same general character. All earnings above this point may be considered as true profit.

To carry out his engineering design most ideally, the engineer should know still more about his client's business affairs. In working for a public-service enterprise whose books are a matter of public record, he can have this knowledge without any difficulty, and in order to save his client's money he should be permitted, subject to the usual confidential relations of engineer and client, to have such knowledge when retained in a non-public undertaking. If the business for which a power plant is projected is incapable of extension, *i.e.*, if the ultimate product is sold in a restricted field, the owner will not need to demand so big a return on every dollar invested in the plant, as he would in case his business were capable of further extension without sacrifice of profits. In order to justify the investment in the case of the inextensible business, a dollar expended for power plant would be required to pay only the interest and depreciation, taxes, risk insurance, and operating costs. On the other hand, that same dollar invested in a power plant incidental to a very profitable and extensible enterprise would be deflecting money away from a source of real profit, unless in addition to the foregoing fixed charges it earned as good a profit as it would have earned if invested in manufacturing or merchandising. The engineer, in cooperation with his client, should determine whether any refinement in detail, or the power plant as a whole, is as good an investment as the client could otherwise make for his money. This sounds like a very complex problem, but with the constants once and for all established it is extremely simple. The difficulty is likely to be in getting at the constants, as in many cases the client may be unwilling to divulge facts about his own affairs which he may consider to be none of the engineer's business.

13. Proper Balance between Fixed and Operating Costs.—We have stated that each refinement in detail must pay its own fixed charges. This statement will bear a little further analysis. For a given load and fixed income, any power plant might be designed

so badly, through an effort to keep down investment, that operating costs, together with the fixed charges on this slight investment, would be much in excess of the value of the product. That is to say, the power could be purchased at less than it is costing to manufacture. In this case in Fig. 6, the ratio of profit to investment, the investment and the losses, or negative profits, would be

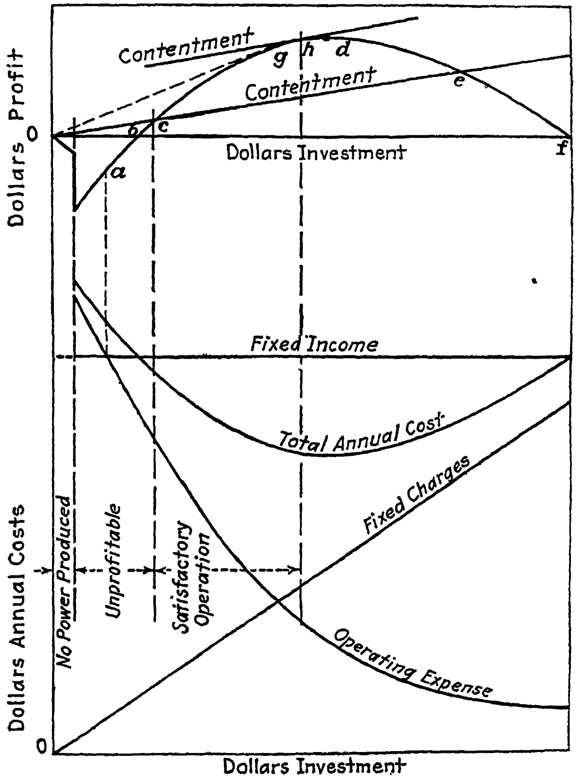


FIG. 6.—Ratio of profit to investment for a given load and income.

shown at such a point as a. A more generous design, providing for a more efficient plant, might, as at b, bring the losses to zero, so that the investment would be just carrying itself and the operating costs. The total of fixed and operating costs would just equal the value of the output. Still more generous design would begin to show a positive excess of value above the cost of production, so that at c the plant is paying a profit. By continuing the investment, we might reach such a point as d, where the profits

have become a maximum. At this point, further elaboration might still be reducing operating expenses, but no more rapidly than the fixed charges on the investment were increasing, so that the total cost of the output would be unaffected one way or the other by the expenditure or nonexpenditure of one additional dollar. Beyond this point, as at *e*, additional investment might actually involve fixed charges considerably in excess of the savings effected, thereby decreasing the total profits, and this process might continue until at such a point as *f* the plant is actually eating itself up in fixed charges, again showing an actual loss. In attempting to increase the profits indefinitely by increasing the investment, there comes a stage where the profits increase from *h* to *d* at a rate less than proportional to the increase in investment. This economic principle is called by the economists "the law of diminishing returns."

TABLE 6.—RELATION BETWEEN CAPITAL AND REVENUE*

Year	Investment, in thousands of dollars	Total revenue, in thousands of dollars	Ratio	Residential	
				Kw.-hr. per customer	Revenue, cts. per kw.-hr.
1918	$ 3,600,000	$ 664,850	5.42	272	8.27
1920	4,400,000	882,750	4.98	339	7.45
1923	5,800,000	1,269,550	4.56	368	7.20
1926	8,400,000	1,520,159	5.52	428	6.98
1927	9,500,000	1,661,032	5.72	444	6.80
1928	10,300,000	1,743,809	5.91	460	6.60
1929	11,100,000	1,938,520	5.72	499	6.30
1930	11,800,000	1,990,955	5.92	543	6.00
1931	12,400,000	1,975,944	6.28	578	5.74
1932	12,660,000	1,813,717	6.97	597	5.57
1933	12,800,000	1,754,366	7.30	595	5.49
1934	12,660,000	1,831,871	6.92	624	5.30
1935	12,660,000	1,911,989	6.62	672	4.99
1936	12,660,000	2,044,587	6.18	727	4.65
1937	12,660,000	2,180,788	5.81	793	4.39
1938	12,780,000	2,168,495	5.88	845	4.21
1939	12,790,000	2,293,644	5.57	890	4.03

* See *Elec. World*, Jan. 13, 1940, and *E.E.I. Bull.* H2, March, 1940.

Something like this condition seems to have prevailed in the electric light and power industry in the period up to 1933 when the extensions on the systems were made more rapidly than the returns were realized. Since then the ratio has decreased as excess plant capacity has been used to carry the increased loads. Table 6, Relation between Capital and Revenue, for the industry shows the variations for capital and revenue from ultimate consumers.

The reduced use of the industrial demand during the period of the depression and the increased investment in extensions and rehabilitations planned and under way on the expected growth of load, as predicted during the period 1925 to 1929, have produced the startling increases shown. Also, it must be remembered that changes in the rates charged for service affect this ratio quite as much as investment policy does, and that the universal experience is that rates are being continually reduced.

Having in mind the way in which, in a particular development, the more generous proportioning of parts reacts on the operating cost, the engineer will have to decide where, between the points of zero profit, he will effect his compromise between highly efficient and expensive development, and inefficient equipment of low investment. At a glance it might seem that *d*, the point of maximum aggregate profits, would be the desirable end to strive for in design, but as has been noted, at this point an additional dollar put into plant refinement earns nothing beyond its bare fixed charges. Thus there is no incentive to make this "marginal" investment, and there is every incentive to avoid investment between *d* and *f*, since at such a point as *e* an additional dollar put in is not carrying itself but is actually losing money. Yet it is frequently necessary, for the public service, to make such an investment. It is interesting to note that this is a loss, even though point *e* shows an average profit on the total investment, which is quite to the satisfaction of the owner of the enterprise. The tangent of the angle *eO$* is assumed to have been taken as the best profit that the owner can derive from any part of his business still capable of extension. The difficulty is, of course, that the owner would have made a much better return on his investment, had it been stopped at *g*, the point of tangency of a line drawn through the origin, which gives the maximum possible ratio between gross profit and money invested.

Point g is not, however, representative of the proper limit of design, since although the profit per average dollar invested is here a maximum, the profit to be obtained on the next dollar invested is still better than that shown by tan $eO\$$, which is the owner's margin of contentment. The design should proceed to such a point as h, where a line parallel to Oe becomes tangent to the curve. Here then, the last dollar put into the enterprise is doing as well as the best investment that the owner could otherwise make. If the line Oe lies entirely above the curve, *i.e.*, if the average profits cannot be made at least as great as the owner's margin of contentment, then the plant should not be undertaken at all. But if the investment-profit curve does cut above Oe, elaboration should cease as stated, where the last dollar spent for refinements earns a return just equal to the margin of contentment. Stated in terms of the calculus, this is where the derivative of profit with respect to investment is just equal to this margin of contentment. This will, in general, give aggregate profits less than the maximum that could be obtained, and a ratio of profit to investment less than the best that could be obtained from the plant as indicated by the fact that the slope of line Oh is less than that of line Og. In any engineering undertaking, there will naturally be some portions of the investment which, like the niggardly development shown at a, are nonproductive, being essential to the undertaking, but in themselves representing sheer loss, but which have had to be paid for by those profitable portions of the investment that could not have been undertaken without the others.

14. Holding Companies.—From the established economies obtained by monopoly service of public electric light and power supply in a given territory due to the saving of duplication of investment in transmission and distribution systems, in office and administrative expense, in better load factor, etc., it was but a logical step further to combine adjoining power plants or systems and apply the same principles to a larger area. Thus the holding companies, so prominent at the present time in the electrical power industry, came into being as the owners of various numbers of operating power companies more or less closely related geographically. Such a unity of ownership for a group of operating companies presents the possibility of economy in the financing, management, and engineering services for the units. Funds may

be obtained in one large block supported by the credit of the holding company rather than in several small independent issues, equipment and material may be purchased at a lower price on a bulk order of standardized units rather than on individual items, and equipment that becomes inadequate in one part of the system may still find useful application in another part. Also higher technical and administrative talent may be retained to direct the composite group than any unit could probably afford to carry of itself. By placing rewards on the basis of managerial and operating efficiencies obtained in one unit of the aggregate as compared with the other like units, good performance is stimulated and competition for efficiency is obtained. If the properties are adjoining, a unified development plan can be evolved for the entire interconnected system. This would ensure the building of most economical power stations and transmission lines to supply power at the least cost to the whole group, taking full advantage of diversity factors of the loads, of pooling reserve capacity, and of economical operation. The extent to which interconnection may be economically advantageous is discussed in Chap. II.

Under the Public Utility Holding Company Act of 1935, Congress directs that all holding companies must register with the Securities and Exchange Commission and that the Commission shall bring about integration, *viz.*, produce compact geographical units not so large as to impair the advantages of localized management, efficient operation, and the effectiveness of regulation. Section 11(B) directs that a registered holding company shall confine its interests to a single integrated public-utility system, and to such other businesses as are reasonably incidental, or economically necessary or appropriate thereto. Additional integrated systems may be retained if they cannot be operated as independent units without loss of substantial economies and if they are located in one state, in adjoining states, or in a contiguous foreign country.

There is considerable variety in the charges made by the holding companies for furnishing the services mentioned above, perhaps because of differences in the amount of management provided. The trustee appointed by the Federal District Court, Illinois, for the reorganized Middle West Utilities Company charged a supervision fee of six-tenths of 1 per cent of the operat-

ing revenues (as defined in the contracts) of the affiliated companies.[1] Electric Bond and Share reduced fees to companies that subscribed to its management service to a maximum charge of 1 per cent on the first million of operating revenue.[2]

In general, the holding company acquires control of its operating subsidiaries by the ownership of their common stock, which it may sell directly or may hold in its treasury as the basis for its own security issues. Because its profits come from the public utilities under its control, it has become a matter of public concern as to how the profits are acquired, the extent of the same, and how the gains of the improved operation have been distributed. If securities have been pyramided so as to concentrate all the profits of the group in the limited investment at the top, this action will have an important influence on the financial status of the operating companies. The government and state commissions, therefore, are giving keen attention to the future of the holding company and will doubtless take action to prevent past abuses in "banking transactions," in payment of excessive common dividends at the expense of adequate depreciation and surplus allowance and in charges for unnecessary or unreasonable management and supply contracts. The recommendations outline the holding company in marked resemblance to an investment trust; eliminate all intermediate holding companies, bonds, fixed interest-bearing obligations, and fixed dividend preference stocks; and provide for profitless service of subsidiaries and the inclusion of operating executives on the holding-company board.[3]

Under the act mentioned above, the Security Exchange Commission prescribed a uniform system of accounts for public-utility holding companies effective Jan. 1, 1937, based on original cost which is designed to eliminate "write-ups," and to segregate surpluses. The operating companies continue to be regulated by the appropriate public-utility commission as to security issues, rates, and service.

15. Equivalent Values.—Throughout the text, and particularly in the consideration of obsolescence, we shall have to compare distributed sums of money with lump sums, and to compare lump sums at different times. For example, if it is desired to

[1] See *Elec. World*, Mar. 2, 1935, p. 31.
[2] See *Elec. World*, July 11, 1936, p. 6.
[3] See *Elec. World*, July 14, 1934.

build up a fund by accumulating $\$p$ per year in semiannual installments of $\$p/2$ each, the money being set aside at the end of each period, then the case may be stated as follows:

Proposition 1.—The equivalent of $\$p$ per year is $\$P$ at the end of n years; where $P = p/a_n$, or $p = a_nP$. For by similarity to Eq. (3),

$$P = \frac{a_nP}{e}\left[\left(1 + \frac{e}{2}\right)^{2n} - 1\right]. \tag{18}$$

Let the $\$p$ per year $= a_nP$.
But from Eq. (4)

$$a_n = \frac{e}{\left(1 + \frac{e}{2}\right)^{2n} - 1}.$$

Therefore,

$$P = \frac{p}{a_n}. \tag{19}$$

For the expenditure of an increased lump sum now, a more efficient machine may be purchased which will make a saving in operation each of the following years of its life. Will the value of the ensuing annual savings equal the increase in the lump sum? The savings will be set aside at the end of each period. Let the matter be stated thus:

Proposition 2.—The equivalent of $\$C$ today is $\$c$ per year for the ensuing n years; where $C = \dfrac{c}{a_n + e}$.

$\$C$ invested at an earning rate of e per cent, and compounding semiannually, will produce at the end of n years $\$C\left(1 + \frac{e}{2}\right)^{2n}$.
But from Proposition 1, $\$p$ per year $= a_nP$.

Now from Eq. (4)

$$a_n = \frac{e}{\left(1 + \frac{e}{2}\right)^{2n} - 1}.$$

Then

$$\left(1 + \frac{e}{2}\right)^{2n} = \frac{e}{a_n} + 1 \text{ and } c = a_n\left[C\left(\frac{e}{a_n} + 1\right)\right], \tag{20}$$

or

$$c = C(e + a_n). \tag{21}$$

If a lump sum, as a reserve, is today invested in the business, to grow by the accumulations of its earnings semiannually during the next n years, then we may state the case as follows:

Proposition 3.—The equivalent of $\$E$ in hand today is $\$D$, n years hence; where $D = E\left(\frac{e + a_n}{a_n}\right)$.

If we had a sum $\$E$ in hand today, it could be invested to earn at e per cent compounded semiannually, and after n years would equal $E\left(1 + \frac{e}{2}\right)^{2n}$.

But from Eq. (20)

$$\left(1 + \frac{e}{2}\right)^{2n} = \frac{e}{a_n} + 1 = \frac{e + a_n}{a_n}.$$

Therefore,

$$D = E\frac{(e + a_n)}{a_n}. \tag{22}$$

If an amount which is in hand today is to be expended in part as a certain lump sum payment n years from now and the balance of the amount is to be set up as an annuity paying equal annual sums for the next m years, then the following is true:

Proposition 4.—For $\$F$ in hand today, and $\$G$ to be paid in a lump sum n years from now,

$$\text{The present value of } \$G = \frac{\$G(a_n)}{(e + a_n)} \text{ by Proposition 3.} \tag{23}$$

$$\text{The balance of } \$F \text{ for the annuity } = F - \frac{Ga_n}{e + a_n}. \tag{24}$$

Then $\$f$ per year for ensuing m years, by Proposition 2,

$$= (e + a_m)\left[F - \frac{Ga_n}{e + a_n}\right]. \tag{25}$$

16. Problems.

1. A merchant contemplates the purchase of a delivery truck at a cost of $750. The machine is expected to last 4 years with a final scrap value of $50. The fixed and operating expenses will be as follows:

Annual license \$20 paid at beginning of year.
Insurance \$50 per year paid at beginning of year.
Cost of money use on investment 7 per cent.
Driver's wages at \$1,200 per year.
Maintenance and repairs at \$100 per year.
Gas and oil at 1.5 cts. per mile for 15,000 miles per year.

What is the maximum annual price at which a cooperative delivery service could compete?

2. What is the annual difference in cost between these two plans?

a. Buying a \$20 watch which will last 15 years, if cleaned every second year at a cost of \$1.

b. Buying a \$2.50 watch which will have to be replaced every 2 years.

Money costs 6 per cent with semiannual interest payments.

3. A central station is erected at the following costs:

	Cost	Life, years	Scrap value, per cent
Building, complete	\$1,250,000	40	5
Boiler plant	1,300,000	20	5
Turbines and generators	800,000	15	10
Real estate	450,000	..	100

Taxes are \$35 per thousand on an 80 per cent valuation. Insurance on building, 0.5 per cent; boilers, 2 per cent; turbine and alternators, 0.8 per cent. Money is raised by the issue of 35-year 6 per cent bonds which sell in the open market for 96.

Determine the annual fixed charges on the plant.

4. A transmission line is built from funds secured by the sale of 30-year bonds bearing 5 per cent interest, sold at 98. Taxes are at the rate of \$20 per thousand on an 80 per cent valuation. No insurance is carried. The detailed costs for a 35-ft. pole are

Easement per pole (*i.e.*, rent of ground space, paid only once during life of line)	\$ 2.00
Digging hole	0.95
Erecting and setting pole	1.13
Framing 24 cts., roofing 8 cts., shaving 84 cts.	1.16
Unloading and storing	0.23
Treating top, gains, and butt, carbolineum	0.36
Distributing pole to job	0.63
Pole, on cars in town	10.60
(When worn out after 15 years the pole is jacked out and replaced.)	
Loosening soil and jacking out	0.75
Value of old pole for a stub	3.00

The transmission line will be required for 60 years, at the end of which time the poles will be jacked out and the holes filled with gravel at a cost of $1.10 each.

What is the annual cost per pole?

5. A machine has an original value of $2,000. Its depreciation is to be covered by a sinking fund earning 5 per cent under the assumption that the scrap value at the end of 10 years will be $200. What is the value of the depreciation reserve at the end of 6 years?

6. *a.* Decide between these two steam plants on the basis of annual cost:

Item	Non-condensing	Condensing
First cost	$135,500	$155,500
Life, years	25	20
Salvage value at end of life, per cent	10	15
Earning rate on depreciation reserve, per cent	5	5
Insurance and taxes, per cent	4	4
Yearly cost of operation	$60,000	$52,000
Yearly cost of maintenance	$1,400	$1,500

Funds may be obtained from the sale of 5 per cent bonds at 95, 25 years.

b. What would be the present value of the saving, for the life of the cheaper plant?

c. In how many years would the saving retire the extra cost?

7. From an estate consisting of twenty-five $1,000, 25-year bonds, bearing 6 per cent coupons, issued 15 years ago, it is desired to provide today for the following:

a. An inheritance of $15,000, to be paid to my son on his twenty-first birthday in 5 years.

b. A uniform yearly income for myself for each of the coming 20 years.

What is this yearly income if the bonds sell today at 103?

c. What is the amount to my credit 5 years from today, just after the fifth annual payment?

8. Two methods of accumulating a depreciation reserve are proposed, which will involve the following annual costs:

Method 1.—$12,000 per year for the first 5 years, $10,000 per year for the next 7 years, and $9,000 per year for the next 18 years.

Method 2.—$14,500 per year for the first 8 years, and $8,000 per year for the next 22 years.

Compare the total amounts raised by each plan as of a date 25 years hence. Money earns 6 per cent.

9. If an estate consisting of fifty $1,000, 30-year, 5 per cent bonds issued in 1917, the market value of which today is 96, is left to a widow whose expectancy is 17 years, what maximum uniform annual income could she buy?

10. If a projected development will cost $12,000 a year for the first 10 years and $18,000 a year for the next 10, what is the equivalent uniform cost? Money is worth 6 per cent.

11. *A* lived in a house from Jan. 1, 1915, when he purchased it for $10,000 to Jan. 1 of this year when he sold it for $15,000. On Jan. 1 of each year, *A* paid $100 for insurance for the following year, and $450 for the past year's repairs and taxes. If *A* had chosen to rent the house for himself, he could have done so by paying $900 on the last day of each year. Assuming cost of money use at 6 per cent (annually), state whether, and how much, *A* gained or lost on the date of sale by the transaction.

12. *B* bought a lot for $5,000 cash on Jan. 1, 1920. Taxes are payable Dec. 31 each year at $30 per $1,000 of assessed valuation, which was 80 per cent of real value at purchase date and has not varied since. If the cost of money use has been 6 per cent continuously, compounded annually, for what price cash might *B* have sold on January of this year in order not to have lost money?

13. Find the accumulation at the end of 40 years by these two plans:

a. $100 per year, invested and compounded quarterly at 6 per cent per annum.

b. $200 per year, invested and compounded semiannually at 4 per cent per annum.

14. A $1,000 20-payment endowment policy maturing at age 65 costs a man, aged 20, $24.76 per year. There are 20 annual payments beginning on date of policy issue. Assume money cost at 6 per cent with semiannual compounding.

a. Find total *actual* amount of cash paid to the insurance company.

b. The company states the profit is $1,000 minus the total actual cash paid in. How much is this?

c. Had the insured set aside these 20 payments himself, how much would he have accumulated at age 65?

d. In addition to making the agreement to pay the insured $1,000 at age 65, the company has been ready to pay him $1,000 at any time in case of death. For this service, they ought to receive $12 per year payable in advance, *i.e.*, the cost of straight-life insurance for age 20. Considering this, how much has the insured gained or lost, when he reaches age 65, by having taken the endowment rather than the straight-life plan?

15. On the basis of annual costs, decide whether it is more economical to purchase 250-kva. and 500-kva. 11-kv. single-phase transformers from the *A* Co. or the *B* Co. The guarantees are the same, but tests on the units show the following results.

Size, kva.	500	250
Iron losses B above A, kw	2.17	1.2
Exciting current B above A, amperes	1.0	0.637
Cost of transformer A above B	$200	$150

Interest and depreciation are 15 per cent annually. Power costs $0.006 per kilowatt-hour. One ampere of exciting current circulating in the system represents an investment cost of $7.[1]

[1] Westinghouse Prob. 4, Series 5.

16. The water rates for a 2,500-kw. turboalternator to operate at full load, 80 per cent power factor, with steam at 125 lb., 0° superheat, 12 lb. back pressure are for the *A* Co. turbine 40.6 lb. per kilowatt-hour, for the *B* Co. turbine 38.5 lb. per kilowatt-hour. The price of the *B* Co. turbine is $35,200. Steam cost chargeable to power is 2 cts. per 1,000 lb. Life of turbines estimated at 20 years, salvage 5 per cent, interest 5 per cent, taxes and insurance 4 per cent. Assume unit runs at full load for 4,380 hr. per year. What purchase price for the *A* Co. turbine would place it on a par with the cost of the *B* Co. machine?

17. Compare the total annual costs of the two designs for motor drive for a manufacturing machine shop in the automotive industry. Money costs 5 per cent, taxes and insurance 4 per cent, service life 10 years, neglect salvage. The plant works 250 days per year, 16 hr. per day. Power costs 1.3 cts. per kilowatt-hour.[1]

Item	Individual drive	Group drive
Total number of motors	500	48
Total connected hp	3,614	2,160
Cost of motors installed	$81,000	$47,340
Cost of power-factor correction equipment 1,200 and 600 kva	7,200	4,200
Cost of spare motors, 17 and 4	2,020	2,030
Cost of power transformer equipment, shafts, pulleys, belts, installed	16,800	57,300
Operating cost per year:		
Maintenance labor	5,200	13,000
Maintenance material	1,200	4,800
Repair labor and material	400	800
Power demand, kw., average	1,200	1,400

18. A hydroelectric utility system has elements of property as follows:[2]

	Depreciable cost	Estimated service life, years
A	$150,000	10
B	450,000	20
C	400,000	40

[1] McGuire, D. C., Individual or Group Drive, *Elec. Jour.*, August, 1931.

[2] See Thomas, R. L., Sinking Fund Best for Hydro Depreciation, *Elec. World*, May 9, 1936.

Determine the total annual charge to operation for depreciation:

a. For straight-line depreciation.

b. For sinking-fund depreciation at 4 per cent earning rate compounded semiannually.

Draw a graph—ordinates, dollars; abscissas, years—showing the credit balance in the depreciation reserve year by year including deductions paid out for retirements:

c. For straight-line depreciation.

d. For sinking-fund depreciation as in (*b*).

Compute only the significant points at 10, 20, 30, and 40 years, and sketch curves in between these points.

CHAPTER II

COST OF STATIONS

17. The Investment Cost.—In Sec. 13 we discussed the returns to be earned on investment. It may now be desirable to say a word about the investment itself. The engineer must have fully in mind the fact that the investment in any property is not represented by the aggregate cost of the individual pieces of apparatus and material entering into the property. After these have been shipped from the factory mills, freight and cartage must be paid for in order to get them to the site and labor must be expended in placing them. This labor involves accident insurance and supervision, tools must be used which are either partly depreciated, "lost," or worn out, and an interest-paying period must elapse between payment for the first piece of work or material going into the enterprise and its arrival at operating condition. To these items must be added those costs for bookkeeping and managerial supervision incident upon the employment of labor and the purchase of equipment. Even the engineer's own services are a part of the cost of the plant, and every one of these items, so far as its value attaches only to the first installation, must be retired therewith. A modest engineer will feel that the value of his own designs will probably not outlive the apparatus installed, and that this cost should, therefore, be retired with the equipment, though certain of the preliminary costs of any enterprise, such as surveying and legal expense, together with the general features of the work of the engineering organization, may be spread over as long a period as the plant as a whole is likely to exist. The author does not consider it judicious to estimate a life of greater than 40 years for any piece of engineering work, even for a thing so slowly depreciable as an hydraulic tunnel driven in solid rock.

18. Cost of Central-station Systems.—It was estimated that the capital invested in the electric light and power industry in the United States at the end of 1939 was $12,790,000,000.[1] There

[1] See Table 6 and *Elec. World*, Jan. 13, 1940.

was installed then a generator rating of 37,466,200 kw.; hence the average system investment per kilowatt of generator capacity was $342. The initial 1940 increase of not quite 1,300,000 kw. in generator capacity at a budget expenditure of $605,000,000 gives an installation cost of $465 per kilowatt and brings the average cost for total investment to $346 per kilowatt. The actual investment may range from $233 per kilowatt for a compact, densely loaded, primary urban supply system to $450 per kilowatt for a widely distributed secondary suburban system.

Later reports show that privately owned electric utilities have increased their 1940 schedules to bring the total of new units to 1,520,811 kw., while specific additions scheduled for 1941 and 1942 have moved up to a total of 2,877,150 kw. Including municipal and government generating additions, 5,802,100 kw. are to be added by the end of 1942.[1]

The 1939 generator rating was composed of the following elements:[2]

Region	Steam kw.	Hydro kw.	Internal combustion kw.	Total kw.
New England	1,876,000	829,000	13,000	2,718,000
Middle Atlantic	7,523,000	1,462,000	34,000	9,019,000
East North Central	8,186,000	655,000	67,000	8,908,000
West North Central	2,195,000	450,000	281,000	2,926,000
South Atlantic	2,655,000	1,789,000	66,000	4,510,000
East South Central	739,000	1,118,000	26,000	1,883,000
West South Central	1,691,000	102,000	181,000	1,974,000
Mountain	408,000	997,000	86,000	1,491,000
Pacific	1,459,000	2,533,000	45,000	4,037,000
United States	26,732,000	9,935,000	799,000	37,466,000

The energy sales to ultimate consumers for 1939 were estimated to be 107 billion kw.-hr., subdivided as shown in the table on page 50.[2]

This first year of the sale of over a hundred billion kilowatt-hours saw the continued steady increase of use in the divisions of farm service, residential, small power and light, municipal, and miscellaneous. The great increase, however, was with the

[1] See *Elec. World*, July 27, 1940.
[2] See *Elec. World*, Jan. 13, 1940.

	Million Kw.-hr.
Farm	3,320
Residential	19,820
Commercial small light and power	19,900
Commercial large light and power	53,400
Municipal street lighting	1,830
Street and interurban electric railways	3,900
Electrified divisions steam railways	1,900
Municipal and miscellaneous	2,930

large light and power division which almost regained its previous peak of 1937 and which is responsible, along with national defense, for the substantially larger construction budget for 1940. The total energy generation for the year is estimated at 123 billion kw.-hr. of which 83,600 million kw.-hr. were from fuel (68 per cent) and 39,400 million kw.-hr. were from water power (32 per cent). The total, including imported and purchased energy, was 128,320 million kw.-hr.[1]

Table 7, analysis of the Capital Expenditures for New Construction for the past 12 years, shows distinctly how, with the exception of 1940, the emphasis has shifted from power-plant expenditures to extensions of the transmission and distribution systems, with very heavy allotments to the latter item.

The large amount required for distribution extensions (45.7 per cent of the 1940 budget) is worthy of note, indicating the increase in distribution cost per customer which is offset against the lowering of the production cost in the new power stations.

19. Cost of Steam-electric Power Plants.—Table 8, Unit Investment Cost, Steam Stations, gives a sampling, one plant from each of the eight groups, from the 56 stations reported by A. E. Knowlton in the Fourth Steam Station Cost Survey. The eight stations listed were built or extensively enlarged in the interval 1929–1938. In discussing the survey, G. G. Post[2] points out that $2 per kilowatt is a top figure in favor of 600- to 900-lb. stations as compared with 1,200- to 1,450-lb. plants. On a 50,000-kw. station, the extra investment of $100,000 would mean, say, $15,000 annually in fixed charges to offset the annual saving in production because of the better heat consumption. Concerning the performance of these stations (see Table 9), he remarks that most of the high-pressure units are superposed on

[1] See *Elec. World*, Jan. 13, 1940.
[2] See *Elec. World*, Apr. 6, 1940, p. 65.

TABLE 7.—CAPITAL EXPENDITURES FOR NEW CONSTRUCTION[1] ELECTRIC LIGHT AND POWER INDUSTRY
(In Thousands of Dollars)
(Excluding Federal Hydro Projects, but including Rural Cooperatives)

	Steam	Hydro	Transmission	Substations	Distribution	Miscellaneous	Total
1929	188,000	51,000	145,000	120,000	261,000	88,000	853,000
1930	176,496	117,565	139,533	123,482	258,699	103,642	919,417
1931	104,386	60,317	101,031	87,564	182,158	61,284	596,740
1932	40,000	20,000	60,000	30,000	110,000	25,000	285,000
1933	10,400	4,000	16,100	14,900	72,200	11,700	129,300
1934	10,258	5,704	30,180	11,921	76,299	13,292	147,654
1935	16,172	6,337	35,349	14,766	103,104	17,127	192,855
1936	36,820	9,030	45,220	20,170	174,640	24,000	289,710
1937	94,870	13,025	80,030	40,740	203,250	34,055	455,480
1938	133,000	17,600	36,000	46,400	215,000	34,000	482,000
1939	70,560	10,350	46,890	32,140	270,100	32,370	462,410
1940	183,700	10,700	58,700	61,300	294,000	35,500	643,900

[1] See *Elec. World*, Jan. 13, 1940, and *E.E.I. Bull.* H2, March, 1940.

existing stations of old design and relatively poor efficiency, whereas fewer of the 600- to 800-lb. units are superposed. Attention is called to the average plant factor of the stations costing less than $85 per kilowatt, which is 21.5 per cent, as compared with 40.5 per cent for the plants costing between $85 and $100 per kilowatt, showing that the cheaper plants are not designed to operate at as high plant factors as those which cost more.

In his paper "Design Features of the Port Washington Power Plant" for the Milwaukee Electric Railway and Light Company, G. G. Post gives the following cost studies:[2]

1. Comparison of actual 1,200-lb. generation with 300-lb. generation at Lakeside Station, 1929.

2. Estimated comparison of 1,200-lb. generation with 600-lb. generation for Port Washington.

3. Estimated comparsion of 825 and 750°F. at both throttle and reheat for Port Washington, together with other studies on boilers, heaters, and pulverizing systems. The unit arrangement planned for Port Washington, a single boiler for each single turbo-

[2] *Trans. A.I.E.E.*, September, 1933.

TABLE 8.—UNIT INVESTMENT COST STEAM STATIONS*

Station	3(R)	10(A)	28(H)	30(T)	38(I)	47	50	52
Rating, M kw	To 20	20 to 40	40 to 60	60 to 75	75 to 100	100 to 150	150 to 200	Over 200
Normal function	Base	Trans	Standby	System	Base	System	System	Base
No. units condensing	1	2	1	3	1	2	5	5
Superposed	0	0	0	0	0	0	2	1
No. boilers	2	2	2	5	1	5	14	11
Steam pressure, lb	400	525	475	400–825	1390	415	1250	350–1,250
degrees F	720	825	725	740–825	835	750	820	680–935
Fuel used	Coke	Coal, oil	Coal	Coal, gas	Coal	Coal	Coal, oil, gas	Coal
Fuel equipment	Pulver.	Pulver.	Pulver.	Pulver.	Pulver.	Stokers	Stokers &	Pulver.
Unit investment, $ per kw.								
Land	$ 2.00	$ 1.42	$ 0.72	$ 0.91	$ 5.24	$ 1.73	....	$ 2.24
Buildings and foundations	32.00	21.25	24.84	14.88	21.29	22.35	....	31.10
Condenser supply works	....	4.45	8.05	1.08	1.83	7.04		
Fuel handling, storage	8.41	2.50	6.13	3.79	19.03	6.97		
Ash handling	2.23	0.90	0.30	1.03	0.75	1.22		
Boiler plant	20.20	22.30	13.52	12.55	13.24	12.52	....	37.55
Draft system	6.13	3.98	4.08	1.98	1.48	3.31		
Feed-water system	3.11	3.73	1.66	1.36	3.01	1.68		
Piping system	7.67	7.50	4.40	4.62	3.02	3.53		
Turbine foundations	....	1.09	0.58	0.84	0.93			
Heat-recovery apparatus	....	....	......	1.74	1.79			
Generator coolers	0.52	....	0.79	0.43	0.24			
Turbogenerators	25.20	18.82	12.39	17.45	17.01	14.24	....	20.70
Turbine auxiliaries	2.18	0.64	0.34	4.03	3.22	....	....	10.58
Main condensers	....	4.36	3.23					
Switchgear, wiring	9.43	9.12	8.66	8.52	8.05	12.87		
Miscellaneous	11.20	....	......	9.22	....	30.80	....	2.31
Total station investment	$130.28	$102.06	$89.69	$84.43	$100.13	$118.26	....	$104.48
Outdoor switching	14.25	13.35	5.75	2.61	5.69	....	....	7.10
Total station and switchyard investment	$144.53	$115.41	$95.44	$87.04	$105.82	....	$109.60	$111.58

* From A. E. Knowlton, Fourth Steam Station Cost Survey, *Elec. World*, Dec. 2, 1939.

generator (80,000 kw.), with one set of auxiliaries, one set of transformers, and one 132-kv. transmission line, offers a most attractive design for the reduction of investment cost.

In an article, Trend of Steam Power Station Costs,[1] James E. Burkross, mechanical engineer, Sargent & Lundy, Inc., analyzes the effect of increasing construction costs during the last decade and the counterbalancing advantages of modern development and design. He allocates the construction cost (land, interest during construction, engineering and administrative expense not included) of a condensing station into the following sections:

	Per Cent
a. Structures	13.7
b. Boiler plant	40.2
c. Turbine plant	37.4
d. Electric plant	8.7

Appropriate index figures for the various items are presented as follows:

	Structures	Boiler plant	Turbine plant	Electric plant
1927	0.98	1.02	1.00	1.00
1928	1.01	1.00	1.00	.97
1929	1.00	1.00	1.00	1.03
1930	0.99	1.00	1.00	1.04
1931	0.89	0.94	1.00	1.04
1932	0.83	0.93	1.02	1.04
1933	0.86	0.92	1.04	1.06
1934	0.94	0.95	1.07	1.20
1935	0.94	1.02	1.18	1.21
1936	0.98	1.07	1.19	1.23
1937	1.08	1.26	1.24	1.32
1938	1.05	1.24	1.24	1.33
1939	1.09	1.30	1.19	1.35

As against the marked increases in the index values are placed the following offsetting factors:

The adoption of the unit plan, one turbine and one boiler, saving in extra equipment, building, and piping costs.

[1] *Power Plant Eng.*, January, 1940.

The increased work done in the pressure parts of the boiler because of better furnace design.

Less heating surface installed per kilowatt of capacity.

Higher speeds in the prime movers with savings in foundations and building space.

Improvements in electrical design, auxiliaries fed from main generator.

In the final balance then, the trend for the period indicated is shown in Fig. 7, Trend in Condensing Steam Station Costs per Kilowatt.

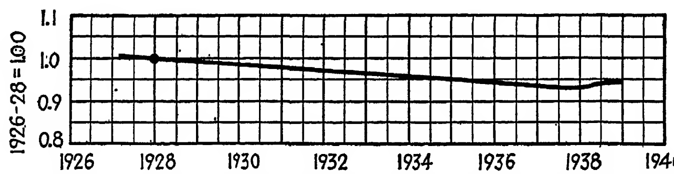


FIG. 7.—Trend in steam power station costs as referred to kilowatt capacity. (*Courtesy of James E. Burkross, Sargent & Lundy.*)

For steam-electric plants built prior to 1932, Philip Sporn, vice president and chief engineer of the American Gas and Electric Co., found that "the plant investment ranged from $85 to $140 per kw. of installed capacity, with the greatest number of the plants lying in the range between $100 and $110. However, the wide variation in costs prevents any rational deduction as to 'average' cost."[1]

20. Cost of Hydroelectric Plants.—Owing to the wide differences in conditions governing the developments, the construction cost of hydroelectric stations may cover a wide range. In general, the cost will decrease for increase in head and with increase in the size of the plant. The investment for the dam with pondage and control works is such a large part of the cost of development that a low cost will be obtained only when large capacity is installed. "Data published in reports of the Federal Power Commission indicate that the cost per kilowatt of installed capacity of hydro plants varies from $124 to $348."[2]

Outstanding among the modern hydroelectric developments are the projects undertaken by the Federal Government. Table

[1] See *Civil Eng.*, January, 1937.

[2] See *Civil Eng.*, January, 1937, p. 52.

TABLE 9.—FEDERAL HYDRO POWER PROJECTS

Project	Estimated cost ($1,000)	Ultimate capacity, kw.	Added in 1939, kw.	In operation end of 1939, kw.	Expected to be added in 1940, kw.	Approx. % completed end 1939
1. TVA (A):						
Gilbertsville	$ 95,000 (B)	160,000	....	....	....	3
Pickwick	31,837 (C)	216,000	72,000	72,000	....	100
Wilson	34,989 (D)	444,000	....	184,000	....	100
Wheeler	31,586 (C)	259,200	....	64,800	....	100
Guntersville	33,400 (E)	97,200	72,900	72,900	....	100
Chickamauga	37,000 (E)	108,000	....	....	81,000	95
Watts Bar	35,000 (E)	150,000	....	....	....	3
Norris	30,749 (F)	100,800	....	100,800	....	100
Hiwassee	20,000 (G)	115,200	....	....	57,600	74
Corps of Engineers:						
2. Fort Peck	123,750 (H)	105,000	....	....	....	90
3. Bonneville	78,120 (H)	518,400	86,400	86,400	108,000	100
4. Denison	48,290 (H)	125,000	....	....	....	1
Reclamation Bureau:						
5. Boulder Dam	132,160 (I)	1,317,000	165,000	700,000	....	100
6. Kendrick	20,000 (J)	32,400	32,400	32,400	....	100
7. Central Valley	170,000 (K)	375,000	....	....	....	35
8. Colorado Big Thompson	44,000 (L)	145,000	....	....	....	30
9. Grand Coulee	394,500 (M)	1,944,000	....	....	....	35
PWA Projects:						
10. Platte River	11,226	26,000	26,000	26,000	....	100
11. Loup River	12,814	47,000	....	45,225	....	95
12. Central Nebraska	35,516	54,000	....	....	54,000	80
13. Grand River	20,000	86,400	....	....	57,600	80
14. Santee-Cooper	34,300 (N)	163,000	....	....	....	20
15. Lower Colorado River Auth.	22,350 (O)	98,000	33,750	33,750	53,250	80
16. Brazos	50,000 (P)	75,000	....	....	22,500	86
Totals	$1,546,587	6,761,600	488,450	1,418,275	438,950	

(A) TVA joint investment, covers navigation, flood control, power.
(B) Estimate with no power.
(C) Estimate includes two units.
(D) Reconstruction cost of facilities acquired from War Department, plus additions made by authority to June 30, 1938.
(E) Estimate includes three units.
(F) Costs to June 30, 1938.
(G) Estimate includes one unit.
Bonneville-Navigation $42,600,000, power house and generating facilities $35,520,000. Fort Peck-Navigation $117,000,000, power $6,750,000. Denison-Allocation as between power and flood control not yet determined.
(I) *Not* including all-American Canal.
(J) Includes Alcora Dam and irrigation district.
(K) Includes whole project; Shasta Dam alone will have power development.
(L) Including $26,000,000 for irrigation.
(M) For dam and irrigation development.
(N) Total estimated cost exclusive of $6,495,691 by WPA for clearing reservoir.
(O) Total exclusive of $18,280,000 allotment to Bureau of Reclamation and direct Congressional authorization and $109,850 WPA.
(P) Possum Kingdom dam, key project in the contemplated $50,000,000 development of the Brazos River Conservation and Reclamation District, has received a $4,200,000 WPA allotment, a substantial portion of which has been expended. Recently RFC purchased $4,200,000 of the district's bonds, of which proceeds approximately $2,500,000 will be used to complete Possum Kingdom. An estimated 75,000 kw. will be generated in the proposed seven-dam hydro development. The RFC bonds will be retired from sale of power supplemented by a state appropriation of $132,000 per year for 16 years.

9, Federal Hydro Power Projects, lists their progress as reported in the *Electrical World*, Jan. 13, 1940.

Figure 10 shows a cross section through the twin wings of Boulder Dam power plant and the canyon walls, and Fig. 11

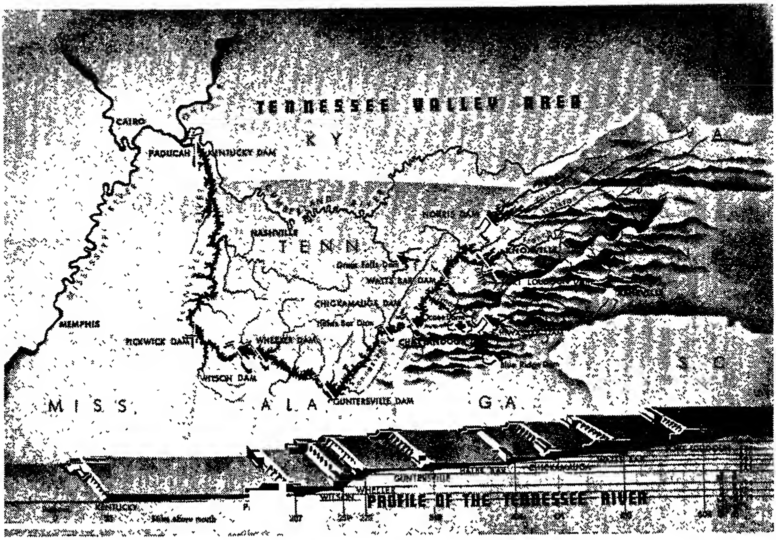


FIG. 8.—Map and profile of the Tennessee Valley area of about 41,000 square miles, showing the nine dams involved in the combined program for power, navigation and flood control. (*Courtesy of Tennessee Valley Authority.*)

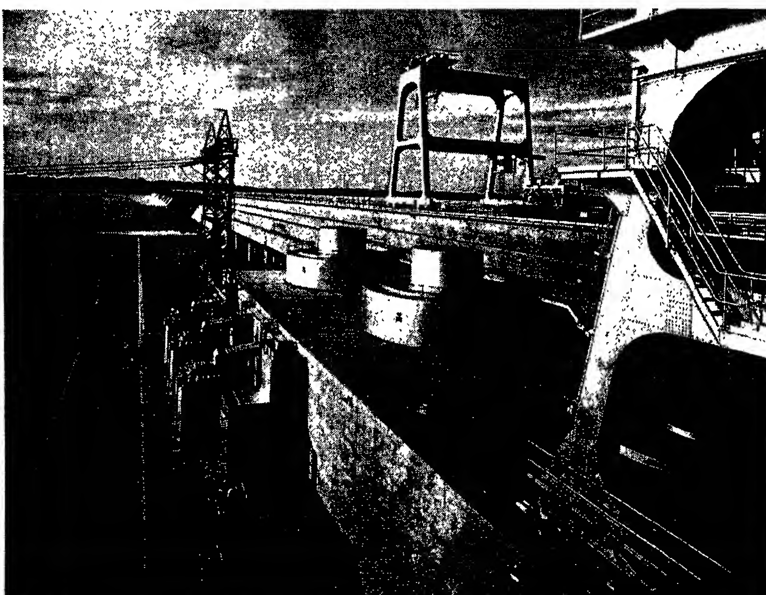


FIG. 9.—Downstream face of Wheeler Dam on Tennessee River, 72 ft. high, 6,502 ft. long, reservoir of 67,000 acres. The outdoor type of generating station with two 36,000-kva. units at 85.7 r.p.m. installed. Navigation lock across the river. (*Courtesy of Tennessee Valley Authority.*)

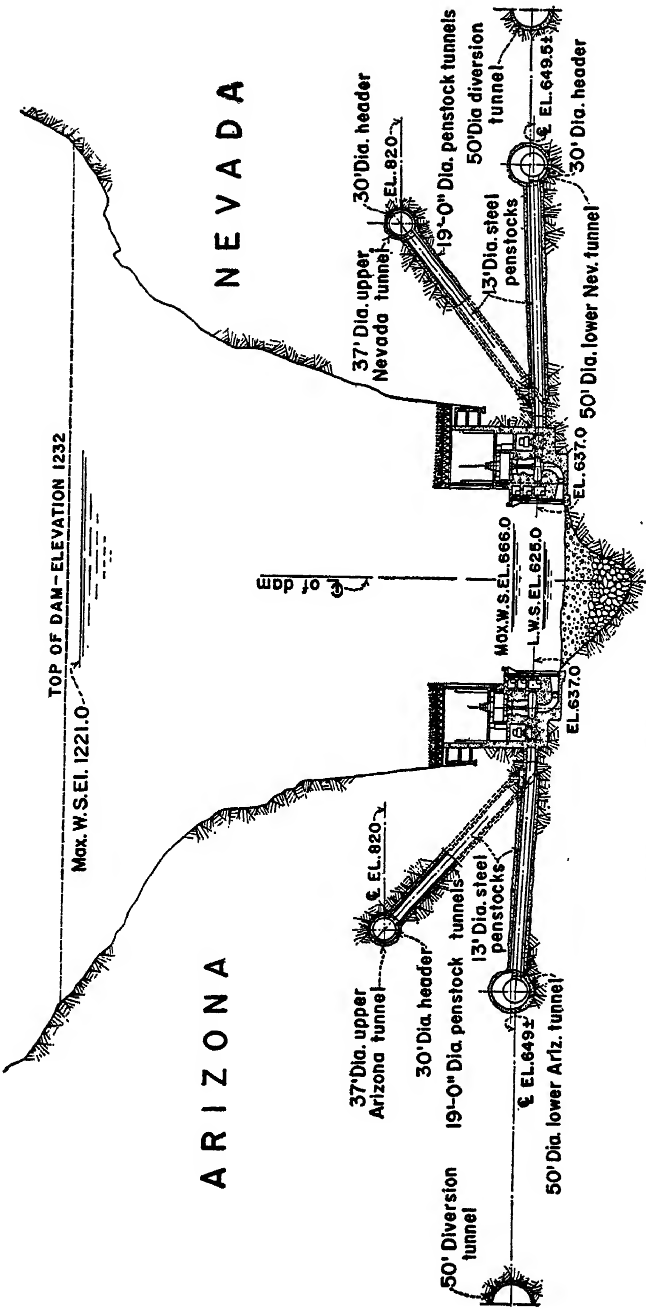


Fig. 10.—Boulder Dam. Cross section through power houses and canyon. (*U. S. Bureau of Reclamation.*)

gives a cross section through one of the power plant wings. Figure 12 shows an assembly view of one of the turbines.

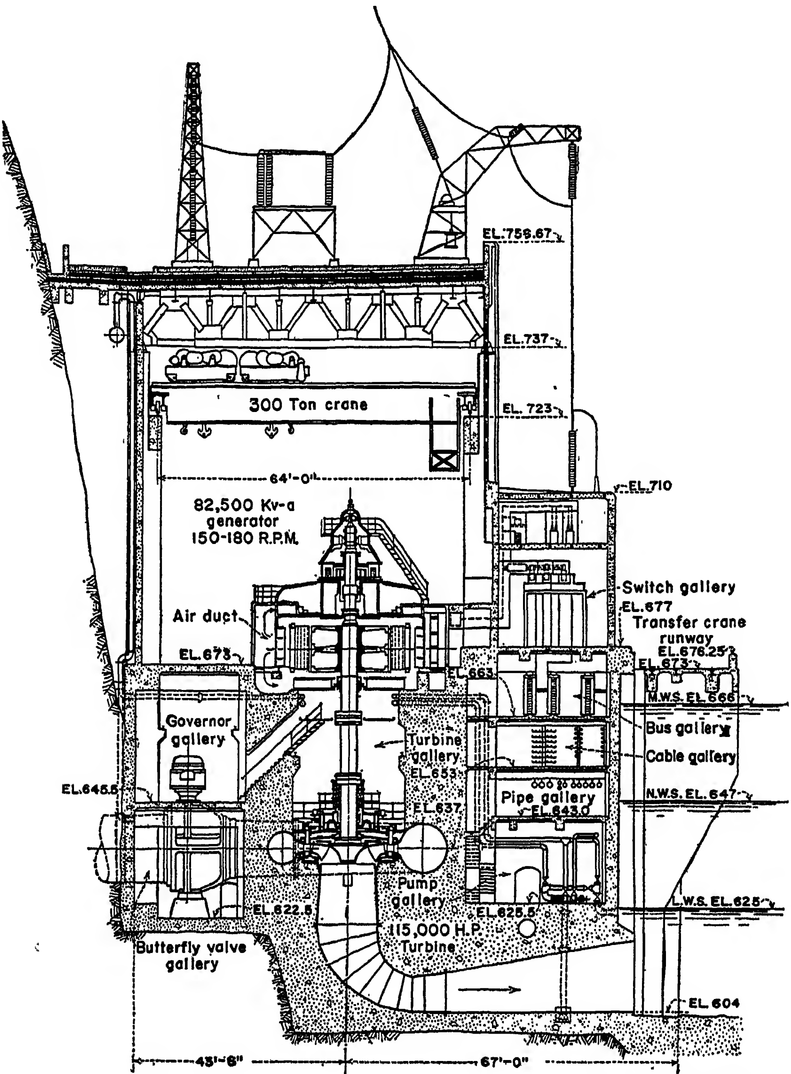


FIG. 11.—Boulder Dam. Cross section through left wing of power house. (*U. S. Bureau of Reclamation.*)

The energy from Boulder Dam has been sold on a contract basis, the revenue to pay all expenses of operation and main-

tenance with interest at 4 per cent and to amortize the investment in 50 years. The charge for primary power at the switchboard has been 1.63 mills per kilowatt-hour and for secondary power 0.5 mill. The users of power from Boulder Dam have maintained that these rates are out of line "in view of the progressive lowering in the cost of steam-generated power and the different

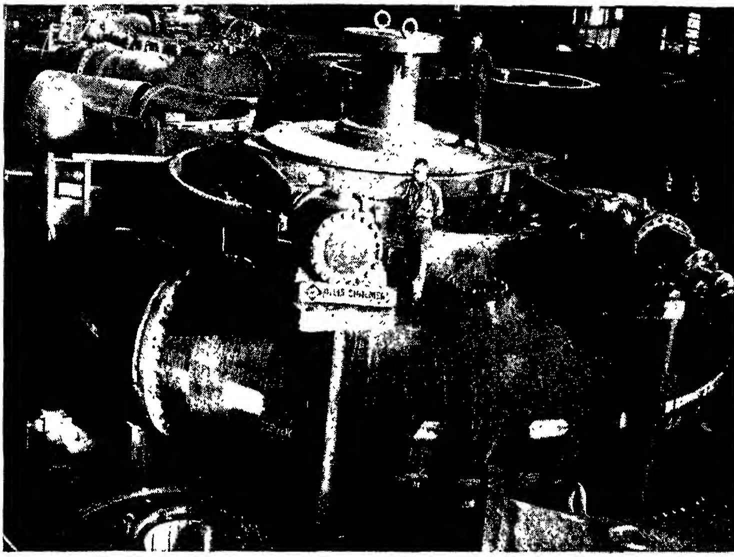


FIG. 12.—Assembly view of 115,000-hp. turbines for Boulder Dam; 490-ft. head, 150 to 180 r.p.m., using about 2500 c.f.s. Cast-steel, one-piece runner 166 in. nominal diameter; shaft 38 in. in diameter. Cast-steel casings in six sections, total weight about 500,000 lb. Casing inlet, 10 ft. diameter. Each turbine uses a pressure regulator to prevent surges when load is suddenly removed. (*Courtesy of Allis-Chalmers Manufacturing Co.*)

rate bases used on other federal power projects."[1] An Adjustment Act passed by Congress in 1940 will

1. Amortize in 50 years the cost of the dam (less $25,000,000 allocated to flood control to be repaid after 1987) at 3 per cent.
2. Pay $300,000 annually each to Arizona and Nevada.
3. Pay $500,000 annually into a fund for development of the upper Colorado basin.[2]

[1] *A.E.C. Bull.*
[2] *Elec. World*, June 22, 1940.

Ontario Hydroelectric Power Commission.—As indicative of what a pioneer government project may grow to, this system is of interest. The Commission's thirty-second annual year ended Oct. 31, 1939:[1]

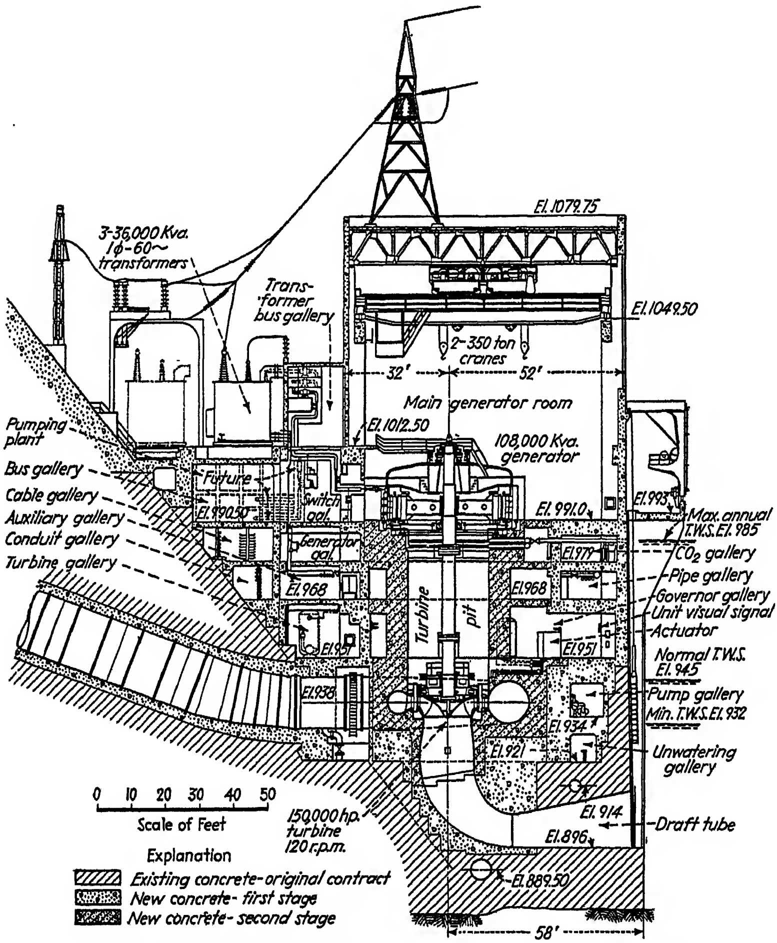


FIG. 13.—Grand Coulee power plant, left power house. Typical section through main unit. (*U. S. Bureau of Reclamation.*)

The capital investment of the Commission in the respective systems, districts, and municipal undertakings is as follows:

[1] See report of the Commission, 1940.

Niagara system (including Hamilton street railway)	$217,771,972
Georgian Bay system	11,634,124
Eastern Ontario system	22,754,586
Thunder Bay system	19,935,848
Manitoulin rural power district	92,793
Nipissing rural power districts	58,809
Sudbury rural power district	33,545
Bonnechere storage	51,742
Office and service buildings	3,267,086
Construction plant and inventories	3,379,952
	$278,980,457
Northern Ontario Properties, operated by H.E.P.C. on behalf of the Province of Ontario	39,888,835
Northern Ontario Properties, construction plant and inventories	143,898
Toronto-Port Credit-St. Catharine's Radial Railways	2,201,775
	$321,214,965
Municipalities' distribution systems	$ 99,489,755
Other assets of municipal hydro utilities	25,417,826
	$446,122,546

The Commission owns and operates some 46 generating stations with a maximum normal plant capacity of 1,525,000 hp. and in addition purchased under contract 622,000 hp. The 1939 distribution of primary power to systems, system coincident primary peaks, is as follows:

System	20-min. Peak Hp., October, 1939
Niagara system, 25 cycle	1,171,582
Dominion Power & Transmission division, 66⅔ cycle	56,970
Georgian Bay system	34,756
Eastern Ontario system	141,908
Thunder Bay system	96,160
Manitoulin rural power district	273
Northern Ontario properties:	
Nipissing district	5,188
Sudbury district	19,740
Abitibi district	130,968
Patricia-St. Joseph district	11,792
Total	1,669,337

McIndoes Plant

	Per Kva.
Preliminary work	$ 14.42
Local general expenses	12.40
Construction plant	8.07
Spillway and abutments	16.82
Intake and power-house substructure	16.73
Power-house superstructure	7.02
Hydraulic equipment	15.14
Indoor electrical equipment	17.20
Step-up substation	3.34
Operators' quarters (remodeling work)	0.17
Landscaping and power-house yard	0.68
Tailrace	2.79
Total direct construction costs	$114.78
Interest during construction (2 years 13 days from beginning of detail survey of river bed to completion of project cleanup)	6.38
Contractor's expense	9.84
Grand total cost	$131.00

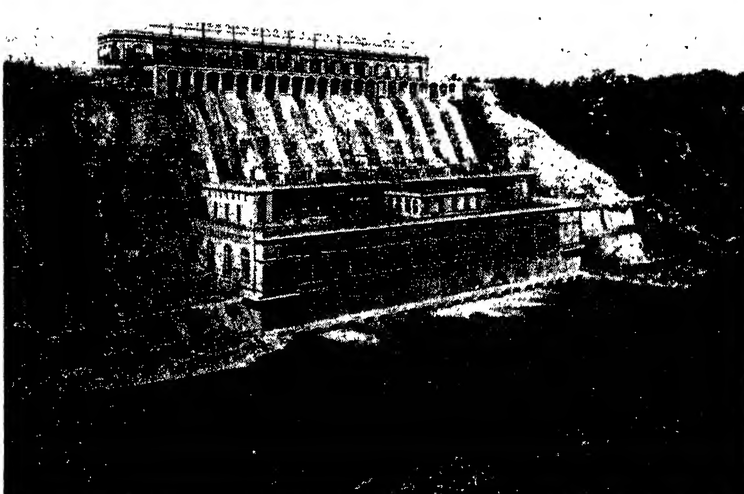


Fig. 14.—Queenston generating station, Niagara River, completed for ten units, rated capacity 497,000 kva. Head 294 ft. Screen house and administration building on top of 300-ft. bank. Generator voltage 12 kv. transformed to 110 kv. for transmission. 25 cycles. (*Courtesy of the Hydroelectric Power Commission of Ontario.*)

Figures 14 and 15 show the Queenston station of the Commission.

McIndoes Development.—An attractive 27-ft. head plant at McIndoes Falls on the Connecticut River, completed in 1931, is described in the *Electrical World* of May 20, 1933. This project

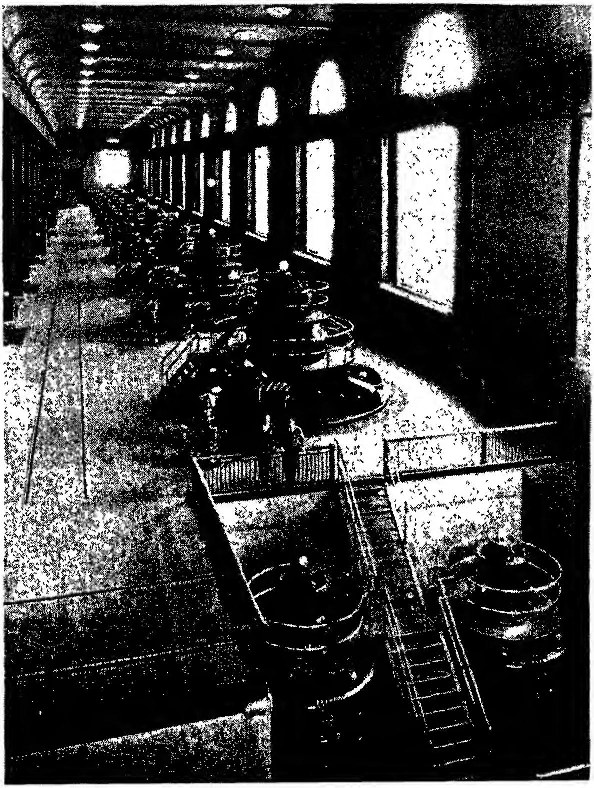


Fig. 15.—Interior of Queenston station, 135 by 590 ft. Ten main units installed, 497,000 kva., with two service units, each 2,200 kva., in foreground. Only the upper frame and exciter of each main unit are above floor level. (*Courtesy of the Hydroelectric Power Commission of Ontario.*)

has a pond area of 540 acres with an available pondage of 3,000 acre-ft. in a 6-ft. drawdown. Four units, each of 3,300 kva., are installed. The total cost is reported as $131 per kilovolt-ampere, very complete cost subdivisions being shown which are summarized at the top of page 62.[1]

[1] See *Elec. World,* May 20, 1933.

Comerford (15-mile) Plant on the Connecticut River was completed in 1930 with four units each of 39,000 kva. at 180-ft. head. It has a pond providing 5,500 acre-ft. volume. Exclusive of the transmission line beyond the switching station, the costs are detailed as shown:[1]

COMERFORD PLANT

Item	Per cent of total		Per kva.
Local general	8.4	23.6	$ 24.84
Construction plant	7.0		
Preliminary work	4.5		
Diversion channel	2.9		
Operators' quarters	0.8		
Landscaping and cleaning up	0.07		
East retaining wall and earth fill dam	7.7	35.6	37.58
Non-overflow and spillway sections	12.1		
West retaining wall and earth fill	0.4		
Intake	10.5		
Tailrace	4.9		
Power house:			
Substructure	2.6	6.4	6.66
Superstructure	3.8		
Equipment:			
Hydraulic	3.7	21.1	22.17
Indoor electrical	7.6		
Step-up substation	9.8		
Interest during construction	6.0	13.3	14.09
Contractor's expense	7.3		
Total	....		$105.34

For detailed costs of other water-power plants covering a wide range of heads and sizes, the reader is referred to H. K. Barrows' "Water Power Engineering."[2]

21. Cost of Diesel-electric Plants.—In recent years, the diesel engine has continued to make a place for itself in stationary power applications, the 1939 estimate being 812,755 kw. in internal-combustion engines. Fairly large-sized units are now available, such as the 15,000-kw. set installed along with 130,000 kw. of steam capacity in the Oersted municipal power station in

[1] See *Elec. World,* Mar. 4, 1933.
[2] McGraw-Hill Book Company, Inc.

Copenhagen. This double-acting two-stroke uniflow scavenger engine has a guaranteed fuel consumption of 0.55 lb. per kilowatt-hour at 15,000 kw., and 0.53 lb. per kilowatt-hour at 12,500 kw. (about 10,000 B.t.u. per kilowatt-hour). A notable 1933 American installation of five 7,000-b.hp. units in the municipal plant at Vernon, Calif., is shown in Fig. 16.[1] These engines are eight-cylinder double-acting two-cycle diesels running at 167 r.p.m. Tests on individual engines show that at

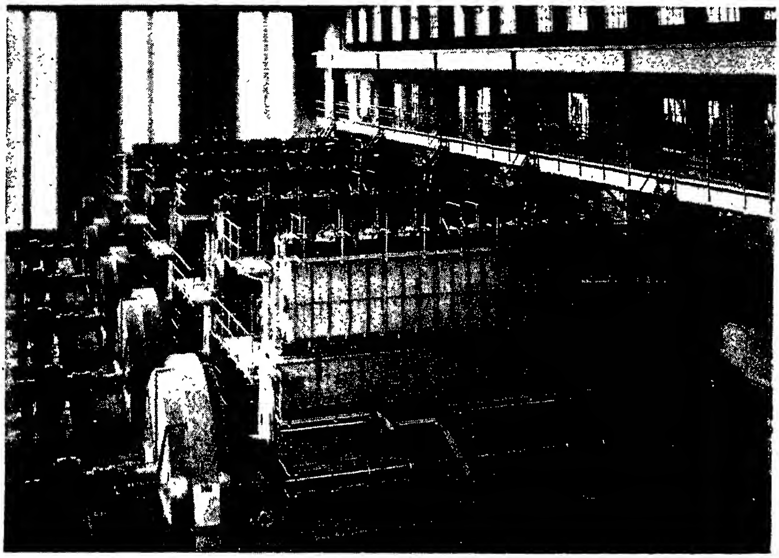


FIG. 16.—Vernon, Calif., diesel-electric power plant, five 7,000-b.hp. units, eight-cylinder, double-acting, two-cycle, running at 167 r.p.m. (*Courtesy of Hooven, Owens, Rentschler Co.*)

75 and 100 per cent rating the over-all thermal efficiencies are 37.5 and 36.3 per cent, respectively, and the fuel consumption per net brake horsepower is 0.371 and 0.383 lb., respectively.[2]

Naturally the widest adoption of the diesel has come in areas that provide cheap oil fuel and satisfactory cooling water and where large steam stations would not be economical because of the adverse fuel and scarce water conditions, as well as the long transmissions needed to supply scattered loads. The high thermal efficiency, around 35 per cent for modern engines, with

[1] See *Diesel Power*, October, 1933.

[2] See 1934 *Rept.* A.I.E.E. Committee on Power Generation.

a present best performance of 41 per cent,[1] the flatness of the fuel-consumption curve, and the relatively high economy of the small units are favorable points for the engine units. Because of the elimination of coal banking losses, as in a steam plant, the diesel generating units are often favored as peak, auxiliary,

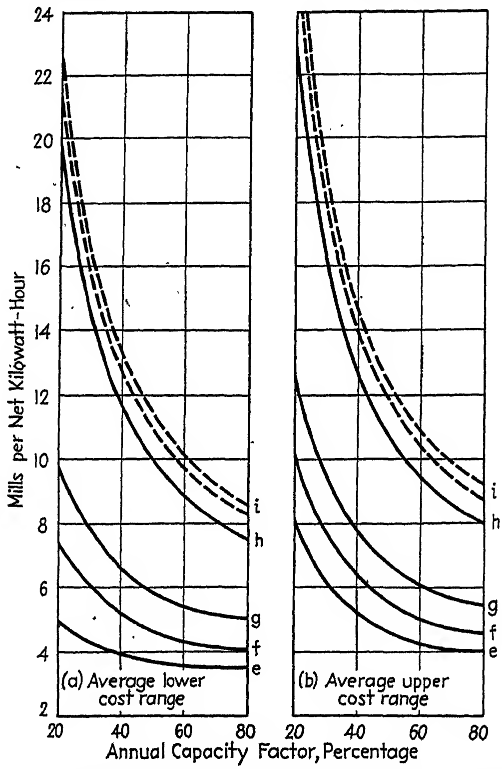


FIG. 17.—Average ranges of power cost for diesel plants. (*Courtesy of R. M. Van Duzer, Jr., Detroit Edison Company.*)

Curve *e* = fuel oil expense, 5 cts. per gal.
f = operating labor and superintendence, wage 70 cts. per hr.
g = lubricating oil (55 cts. per gal.), miscellaneous supplies and maintenance.
h = fixed-charge expense, 13 per cent on $135 per kw. of plant capacity.
i = general overhead expense, 10 and 15 per cent of totals shown by curve *h*.

and stand-by capacity in conjunction with base hydro or steam plants. Significant developments have recently been made in better materials and design, in lighter weights and higher speeds, in air filtering and water softening, and in saving the heat from jacket water and exhaust gases. Proper treatment and cooling

[1] See DEGLER, H. E., Five Years' Progress of Oil and Gas Power, *Trans. A.S.M.E.*, October, 1939.

of the jacket water, suitable silencing, and the elimination of vibration are still important problems for the station designer.

Table 10 gives the total cost per kilowatt-hour for various municipal plants in Wisconsin for 1936 operation as determined

TABLE 10.—COST OF MUNICIPAL DIESEL-ELECTRIC PLANTS, WISCONSIN 1936[1]

Item	Menasha	Lake Mills	Cedar-burg	Ar-cadia	Elroy	Cash-ton
Total engine hp.	3,600	1,650	1,200	705	360	210
Number of engines	4	4	2	3	2	2
Adjusted investment:						
Land	$ 2,964	$ 1,000	$ 240	$ 1,644	$ 200	$ 200
Structures	27,691	10,445	27,629	9,607	10,910	5,000
Generating equipment	287,986	117,279	112,158	63,370	44,233	34,344
Total plant	$318,641	$128,724	$140,027	$74,621	$55,343	$39,544
Investment per hp	88.5	78.0	116.7	105.9	153.7	188.3
Production costs, cts. per kw.-hr.:						
Labor and superintendence	0.123	0.199	0.266	0.362	0.526	0.644
Fuel oil	0.473	0.525	0.717	0.527	0.637	0.522
Lubricating oil	0.062	0.043	0.040	0.096	Inc.	0.122
Supplies and expense	0.022	0.058	0.051	0.085	0.246	0.104
Cooling water	0.021	Inc.	Inc.	0.052	0.018	0.047
Maintenance	0.068	Inc.	0.239	Inc.	Inc.	Inc.
Subtotal	0.769	0.825	1.313	1.122	1.427	1.439
Insurance on plant	0.009	0.014	0.009	0.015	0.038	0.052
Compensation insurance	0.008	0.014	0.018	0.028	0.027	0.050
Fixed charges, 13 %	0.570	0.847	1.104	1.352	1.204	1.612
Credit for heating buildings	....	....	0.030	....	....	0.063
Total cts. per kw.-hr.	1.356	1.700	2.414	2.517	2.696	3.090
Statistics:						
Kw-hr. generated, M	7,689	2,077	1,753	723	679	348
Kw-hr. station use, M	418	102	105	6	82	29
Kw. peak load	2,000	658	555	250	200	105
Annual load factor	43.9	36.0	36.0	33.0	38.75	37.78
Gal. fuel oil used	653,750	185,046	205,980	68,642	68,796	31,732
Fuel oil, cts. per gal	5.272	5.60	5.75	5.597	5.525	5.122
Specific gravity oil	24–30	28–30	24–26	32–36	30–36	28–30
Gal. lubricating oil used	9,031	2,063	1,327	1,210	....	880
Lubricating oil cts. per gal	49.87	41.10	49.40	56.60	....	44.00
M gal. water from mains	73,835	....	....	16,223	1,188	2,000
Water cts. per M gal	2.00	....	....	2.21	9.24	7.5
Running plant capacity factor	74.09	58.45	....	....	....	52.60
Kw.-hr. per hp. installed	2,136	1,259	1,461	1,025	1,886	1,655

[1] Courtesy J. S. Hartt, consulting engineer, Madison, Wis.

by J. S. Hartt from the reports of the Public Service Commission of Wisconsin.[2] A consideration to be kept in mind in using the figures is that four of the six plants are of insufficient peak capacity to assure service continuity in case of failure of a major

[2] See also *Elec. World*, Apr. 23, 1938.

unit; thus costs are lower than they would be if this provision were taken into account.

Figure 17, Average Ranges of Power Cost for Diesel Plants, represents the estimate of the late Dr. Hirshfeld and R. M. Van Duzer, Jr., in their paper "Heat-generated Energy" in the *Proceedings* of the A.S.C.E., April, 1938. The authors note that the thermal performance is influenced more by the type of load carried, the character of operation, and the age of the engine than by the size of the unit.

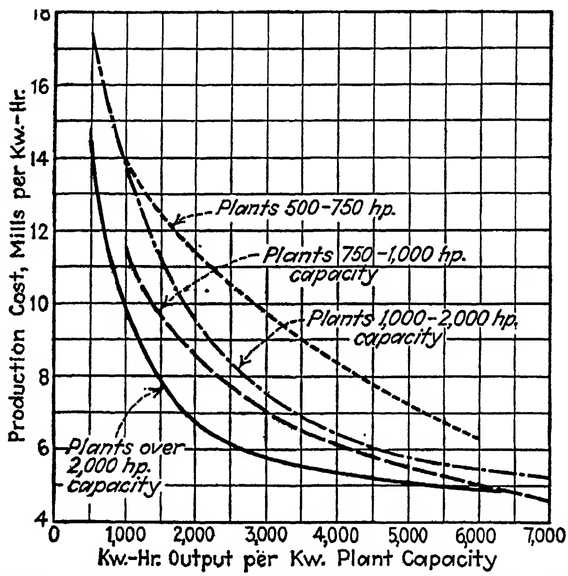


FIG. 18.—Median lines of various capacities. Production costs of diesel power plants. (*Diesel Power, June,* 1934.)

For further data on production cost (no investment costs), the reader is referred to the reports of the Subcommittee on Oil Engine Power Cost, A.S.M.E. The 1938 report includes information from 151 oil-engine generating plants, containing 444 engines, totaling 248,547 rated brake horsepower. The net output for the 151 plants, municipal, utility, and industrial, amounted to 318,636,891 kw.-hr. An analysis of similar data from the report for 1932 is made in *Diesel Power* for June, 1934, from which Fig. 18 is taken, showing the production cost against kilowatt-hour output per kilowatt plant capacity for the median curves of various capacities.

22. Relation between Fixed and Operating Costs, Operating Ratio.—The relative values of the fixed charges and the direct operating costs are shown for the Detroit Edison Company, for 1939 in the "Outgo of Income" in the *Synchroscope* for February, 1940:

	Per Cent
Labor	24.5
Fuel	10.4
Other operating expenses	10.9
Depreciation	13.4
Taxes	14.6
Interest	10.1
Dividends	12.8
Other charges and surplus	3.3
	100.0
Direct operating expenses, before taxes	45.8
After taxes	60.4

The *Electrical World* of Jan. 13, 1940, reports the following:

OPERATING RATIO BEFORE AND AFTER TAXES, PRIVATE COMPANIES AND MUNICIPAL PLANTS

Per cent	1912	1917	1922	1927	1932	1937
Private companies:						
Before taxes	65.2	63.5	52.5	44.4	40.2	42.8
After taxes	70.7	69.7	60.2	52.9	51.1	56.9
Municipal plants:						
Before taxes	Included		68.0	....	55.2	52.0
After taxes	with private		68.8	....	56.1	53.5

23. Operating Costs of Steam-electric Plants.—In Table 11 are given the operating costs of the eight plants, selected from A. E. Knowlton's tabulation, whose investment costs were shown in Table 8.

Since these data are the results of only one year's operation of comparatively new plants in various stages of ultimate completion it will be instructive to examine the operating record of an older station, which had a growing period, reached completion, and operated as a mature station for a number of years, and then was rehabilitated on an advanced thermal cycle. Table 12 shows the production expense per kilowatt-hour for the Connor's

TABLE 11.—1938 PRODUCTION COST STEAM STATIONS*

Station	3(R)	10(A)	28(H)	30(T)	38(I)	47	50	52
Rating, M kw	To 20	20 to 40	40 to 60	60 to 75	80	100 to 150	150 to 200	Over 200
Generation, million								
Gross, kw.-hr	108.68	116.62	282.72	167.33	415.72	390.53	495.15	1,111.72
Net, kw.-hr	105.43	109.75	270.07	157.47	393.71	381.56	466.76	1,035.09
30-min. peak, kw	17,568	33,000	66,000	43,700	80,000	132,700	114,000	196,300
Billion B.t.u. used	1,620	1,558	3,685	2,555	4,250	5,450	6,980	17,600
Cost, cts. per million	6.45	15.19	10.70	11.31	16.24	10.08	10.7	9.26
B.t.u. net kw.-hr	15,375	14,215	13,625	16,128	10,788	14,255	14,964	17,016
Thermal efficiency	22.2	24.1	25.1	21.2	31.7	24.1	22.9	20.1
Annual factors								
Plant, per cent	72.4	44.3	53.8	28.0	59.3	37.2	36.4	60.8
Utilization, per cent	117	110	110	64.2	100	111	73.3	93.5
Men per M kw. per day	1.33	1.33	1.05	0.95	0.61	1.19	0.98	1.11
Operating cost, mills per net kw.-hr.								
Fuel	0.960	2.140	1.450	1.840	1.751	1.482	1.680	1.580
Wages and supervision	0.600	0.413	0.312	0.510	0.302	0.274	0.590	0.369
Water, lubrication, supplies	0.040	0.064	0.024	0.060	0.023	0.094	0.060	0.049
Maintenance and repairs								
Buildings	0.034	0.029	0.066	0.070	0.011	0.013	0.020	0.013
Boiler plant	0.093	0.101	0.065	0.130	0.070	0.051	0.190	0.090
Fuel handling	0.065	0.027	0.021	0.060	....	0.031	0.040	0.028
Turbogenerator and auxiliary	0.030	0.033	0.060	0.060	0.034	0.023	0.090	0.048
Electric equipment, miscellaneous	0.026	0.020	0.014	0.050	0.030	0.038	0.010	0.032
Supervision	0.037							
Total production	1.885	2.827	2.012	2.780	2.221	2.006	2.720	2.209
Fixed charges, 15%	3.08	4.74	3.18	5.67	3.32	5.58	5.48	3.41
Total cost	4.97	7.57	5.19	8.45	5.54	7.59	8.20	5.62

* From A. E. Knowlton, Fourth Steam Station Cost Survey, *Elec. World*, Dec. 2, 1939.

TABLE 12.—PRODUCTION EXPENSE PER KILOWATT-HOUR OUTPUT AT CONNOR'S CREEK STATION OF THE DETROIT EDISON COMPANY FOR 12-MONTH PERIODS ENDING JUNE 30

A. Stoker-fired, steam at 200 lb. 600°F. Three 20,000-kw., one 30,000-kw., two 45,000-kw. units. 1916–1932. *B*. Stoker-fired, steam at 600 lb. 825°F., units of 30,000 and 60,000 kw. 1935–1939

Item	*A*							*B*				
	1916	1918	1919	1920	1921	1928	1932	1935	1936	1937	1938	1939
Operation, cts.:												
Supervision, engineering	......	......	......	......	......	......	......	0.0243	0.0159	0.0114	0.0101	0.0090
Wages, superintendent	0.057	0.064	0.072	0.072	0.094	0.071	0.109	0.0938	0.0584	0.0446	0.0470	0.0397
Fuel	0.158	0.368	0.394	0.471	0.682	0.286	0.240	0.1948	0.1704	0.1729	0.1708	0.1686
Water	......	......	......	......	......	......	......	0.0002	0.0003	0.0003	0.0005	0.0004
Lubricants	0.001	0.001	0.002	0.001	0.002	0.001	0.001					
Supplies, expense	0.005	0.006	0.010	0.011	0.010	0.012	0.012	0.0117	0.0095	0.0083	0.0062	0.0060
Maintenance, cts.:												
Supervision, engineering	......	......	......	......	......	......	......	*	*	*	0.0017	0.0015
Station buildings	0.006	0.008	0.008	0.010	0.012	0.012	0.012	0.0087	0.0122	0.0104	0.0056	0.0053
Steam equipment	0.019	0.025	0.024	0.036	0.043	0.028	0.022					
Boiler plant	......	......	......	......	......	......	......	0.0169	0.0165	0.0182	0.0128	0.0152
Electric equipment	0.003	0.002	0.001	0.001	0.003	0.004	0.002					
Generating and electric equipment	......	......	......	......	......	......	......	0.0057	0.0085	0.0160	0.0068	0.0050
Miscellaneous equipment	......	......	......	......	......	0.004	0.004	0.0005	0.0006	0.0001	0.0004	0.0003
Total operation and maintenance, cts.	0.249	0.474	0.511	0.602	0.846	0.418	0.402	0.3566	0.2923	0.2822	0.2619	0.2510
Kw.-hr. output, thousands	125,159	280,815	383,252	488,061	485,189	660.909	429,509	370,606	631,627	999,214	995,396	1,194,529
Maximum demand (30 min. kw.)	35,000	59,000	82,000	104,000	107,500	138,000	92,000	85,000	150,000	210,000	212,500	270,000
Average load, kw.	14,000	32,100	43,700	55,600	55,300	75,400	48,900	42,300	71,900	114,100	113,600	136,400
Load factor	0.409	0.544	0.553	0.534	0.515	0.546	0.532	0.498	0.479	0.543	0.535	0.505
Coal, lb. per kw.-hr.	1.44	1.63	1.67	1.83	1.78	1.49	1.44	1.07	0.93	0.91	0.91	0.90
B.t.u. per kw.-hr.	19,700	20,940	21,200	22,800	21,800	19,684	19,020	14,100	12,600	12,500	12,300	12,200
B.t.u. per lb. coal	13,670	12,840	12,700	12,460	12,250	13,250	13,210	13,200	13,500	13,600	13,600	13,600
Average coal cost per ton, dollars†	2.19	4.52	4.72	5.14	7.66	3.85	3.33	3.66	3.68	3.79	3.77	3.77
Installed capacity, kw., December	40,000	60,000	105,000	135,000	180,000	180,000	180,000	180,000	240,000	300,000	255,000	315,000

* Included in Operation, supervision, engineering for 1935 to 1937. Federal Power Commission classification of accounts (adopted 1938) provides for separation between operation and maintenance expense.

† Does not include labor charges for handling and preparation.

NOTE.—Connor's Creek power plant rebuilt 1934 to 1939. Old No. 1 20,000-kw. unit and No. 3 30,000-kw. replaced by new 30,000-kw. in 1934. No. 2 and No. 3 20,000-kw. replaced by 30,000-kw. units in 1935. In 1936 and 1937, added one new 60,000-kw. unit each. In 1938, withdrew one old 45,000-kw., and in 1939 added a 60,000-kw. unit.

Creek Station of the Detroit Edison Company from 1916 to 1939. Such a tabulation is the honorable service record of a gallant fighter who worked to his utmost capacity during the first World War, who fought a losing fight against wild prices for fuel and labor in the immediate postwar years, but who persevered until he conquered and, in 1928, regained the 19,700 B.t.u. per kilowatt-hour with which he began his work. Then, with a better load factor he improved the efficiency steadily for the next four years. Unfortunately for the economic results of 1916, a new era of fuel, labor, and material prices prevailed, but that detracts nothing from the glory of the fight to maintain the station efficiency. The record plainly shows how persistent and relentless were the adverse forces and what magnitude they reached. Incidentally, the engineer may read here how his carefully prepared estimates and plans will be turned topsy-turvy in such a troublesome time.

In 1933, the first two of the old boilers were removed in the work of rehabilitating Connor's Creek station and of bringing it to a simple regenerative cycle with steam at 600 p.s.i., and a temperature of 825°F. The first rebuilt turbine units are 30,000 kw. each and are being followed by 60,000-kw. units.

The complete rehabilitation plans for a capacity of 330,000 kw. performing at less than 13,000 B.t.u. per kilowatt-hour at a total book value of completed plant (including nearly $10,000,000 in land, buildings, and equipment retained from the old plant) estimated at $30,367,200, or $92 per kw.[1]

Of course, the real function of the engineer is to deliver the power at the station bus bars at the lowest possible cost, and it is in the analysis of the elements that make up the cost that the real answer as to the most profitable trend for future power-station development is found. The important elements are the marked reductions in fuel cost and the definite increase in the fixed charges. Hence if lower fuel costs are sought for by the use of more efficient stations, unless there is a large energy output the increase in fixed charges may more than offset the decrease in the fuel cost.

As indicative of the gain in economical performance due to moderately higher steam temperature and pressure, and larger

[1] See Rehabilitation of the Connors Creek Plant, by R. E. Greene, *Elec. Eng.*, June, 1935.

units than for old Connor's Creek station, Table 13 gives the production expense per kilowatt-hour for the Trenton Channel Power House of the Detroit Edison Company. For 1932 it

TABLE 13.—TRENTON CHANNEL POWER HOUSE PRODUCTION EXPENSE PER KILOWATT-HOUR OUTPUT, 12 MONTHS ENDING JUNE 30

Pulverized fuel, regenerative cycle, steam at 375 lb., 700°F., 50,000-kw. units

Item	1926	1927	1929	1930	1932	1934	1936	1938	1939
Production operation, cts.:									
Supervision, engineering	......	......	......	......	......	......	0.0111	0.0147	0.0161
Superintendence	0.013	0.010	0.013	0.012	0.018	0.013			
Wages	0.045	0.040	0.027	0.031	0.039	0.032	0.0289	0.0500	0.0601
Fuel	0.235	0.236	0.200	0.199	0.176	0.171	0.1907	0.1974	0.1991
Water	......	......	......	......	......	......	0.0006	0.0006	0.0006
Lubricants	0.001	0.001	0.001	0.001					
Supplies, expense	0.017	0.015	0.009	0.009	0.009	0.007	0.0057	0.0087	0.0090
Maintenance, cts.:									
Supervision, engineering	......	......	......	......	......	......	*	0.0033	0.0038
Station buildings	0.009	0.006	0.004	0.004	0.006	0.004	0.0032	0.0049	0.0062
Steam equipment	0.026	0.023	0.025	0.022	0.016	0.018			
Boiler plant	......	......	......	......	......	......	0.0154	0.0236	0.0225
Electric equipment	0.002	0.001	0.002	0.002	0.002	0.001			
Generating and electric equipment	......	......	......	......	......	......	0.0133	0.0094	0.0140
Miscellaneous equipment	0.002	0.004	0.003	0.002	0.001	0.001	0.0002	0.0003	0.0004
Total, cts.	0.350	0.336	0.284	0.282	0.267	0.247	0.2691	0.3129	0.3318
Kw.-hr. output, thousands	624,447	807,372	1,133,538	1,028,529	780,763	908,475	1,108,827	764,568	641,713
Maximum demand (30 min. kw.)	140,000	200,000	243,000	233,500	180,000	206,500	285,500	232,500	225,000
Average load, kw.	71,300	92,200	129,400	117,400	88,900	103,700	126,200	87,300	73,300
Load factor	0.509	0.461	0.532	0.503	0.494	0.502	0.442	0.375	0.326
Coal, lb. per kw.-hr.	1.18	1.16	1.10	1.10	1.06	1.05	1.03	1.04	1.05
B.t.u. per kw.-hr.	15,986	15,353	14,470	14,460	14,280	14,080	13,800	14,000	14,100
B.t.u. per lb. coal	13,594	13,265	13,150	13,110	13,500	13,400	13,500	13,500	13,500
Average cost of coal per ton burned, dollars†	3.99	4.07	3.65	3.61	3.33	3.27	3.68	3.77	3.77
Installed capacity, kw., December	150,000	250,000	300,000	300,000	300,000	300,000	300,000	300,000	300,000

* Included in Operation, supervision and engineering for 1935 to 1937. Federal Power Commission classification of accounts (adopted 1938) provides for separation between operation and maintenance expense.

† Does not include labor charges for handling and preparation.

required only 14,280 B.t.u. per kilowatt-hour instead of the 19,020 required at Connor's Creek. By 1938, load had begun to be shifted to the newer station at Connor's Creek, which

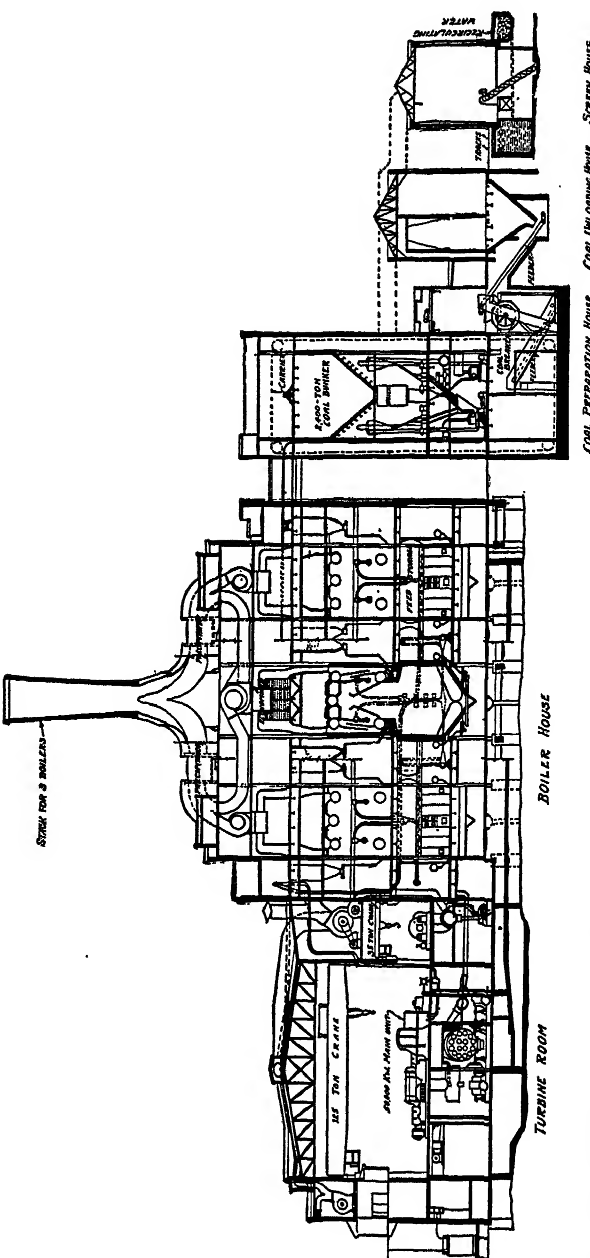


FIG. 19.—Trenton Channel Power House, section through plant looking north. Pulverized fuel. Each boiler has 29,343 sq. ft. of heating surface. Steam delivered at 375 lb., 700°F. (*Courtesy of Detroit Edison Company.*)

accounts for the reduced load factor for Trenton Channel for that year and the following ones.

Representing the latest thermal cycle adopted by the Detroit Edison Company, Table 14 shows the effect on the production expense of the first 75,000-kw. unit in Delray No. 3 Extension working with steam at 815 p.s.i. and 900°F.

TABLE 14.—DELRAY POWER PLANT PRODUCTION EXPENSE PER KILOWATT-HOUR OUTPUT AND OPERATING STATISTICS

Item	1935	1936	1937	1938	1939
Production, steam generation operation, cts.:					
Supervision and engineering	0.0196	0.0186	0.0210	0.0215	0.0140
Station labor	0.0649	0.0603	0.0709	0.0780	0.0453
Fuel	0.1921	0.1910	0.1927	0.1926	0.1777
Water	0.0009	0.0011	0.0012	0.0014	0.0009
Supplies and expense	0.0118	0.0106	0.0091	0.0090	0.0066
Maintenance, cts.:					
Supervision and engineering	*	*	*	0.0026	0.0014
Structures and improvements	0.0066	0.0086	0.0074	0.0090	0.0044
Boiler-plant equipment	0.0221	0.0233	0.0269	0.0263	0.0183
Generating and electric equipment	0.0051	0.0086	0.0190	0.0153	0.0091
Miscellaneous equipment	0.0002	0.0003	0.0002	0.0006	0.0003
Total	0.3233	0.3224	0.3484	0.3563	0.2780
Kw.-hr. output, thousands	559,093	599,765	557,254	503,243	908,444
Maximum demand (30 min.), kw.	145,000	145,000	140,000	200,000	212,500
Average load, kw	63,800	68,300	63,600	57,400	103,700
Load factor	0.440	0.471	0.454	0.287	0.488
Coal per kw.-hr., lb	1.04	1.04	1.04	1.04	0.96
B.t.u. per kw.-hr	14,100	14,100	14,100	14,000	13,000
B.t.u. per lb. coal	13,500	13,600	13,500	13,500	13,600
Average cost of coal per ton burned, dollars†	3.66	3.68	3.79	3.77	3.77
Installed capacity in kw., December	160,000	160,000	160,000	235,000	235,000

* Included in Operation, Supervision and Engineering for 1935 to 1937. Federal Power Commission classification of accounts adopted 1938 provides for separation between Operation and Maintenance Expense.

† Does not include labor charges for handling and preparation.

Note: One 75,000-kw. turbine placed in service in December, 1938, operates at 815 lb. pressure and 900 degrees steam temperature. All other turbines operate at 375 lb. pressure and 700 degrees steam temperature.

24. Fuel.—The first and most obvious operating cost in connection with a steam plant is the expenditure for fuel, with which

should be included the cost of delivery to the plant and the cost of removal of the ash, together with the expense of fuel analyses. In plants employing a large number of firemen, most of the firing labor may be conveniently included with the coal on a tonnage basis, on the approximately correct assumption that almost all

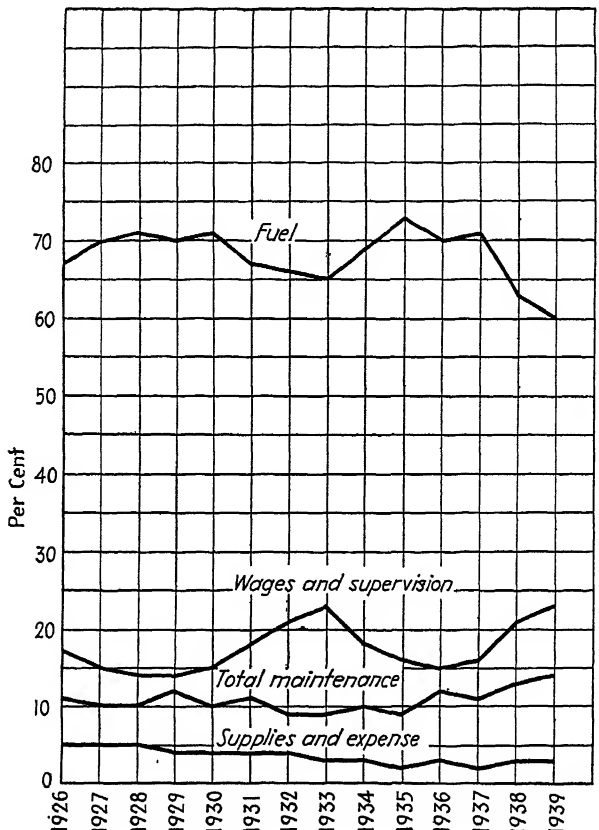


FIG. 20.—Operation and maintenance cost in per cent of total, Trenton Channel Power Plant of Detroit Edison Company.

the fireroom and ash-disposal labor will be proportional to the tons of coal fired. Tables 11, 12, and 13 show that fuel is by far the most important item in the operating expense. Even with the coal at $2.19 a ton in 1916, in Table 12 the fuel is 63.4 per cent of the total cost, whereas with coal at $3.77 in 1939, the fuel is 67.3 per cent of the total cost. Hence the importance of fuel economy is plainly evident.

It is interesting to note that of the amount per ton paid for coal, about 61 per cent goes for freight and 39 per cent to the coal operator. Thus there is but little hope for cheaper coal unless freight rates can be lowered. The artificial increase in the price of coal because of the Bituminous Coal Act of 1937 had not arrived in 1939 but began in 1940, since the minimum mine-mouth prices for all grades of soft coals were determined by the Bituminous Coal Division of the Department of the Interior effective as of Sept. 3. The prices range from 10 cents per ton for low-grade Indiana coal dust to $5.25 for high-grade lump coal in the state of Washington.

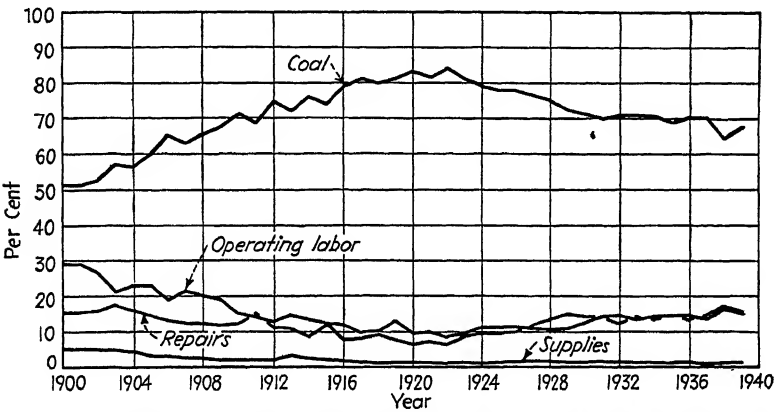


FIG. 21.—Detailed operating costs in per cent for four large Midwest power stations.

The latest power-station designs using mass production of steam in water-cooled furnaces of great size, with pulverized fuel and very high steam pressures, as later described under the heading Developments in Fuel-burning Plants, have been extremely effective in lowering this cost item. It is especially worthy of notice in Table 12 that in the 1932 data a saving of even 1 per cent in fuel is the equivalent of a saving of 2.2 per cent in wages and superintendence, or of 6.0 per cent in maintenance. Figure 20 shows the data of Table 13 in graphical form arranged in percentages of the total operating plus maintenance costs. Similarly, Fig. 21 shows the "Detailed Operating Costs in per Cent," 1900 to 1939, of four large Midwest stations.

The entire operating cost and particularly the fuel cost will depend upon the size of the plant and the plant factor. The

larger the turbines and boilers, the better the efficiency will be except at light load when the fuel consumption will increase per unit of energy generated, since both the turbines and the auxiliary apparatus are less efficient at part loads. The effect of variation of the plant factor on the investment and operating costs will be extensively developed in Chap. IV, Power Plant Load Curves.

25. Wages and Supervision.—The next item in order of expense is station labor, and it is the first item in order of real importance in the efficient operation of any plant. Station labor carries with it the expense of supervision resident in the station. This will vary with the character and size of the plant and with the breadth of managerial judgment. It is rather evident that a plant burning 500 tons of $5 coal a day will not have to show a very great improvement in efficiency to justify a highly paid plant superintendent.

Closely analogous to plant supervision is a portion of the overhead expense of the business, whether a power business as such or some other business to which a power plant is incidental. It is practically impossible to keep a considerable portion of the plant problems from coming to the attention of the manager of the enterprise, and this will be particularly the case if plant labor and supervision are not of a high order. If the manager is on a salary, the amount of his time called for by the plant should be charged to the plant, and not at the actual salary rate paid to him, but at his highest productive value elsewhere in the concerns of the business, be that other place the financial affairs of the concern, sales, manufacturing, or whatnot. The assumption is that no enterprise would employ a manager at a given salary unless his presence provided more profits to the concern than his actual wage unless, indeed, the manager is regarded as one of the nonproductive essentials of doing business at all. If a $15,000 manager is capable of saving or earning for the concern $30,000 a year by attention to certain other details, which he could attend to if relieved from attention to the power plant by a $5,000 plant superintendent, who might through being permanently resident in the plant do even better than could the manager himself, it is absurd to assume that there is any economy in saving on the cost of supervision.

Tables 11, 12, and 13 and Figs. 20 and 21 show wide variations in the cost of labor, both between the same plants for

different years and between different plants for the same year. Table 12 shows plainly the general rise in wages during the war period and the new higher level maintained after the war. In the new plants, the enormous sizes of the boilers and turbines together with the complete mechanical handling of coal and ash are reducing the number of operating men required but are calling for operators of higher skill and more training. A large plant has inherently a better labor efficiency than a small plant in that the former has work enough to keep each individual employed up to

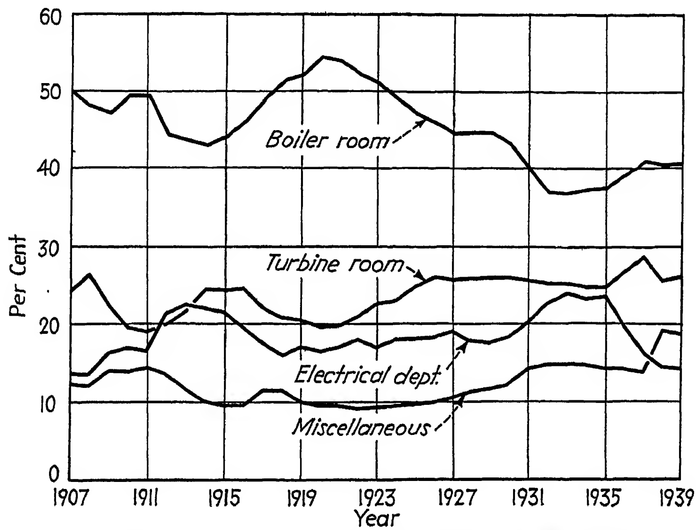


FIG. 22.—Operating labor costs in per cent for four large Midwest power stations. (In 1938 there was a change in the allocation of certain expenses.)

his normal rate all the time, whereas the latter will not achieve such a result. Unless the light load periods of the 24 hr. are of sufficient duration to allow of reducing the crew during standard operating shifts, the labor cost will not be greatly affected by a moderate change in load factor, since the number of men will be determined by the peak load.

Considering the operating labor alone, Fig. 22 gives the division of operating labor costs in per cent. The boiler labor shown in the graph includes labor for coal handling and unloading, handling of ash, and the boiler-room engineers, water tenders, firemen, and cleaners. The turbine-room labor includes the engineers and operators. The increase in the labor cost here is due to the

increased complexity of the equipment in the new stations. In the electrical department are included the shift electricians, the switchboard operators on both the main and auxiliary power boards, the switchhouse attendants, etc. The miscellaneous labor includes the clerks, janitors, watchmen, and other unclassified labor. It is worthy of note that almost one-half of the operating labor cost is in that part of the plant which has to do with the handling and burning of fuel and the making of steam.

TABLE 15.—THE DETROIT EDISON COMPANY TRENTON CHANNEL VS. CONNOR'S CREEK POWER HOUSE PRODUCTION EXPENSE PER KILOWATT-HOUR OUTPUT 12 MONTHS ENDING DEC. 31
(In Cents)

Item	Connor's Creek 1928		Trenton Channel 1928		Trenton Channel 1933	
Production						
Operation:						
Superintendence		0.013		0.012		0.016
Wages						
Coal handlers	0.006		0.007		0.007	
Firemen	0.011		0.006		0.007	
Care of boilers; Economizers and heaters	0.004		0.001		0.002	
Ash handlers	0.002				0.001	
Turbine engineers	0.010		0.005		0.006	
Care of condensers and screens	0.002					
Switchboard operation	0.005		0.004		0.003	
Care of electrical apparatus	0.002		0.001		0.002	
Routine testing	0.001		0.001		0.001	
Care of buildings	0.004		0.002		0.004	
Care of grounds	0.003		0.001		0.001	
Miscellaneous	0.003	0.053	0.002	0.030	0.003	0.037
Fuel		0.272		0.205		0.161
Water						
Lubricants		0.001		0.001		0.001
Station supplies and expense		0.008		0.012		0.008
Maintenance:						
Station buildings		0.015		0.004		0.004
Steam equipment		0.019		0.027		0.015
Electrical equipment		0.003		0.002		0.001
Miscellaneous equipment		0.002		0.002		0.001
Total		0.386		0.295		0.244
Kilowatt-hour output	725,651,700		1,024,565,400		765,643,900	
Capacity kw	180,000		250,000		300,000	
Plant in commission	April, 1915		July, 1924		July, 1924	
Type of firing	Stokers		Pulverized fuel		Pulverized fuel	
Boiler heating surface, sq. ft.	23,630		29,087		29,343	
Turbine units, 1,000 kw	20's, 30's and 45's		50's		50's	

As indicative of the change in labor costs resulting from the use of a newer and larger power station, Table 15 shows in great detail the production expense per kilowatt-hour of Connor's Creek Station as compared with that of Trenton Channel Station. Both plants are worked to capacity at approximately the same load factor. If we eliminate the item of fuel cost, for 1928 Connor's Creek shows 0.114 ct. per kilowatt-hour as compared with 0.090 and 0.083 ct. per kilowatt-hour for Trenton Channel, a saving of 21 to 27 per cent. For total wages, Connor's Creek gives 0.053 ct. per kilowatt-hour, whereas Trenton Channel has 0.030 and 0.037 ct. per kilowatt-hour, a saving of 30 to 43 per cent.

A small detail in the labor cost is the aggregate cost of accidents, including accident insurance or compensation paid to injured employees, medical care, the adjustment of accident claims and legal expenses, together with the prorata share of so-called welfare work undertaken for the employees.

26. Oil, Waste, and Supplies.—Oil, waste, boiler compound, materials for water purification, boiler tubes, and fire bars are included under the general heading of Supplies, each one of which is in itself comparatively small but which in the aggregate are quite appreciable.

27. Repairs.—Repairs are more or less sporadic and with proper inspection and maintenance can be minimized by the proverbial stitch in time, though even with the best of inspection and maintenance certain extraordinary accidents are bound to occasion some repairs.

Figure 23 shows the division of the Maintenance Costs given in the Operation and Maintenance graph of Fig. 20 of the Trenton Channel Station, with its component parts of maintenance for buildings, steam equipment, and electrical equipment expressed as a percentage of total operating and maintenance cost. It is notable that the steam equipment takes about 70 per cent of the total maintenance.

The Repair Costs in per cent for four large Midwest power stations are given in Fig. 24.

Each of the details entering into the operation carries with it its own burden of overhead expense, such as the office force employed in connection with the accounts of the business, the cost of operating the purchasing department, interest on the stores,

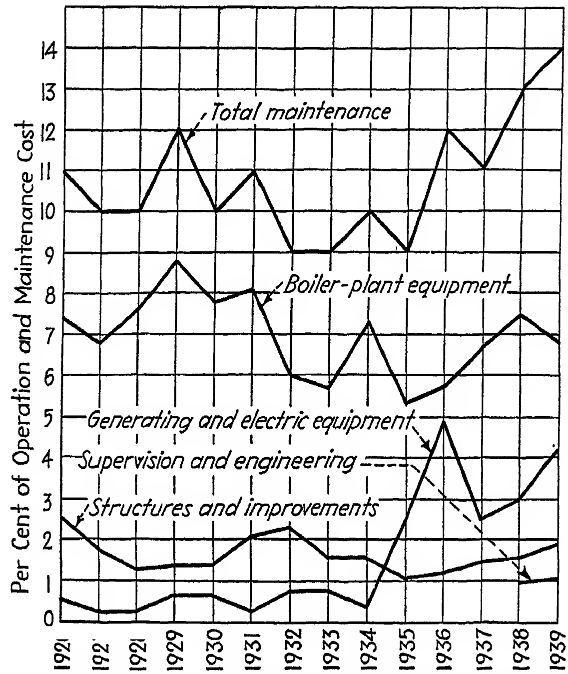


FIG. 23.—Maintenance costs in percentage of the total operation and maintenance, Trenton Channel Station, Detroit Edison Company.

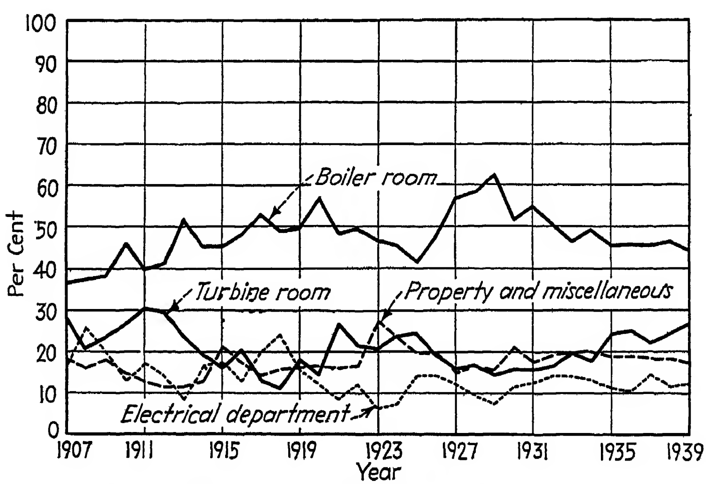


FIG. 24.—Repair costs in per cent for four large Midwest power stations.

stock, and handling materials in the stores. In a well-managed property, these overhead expenses should add a very small percentage to the various supplies entering into plant operation,

TABLE 16.—COSTS IN WATER-POWER SYSTEMS[1]

Item	Wisconsin Valley Electric Company					
	1912	1917	1922	1927	1932	1937
Kilowatt-hours generated, thousands	5,362	13,568	26,549	45,828	54,396	56,342
Capacity, kilowatts	1,800	2,890	6,712	12,552	13,504	12,902
Kilowatt-hours generated per kw. capacity	2,979	4,786	3,956	3,651	4,028	4,367
Superintendence and labor	0.124	0.098	0.109	0.069	0.062	0.077
Water for power	0.018	0.031	0.030	0.031	0.035	0.047
Miscellaneous supplies and expenses	0.017	0.011	0.013	0.007	0.010	0.011
Total operation	0.159	0.140	0.152	0.107	0.107	0.135
Total maintenance	0.090	0.053	0.043	0.013	0.019	0.022
Total generation expense	0.249	0.193	0.195	0.120	0.126	0.157
Conversion and storage	.....	......	0.018	0.014	0.011	
Transmission	0.001	0.006	0.012	0.027	0.031	
Cost to distribution system	0.250	0.199	0.225	0.161	0.168	
Distribution	0.144	0.106	0.151	0.129	0.105	
Cost of energy distributed	0.394	0.305	0.376	0.290	0.273	
Utilization	0.050	0.032	0.021	0.028	0.034	
Commercial	0.061	0.022	0.077	0.110	0.067	
New business	0.198	0.116	0.006	0.055	0.065	
General and miscellaneous	0.365	0.272	0.340	0.357	0.216	
Total expense before fixed charges	1.068	0.747	0.820	0.840	0.655	
Taxes	0.103	0.142	0.312	0.293	0.228	
Total expense before depreciation and interest	1.171	0.889	1.132	1.133	0.883	
Depreciation	0.212	0.313	0.303	0.282	0.225	
Interest, 5%	0.425	0.625	0.605	0.565	0.451	
Grand total cost per kw.-hr, cts.	1.808	1.827	2.040	1.980	1.559	

[1] Courtesy of Wisconsin Public Service Commission.

and therefore no attempt need be made to apportion these individually to the different classes of materials used. It will be quite satisfactory to use a general percentage of loading to

cover such expense, even though thereby one item may be unnecessarily burdened to the benefit of another.

28. Operating Costs of Hydroelectric Plants.—The complete generation, transmission, distribution, and overhead costs for a Wisconsin plant is shown in Table 16, Costs in Water Power Systems.

The wholesale cost of power supplied by the Hydro-electric Power Commission of Ontario in its major system is indicated in the annual report for the year ended Oct. 31, 1939, as follows:

Niagara System, Generated and Purchased, Fiscal Year 1938–1939

Generating plants	Maximum normal plant capacity Oct. 31, 1939, hp.	Peak load during year 1938–1939, hp.	Total output during fiscal year 1938–1939, kw.-hr.
Queenston-Chippawa-Niagara River..	500,000	494,638	2,273,928,000
"Ontario Power"—Niagara River....	180,000	176,944	680,430,000
"Toronto Power"—Niagara River...	150,000	136,059	280,146,000
Chats Falls (Ontario half)—Ottawa River..........................	108,000	114,611	342,874,500
DeCew Falls—Welland Canal........	50,000	46,917	137,088,000
Steam Plant—Hamilton.............	24,000	8,311	21,600
Total..........................	....	....	3,714,488,100

Statement of Operations and Cost of Power, Year Ended Oct. 31, 1939. Niagara System

Municipalities, Rural Power Districts, Companies and Local Distribution Systems

Cost of power purchased.....................	$ 6,800,716
Operating maintenance and administrative expenses...............................	4,100,922
Interest.................................	9,175,389
Provision for renewals......................	1,417,262
Provision for sinking fund...................	2,188,781
(40-year basis to liquidate capital liabilities)....	
Total cost...............................	$23,683,070

The charges for power supplied by the Commission to the various municipalities vary with the amounts of power used, the distances from the sources of supply, and other factors. The entire capital cost of the various power developments and transmission systems is annually

allocated to the connected municipalities and other wholesale power consumers, according to the relative use made of the lines and equipment. . . . Included in the municipality's remittance to the Commission for the wholesale cost of power—besides such current expenses as those for operation and maintenance of plant, for administration, and for interest on capital—are sums required to build up reserves for sinking fund for renewals, for contingencies and obsolescence, and for stabilization of rates."[1]

Under the Rural Hydro-electric Distribution Act, the Treasurer of Ontario may pay to the commission or municipal corporation 50 per cent of the capital cost of lands and works for the supply of power up to the point of delivery.

The power operating costs for the Tennessee Valley Authority are detailed in the annual report for the year ended June 30, 1939, as follows:

Direct hydraulic production expense, Schedule D,

Wilson Dam, 184,000 kw. installed, generated, 669,895,100 kw.-kr.

	Amount	Mills per kw.-hr.
Operation:		
Supervision and engineering	$ 17,636.10	
Hydraulic labor	672.16	
Prime mover and generator labor	39,927.15	
Electric labor	24,176.03	
Miscellaneous station labor	9,044.95	
Lubricants	760.09	
Station supplies	1,289.35	
Station expenses	7,880.33	
Total operation	$101,386.16	0.15
Maintenance:		
Supervision and engineering	5,443.51	
Structures and improvements	7,375.28	
Reservoirs, dams, and waterways	1,483.19	
Prime movers and generators	20,274.76	
Accessory electric equipment	5,107.06	
Miscellaneous power plant equipment	2,306.44	
Total maintenance	41,990.24	0.06
Depreciation of power facilities	265,246.47	0.40
Total	408,622.87	0.61

[1] Commission report, 1940, p. 104.

Norris Dam, 100,800 kw. installed, generated.......	376,249,000 kw.-hr.
Total direct hydraulic production expense........	0.46 mills per kw.-hr.
Wheeler Dam, 64,800 kw. installed, generated......	352,332,000 kw.-hr.
Total direct hydraulic production expense........	0.73 mills per kw.-hr.
Pickwick Dam, 72,000 kw. installed, generated......	332,670,470 kw.-hr.
Total direct hydraulic production expense........	0.78 mills per kw.-hr.

SUMMARY OF PRODUCTION EXPENSE

	Direct production expense			Common expense*	Total
	Hydraulic	Other	Total		
Kw.-hr. generated.......	1,731,146,570	10,537,868	1,741,684,438		
Operation...............	$ 283,852	$10,391	$ 294,243	$163,719	$ 457,962
Maintenance............	96,571	2,565	99,136	22,679	121,815
Depreciation............	718,636	2,428	721,064	232,315	953,379
Miscellaneous...........	....	19,984	19,984	....	19,984
Total production expense.	1,099,059	35,367	1,134,426	418,712	1,553,139
Mills per kw.-hr.........	0.63	3.36	0.65	0.24	0.89

* Maintenance and depreciation allocated in accordance with Authority's allocation of common investment, *viz.*, 36 per cent to navigation, 24 per cent to flood control, and 40 per cent to power.

Details of power operating costs year ending June 30, 1939:

Kw.-hr. generated..................	1,741,684,438 kw.-hr.
Total production expense...........	$1,553,139
Transmission expense...............	1,283,449
Distribution expense................	122,523
Customers' accounting and collection expense........................	39,021
Sales promotion expense............	76,998
General and administration expense charged to power................	746,012

The rates charged and the revenues received are reported in the pamphlet, "T.V.A., Its Work and Accomplishments," 1940, and summarized on page 47, as follows:

Municipalities and cooperatives which purchase only primary power paid slightly more than 5 mills per kilowatt-hour. The average price for municipalities alone was approximately 4.9 mills, that for cooperatives 5.9 mills.

Industrial customers paid an average price of 2.78 mills per kilowatt-hour for power which included large quantities of inferior grades of so-called secondary and interruptible energy. Private utilities pur-

chasing some primary and substantial amounts of secondary, interruptible, and dump power paid an average of 2.92 mills per kilowatt-hour.

For the rates charged for Boulder Dam power see Sec. 20.

The first rates scheduled for Bonneville were for firm power $17.50 per kilowatt-year along the transmission lines and $14.50 per kilowatt-year within a 15-mile zone at the dam site. It was planned to amortize over a period of 40 years and to return to the government 3½ per cent interest on its investment on that part of the project allocated to power, *i.e.*, 57 per cent of total cost completed.[1] Tax payments were not considered. New supplementary rates approved by the Federal Power Commission in 1939 permit the sale of power upon a monthly basis, at a cost of ¼ ct. per kilowatt-hour, plus a demand charge of 75 cts. per month per kilowatt of demand. There is a minimum billing under which a customer is never billed for less than 75 per cent of the highest demand charge created in any of the preceding twelve months.[2]

A very extensive study of cost data for hydroelectric plants is presented in Prof. Barrows' paper, "Hydro-generated Energy," in the *Proceedings* of the A.S.C.E. for April, 1938. This covers 57 plants ranging from 400 to 252,000 kw., with heads from 13 to 2,561 ft., plant costs from $95 to $479, and energy costs from 0.16 to 3.74 cts. per kilowatt-hour with yearly capacity factors of 0.10 to 0.94.

29. Developments in Hydro Plants.—From Boyden's design of a 75-hp. reaction turbine for Lowell, Mass., in 1844, and the development of the impulse wheel in California from 1883 on, hydraulic turbines have grown in size and improved in efficiency, until at the present time any normal power requirement can be satisfactorily supplied. In reaction wheels, three units of 70,000 hp. each were installed in 1925 by the Niagara Falls Power Company to operate under 213.5 ft. net effective head at 107.1 r.p.m. As shown in Fig. 81, these turbines developed a maximum efficiency of 93.8 per cent on test, which with a generator efficiency of 98.1 per cent gave 92 per cent combined efficiency for the set. Each of the turbines has carried a load of nearly 84,000 hp. The Boulder Dam plant will install 15 turbines of 115,000 hp. (see

[1] See *Elec. World,* Feb. 19, 1938.
[2] See *Elec. World,* Sept. 23, 1939.

Fig. 12), 150 to 180 r.p.m., to operate under an average head of 530 ft. (maximum 590 ft., minimum 420 ft.), and the Grand Coulee project is planned for 18 turbines each of 150,000 hp. (see Fig. 13) under an average head of 330 ft. for the high dam.

In impulse wheels, a unit of 56,000 hp. was installed in 1929 in the Big Creek No. 2-A plant of the Southern California Edison Company to operate at 250 r.p.m. under 2,200 ft. effective head.

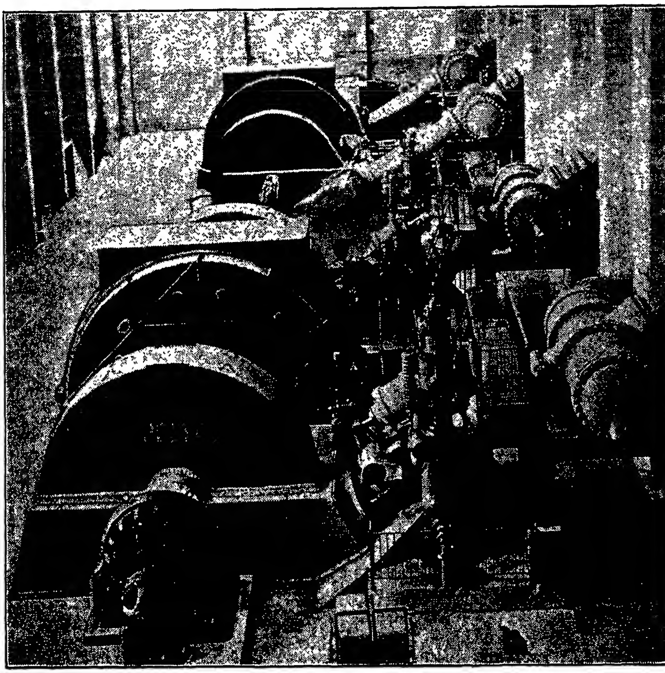


Fig. 25.—Interior view of Big Creek 2-A power house; 56,000 hp. Pelton impulse wheel, 2,200-ft. head, 250 r.p.m. Direct-connected exciter. All main and relief nozzles accessible from galleries. (*Courtesy of Pelton Water Wheel Co.*)

This Pelton double overhung wheel, shown in Figs. 25 and 26, has developed 70,000 hp. during peak-load periods. In efficiency, the impulse-wheel installations have reached 87 per cent, whereas the reaction turbines have reached values around 94 per cent at their best point, together with a marked improvement in developing runners of high specific speed. In view of such splendid performance, any great improvement in efficiency for the hydraulic turbine seems hardly possible, only fractions of a per cent may be gained here and there at the point of best

efficiency. Future efforts should be directed more toward the improvement of efficiency for the conditions of head and loading other than those at the point of maximum efficiency. Along this line, the recent adoption of the Kaplan turbine with its flat efficiency curve (Fig. 27) has improved low-head developments.

The largest Kaplan turbines are those at Bonneville, which has 66,000-hp. units designed to operate under heads varying from 30 to 69 ft. Two additional 74,000-hp. units for this project are under construction.[1]

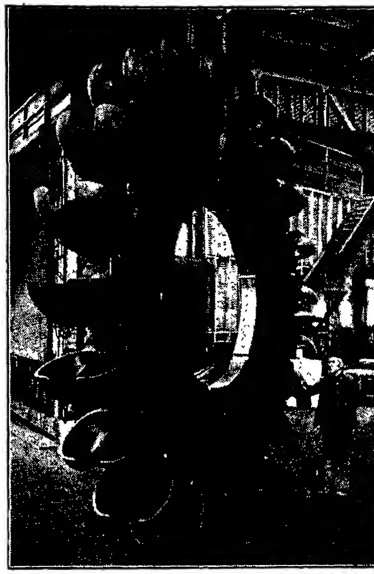


FIG. 26.—Runner for Big Creek Station, 56,000 hp. Pelton wheel, 16-ft. diameter, weight 24 tons, forged-steel ring with cast-steel buckets. Interior bore fits over cast-steel hub attached by six tapered steel bolts. (*Courtesy of Pelton Water Wheel Co.*)

With regard to water-power plants as a whole, however, endeavor should be made to coordinate better flood and pondage control and to reduce the investment cost per kilowatt so that the fixed charges will compare more favorably with those of steam power. The present universal trend to link each new station by a transmission network to the other stations of the same power system increases the total amount of economically installed capacity beyond the formerly accepted limits. The combined sources can then be operated so that such additional hydro capacity can be utilized to render firm peak service to its power system.[2] In addition, such connection makes it possible to use large amounts of the extra flow available part time during the year, the secondary power, and thus tremendously enhance the economic value of the development.

[1] See Hydro-electric Power-plant Design, by A. T. Larned, *Mech. Eng.*, August, 1939.

[2] See ALLNER, F. A., Economic Aspects of Water Power, *A.I.E.E. Trans.*, March, 1933.

1. New constructions in the United States.

Reference to Table 9, Federal Hydro Power Projects, will show the tremendous extent of the water-power expansion in the past few years, in which additional capacity has been installed at the rate of about 500,000 kw. per year. The growth promises to continue for some time to come, as projects now under construction reach completion. Particularly notable for the tremendous size of its dam and its enormous total power capacity of 1,944,000 kw., the Grand Coulee project will serve for power, river regulation, and irrigation. In addition to the governmental

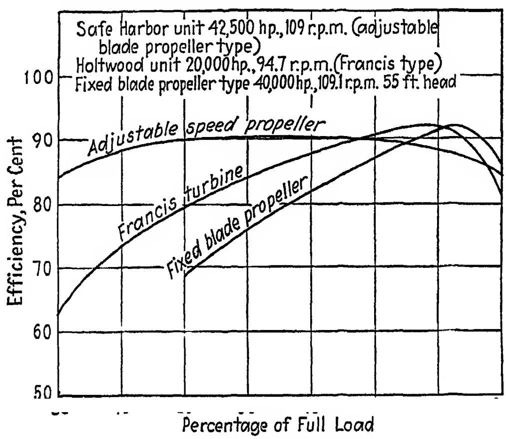


FIG. 27.—Comparison curves for various types of turbines. ("*Safe Harbor Kaplan Turbines*," *by L. M. Davis and G. W. Spaulding, A.I.E.E., October*, 1932.)

program, many projects have been developed by utility and industrial companies, particularly in the Northeastern and Midwestern sections of the United States. For 1937 and 1938, some sixty such projects represented 500,000 hp. These included the four 26,000-hp. units for 110-ft. head in the Clayton development of the Appalachian Electric Power Company on the New River, Virginia, and the 77,000-hp. unit for 188-ft. head in the Kerr power house of the Rocky Mountain Power Company on the Flathead River in Montana.[1]

Several of the recent plants are of special interest.

a. Safe Harbor Power Plant, on the Susquehanna River.

Seven units each of 42,500 hp. at 109.1 r.p.m. under an effective head of 55 ft. are installed (1940) of an ultimate

[1] See Progress in Power Generation, *Elec. Eng.*, January, 1940.

design for 12 units. Because of the wide range in flow and head (36 to 57.5 ft.), the limited storage and the prime importance of the peak load value of this plant, the turbines are automatically adjustable Kaplan machines. The relative flatness and percentage efficiencies of the Kaplan, the fixed-blade propeller, and the

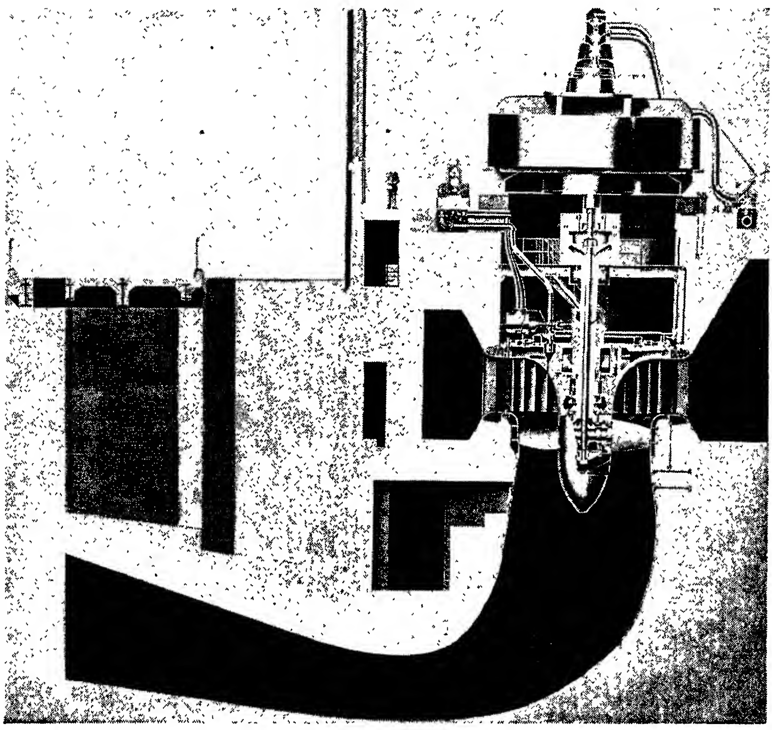


FIG. 28.—Vertical section of Safe Harbor Kaplan turbine, 42,500 hp., 36- to 57.5-ft. head, 109.1 r.p.m. (*Courtesy of S. Morgan Smith Co.*)

Francis-type turbine are shown in Fig. 27. A vertical section of one of the units is shown in Fig. 28, and Fig. 29 shows a cross section of the power house.

The hydraulic and electrical design anticipates the dual use of the units for generation and regenerative pumping, which will be feasible because the Safe Harbor turbines discharge directly into the pond formed by the Holtwood Dam 8 miles downstream. The original

cost of the plant was set at $27,919,516 as of Dec. 31, 1937, by the Federal Power Commission.[1]

b. The Bagnell plant on the Osage River near Bagnell, Mo., has six Francis turbines each of 33,500 hp. under a 90-ft. head (1932). The design is notable for the omission of the power-house superstructure, the generators being

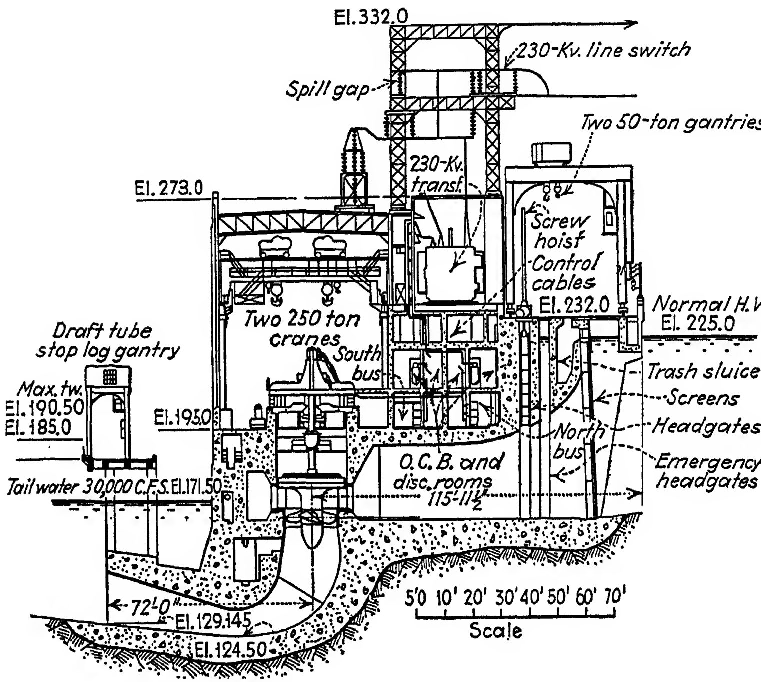


FIG. 29.—Cross section of Safe Harbor power house. (*Courtesy of Pennsylvania Water and Power Co.*)

covered with metal housings, and for a storage capacity of 1,200,000 acre-ft. of water.

c. The Diablo plant of the city of Seattle, built in the Skagit River, has two reaction turbines each of 90,700 hp. under 327-ft. head to operate at a speed of 171.5 r.p.m. (1931).

2. In Canada, 98 per cent of central station output is generated in hydroelectric plants and some very large projects have been constructed, the following plants being especially worthy of note:

[1] See *Elec. World*, Apr. 6, 1940.

a. Beauharnois on the St. Lawrence River, 80-ft. head, with power house designed for fourteen 50,000-hp. units, four units installed initially.
b. Chute-à-Caron on the Saguenay River, 151-ft. head, with four units each of 65,000 hp. installed (1932).
c. Abitibi Canyon on the Abitibi River, 237-ft. head with five units installed of 66,000 hp. each. Tests on four of the units gave maximum turbine efficiencies of 93.6 per cent.[1]
d. Chats' Falls on the Ottawa River, 53-ft. head, with eight units each of 28,000 hp. installed (1933). These turbines use Moody high-speed propellers with six fixed blades to operate under a head of 38 to 58 ft. Actual operating results for weekly periods have shown an over-all plant efficiency as high as 83 per cent.

For an international survey of hydroelectric developments, the reader should refer to the report under that title of the A.I.E.E. Committee on Power Generation, presented at the Summer Convention, June, 1934.

3. Automatic operation:

In addition to this imposing list of recent projects, many of the older small plants have been rehabilitated and converted to automatic operation. A typical case in point is the 10,000-kw., three-unit Borel plant of the Southern California Edison Company.

The remodeling and automatic control have saved approximately 44 per cent in the annual operating expense and at the same time increased the generation by 24 per cent during a normal water year. All the 19 stream-flow plants of this company, of a total capacity of 93,000 kw., are now under automatic or semi-automatic control.[2]

An outstanding example of automatic load control for two 22,500-kw. units, automatic system frequency control and automatic change of one generator to condenser action is embodied in the Morony plant of the Montana Power Company, which has been in commercial service for some years.[3] In 1931, the

[1] See Hydro-electric Practice in Canada, by T. H. Hogg, *A.S.M.E.*, September, 1936.

[2] See *Elec. World*, Dec. 31, 1932.

[3] See *Elec. World*, Apr. 18, 1931.

Hydro-electric Power Commission of Ontario built the three-unit Alexander plant on the Nipigon River for complete automatic operation. The total capacity of the plant is 54,000 hp.

For a discussion of the technical features involved in this type of plant, the reader should refer to the N.E.L.A. *Report* "Automatic and Partial Automatic Hydro-electric Plants" of August, 1930.

4. Pumped storage:

For developments in the utilization of storage to improve flow conditions and increase firm power and for the use of pumped storage, the reader is referred to Sec. 71.

30. Developments in Fuel-burning Plants. Steam.—The history of the fuel-burning plants in the electric light and power

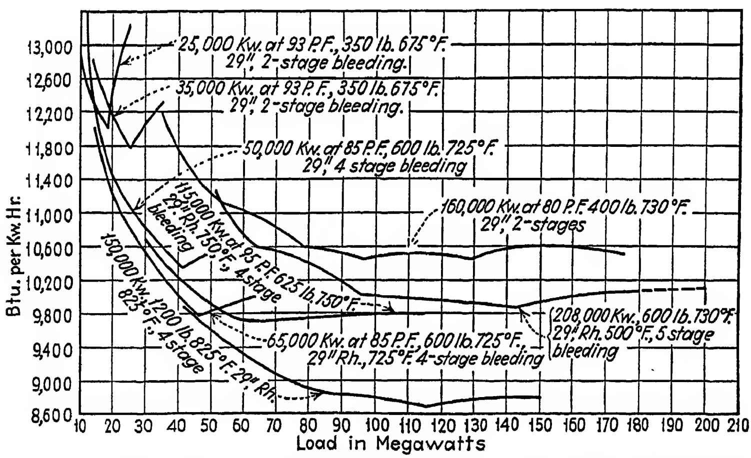


FIG. 30.—Heat rates for various sizes of turbines. ("*Economic Considerations in Application of Modern Steam Turbines,*" *by A. G. Christie, Mech. Eng., July,* 1930, *and E.E.I. Pub.* 3, *Turbines, August,* 1933.)

industry is one continuous story of increases in the capacity of its prime-mover sets together with marked gains in the thermal efficiencies. For the 20-year period following the building of Edison's 1,200-hp. Pearl Street plant in New York in 1882, the prime mover was the steam engine. In these two decades, the engine-generator set developed from Edison's early unit of 185 hp. to the tremendous Manhattan type, duplex vertical-horizontal compound unit of 8,000 hp., in the 59th Street Station of the Interborough Rapid Transit Company.

With the advent of the steam turbine, however, the engine was rapidly superseded as a large prime mover in electric stations. The decisive advantages of lesser floor space, high economy under wide range of loads, ease of application with superheated steam, freedom from oil in the condensate, and the uniform angular velocity were so compelling that the change was made very quickly. Following the initial 5,000-kw. unit installed by the Commonwealth Edison Company in 1903, the size increased to 8,000 kw. in 1906, to 14,000 kw. in 1908, to 20,000 kw. in 1911, to 35,000 kw. in 1915, to 45,000 kw. in 1917, to 60,000 kw. in 1924. From 1925 on, the generators were sometimes developed in double- and even triple-shaft machines as well as in single-shaft units, the increasing sizes in the last classification being indicated

TABLE 17.—AVERAGE CONSUMPTION OF FUEL PER KILOWATT-HOUR BY POWER PLANTS IN THE UNITED STATES*

Year	Consumption of fuel, equivalent short tons	Production by fuel, kw.-hr.	Lb. per kw.-hr.
1920	46,154,000	27,228,000,000	3.39
1921	34,916,000	25,864,000,000	2.70
1922	37,770,000	30,216,000,000	2.50
1923	43,306,000	36,088,000,000	2.40
1924	42,687,000	38,806,000,000	2.20
1925	45,431,000	43,268,000,000	2.10
1926	46,107,000	47,289,000,000	1.95
1927	46,001,000	50,001,000,000	1.84
1928	46,471,000	52,808,000,000	1.76
1929	52,639,000	62,295,000,000	1.69
1930	50,636,000	62,513,000,000	1.62
1931	47,113,000	60,791,000,000	1.55
1932	36,698,000	48,931,000,000	1.50
1933	37,151,000	50,546,000,000	1.47
1934	41,832,000	56,914,000,000	1.47
1935	43,198,000	59,176,000,000	1.46
1936	51,987,000	72,188,000,000	1.44
1937	55,142,000	76,883,000,000	1.43
1938	50,555,000	71,525,000,000	1.41
1939	59,514,000	85,800,000,000	1.39

* From Electric Power Statistics, by Federal Power Commission, May, 1940.

by 90,000 kw. in 1928, 160,000 kw. in 1929, and 165,000 kw. in 1934. Figure 30 shows typical heat rates for various sizes of turbines under their particular operating conditions. As better materials become available, the improved designs utilize higher blade speeds which result in a great saving of weight and reduction in the size of the turbine.

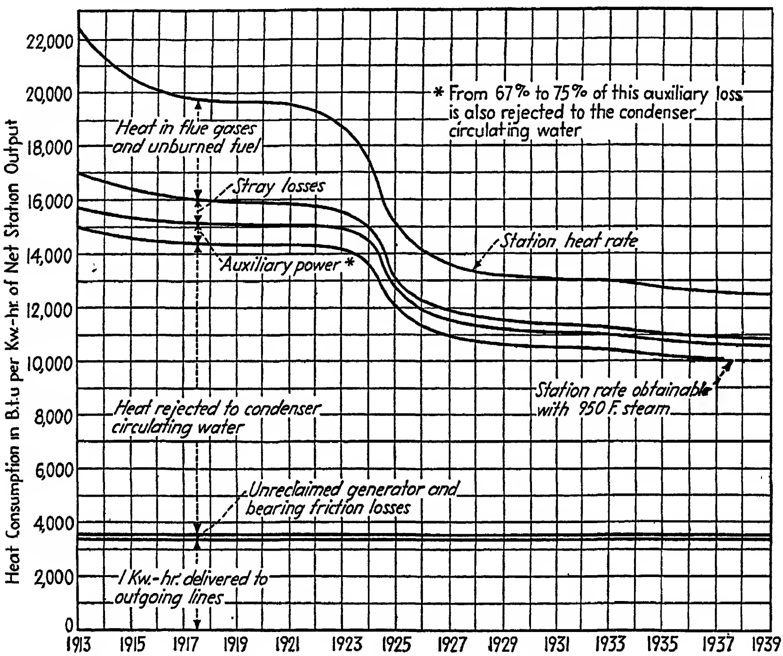


FIG. 31.—Average performance of 50 typical stations, 60,000-kw. capacity and higher, plotted against dates of initial operation. (*Courtesy of Frank S. Clark, consulting engineer, Stone & Webster.*)

The gain in thermal efficiency for power plants as a whole is well shown by the pounds of fuel per kilowatt-hour in Table 17.

The Commission reports, "There was practically no change in the efficiency of the use of fuel by the most efficient steam plant reporting to the Commission. During 1939, 10,770 B.t.u. were consumed per kilowatt-hour in the best such plant operating exclusively for generating electric energy as compared with 10,789 B.t.u. in 1938."

It is interesting to compare the average performance of the whole industry for the various years, as given above, with that of individual stations in Tables 11, 12, 13, and 14.

Fig. 32.—State Line Station, 208,000-kw. turboalternator; 76,000-kw. high-pressure unit in center, steam at 650 lb., 730°F. It exhausts through adjoining reheaters which deliver 110-lb. pressure, 500°F. steam to the two double-flow 62,000-kw. low-pressure units. All generators 22 kv., 1,800 r.p.m. Each low-pressure turbo has four vertical condensers. Five stages of feed-water heating. (*Courtesy of State Line Generating Co.*)

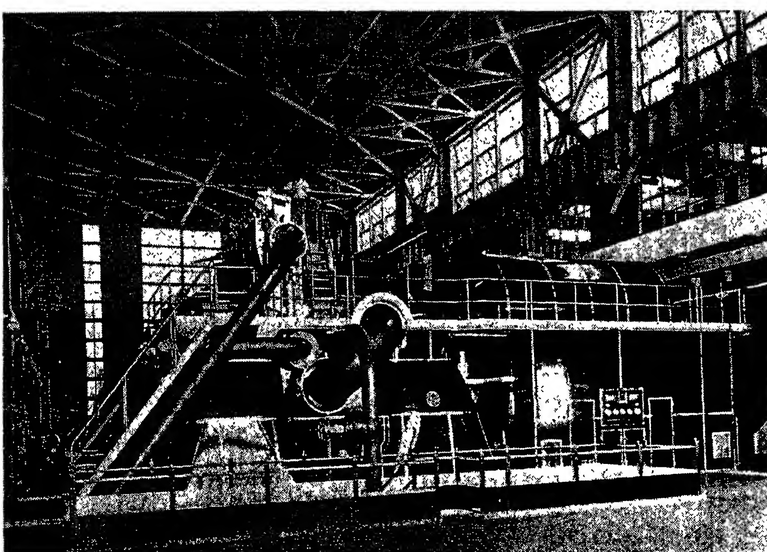


Fig. 33.—Third General Electric vertical-compound steam turbine generator set, 110,000 kw., 1200 lb., 900°F. with hydrogen-cooled generator. In River Rouge plant of the Ford Motor Co.

The trend of American station performance is shown in Fig. 31, which gives the average heat consumption for typical stations of 60,000 kw. capacity and higher, plotted against the dates of initial operation of the stations. The way in which the improvements in the boiler plant and turbine plant have contributed to reduce the over-all fuel consumption is notable. The improvement in the turbine-plant performance has been mainly the

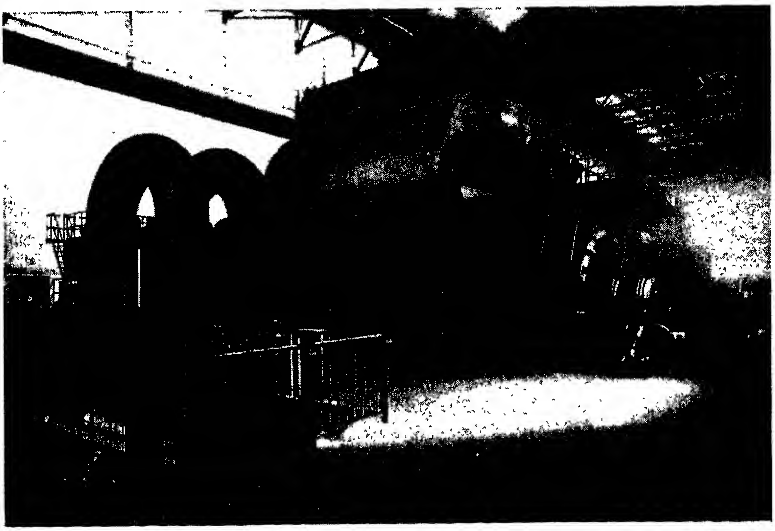


Fig. 34.—160,000-kw., two-cylinder tandem-compound turbogenerator unit, 400 lb., 730°F., three phase, 60 cycle, 16.5 kv., 1,800 r.p.m. (*Courtesy of Brooklyn Edison Co.*)

result of the reduction shown in the amount of heat rejected to the condenser circulating water.

It is evident then that high-pressure high-temperature plants show decided thermal advantages and that the steam cycle has not yet reached its economic limit. Dr. Gaffert[1] has made a study of advanced cycles and estimated the turbine performances shown in Fig. 35, the cycles being based upon the following assumptions:

Over-all efficiency ratio for 50,000-kw. steam turbine, 82 per cent. Maximum of 11 per cent moisture content in exhaust at full load. Terminal difference for feed-water heaters as follows:

[1] Of Sargent & Lundy, Engineers, Chicago.

Feed-water Temperature, °F.	Terminal Difference, °F.
65 to 230	5
230 to 300	10
300 to 400	15
400 to 525	20

Boiler efficiency, including furnace, superheater, air heater, economizer, and reheater, if used; 85 per cent. Pressure drop

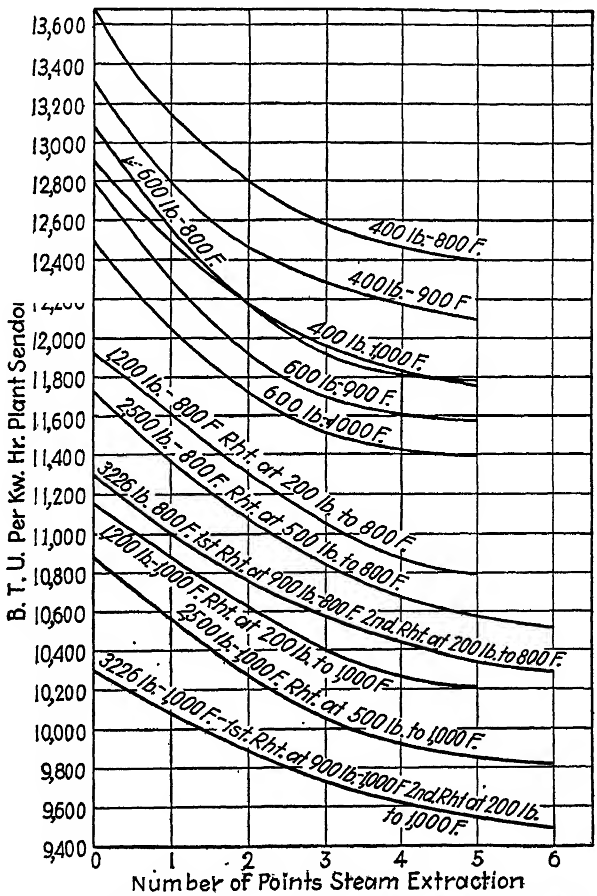


FIG. 35.—Plant-performance steam cycles. (*Dr. Gaffert, A.S.M.E. Trans., October*, 1934.)

between boiler and turbine, bleed points and their respective heaters, reheater piping and reheater of 10 per cent.

Radiation loss of 2 per cent from bleed point to heater and 3 per cent for reheating lines.

Appropriate auxiliary power and efficiencies of pumps and motors. Auxiliary power for a pulverized-fuel system of 20 kw.-hr. per ton of coal prepared and fed to boiler. Feed water heated in equal temperature steps to a maximum of 75 to 80 per cent of saturation temperature corresponding to throttle pressure when the most economical number of feed-water heaters are employed.

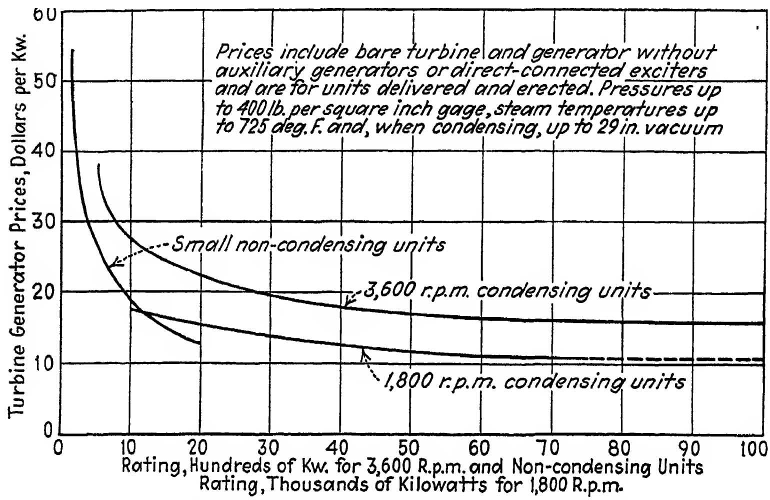


FIG. 36.—United States prices for turbogenerator units. ("*Pressures, Temperatures, Reheat Increase*," *by A. G. Christie, Elec. World, Feb.* 14, 1931.)

In Europe, high-pressure plants have been in service for some time past with installations operating at 815, 850, 1,100, 1,500, 1,700, 1,900, and 3,200 lb. pressure using boilers of the Benson, Loeffler, and Schmidt-Hartmann type. Two of the 3,200-lb. Benson boilers at the Siemens-Schuckert Werke in Berlin have capacities of 55,000 and 100,000 lb. per hour, respectively, and one at the Langerbrugge Station, Belgium, has a maximum capacity of 300,000 lb. per hour.[1]

In 1933, the Trebovice Station in Czechoslovakia began operation with steam at 1,849 lb. and 932°F., supplied to two 21,000-kw. Skoda three-cylinder turbines operating on a reheat-regenerative cycle. The boiler plant consists of three Loeffler boilers,

[1] See Foreign Developments, *N.E.L.A. Rept.* 131, May, 1931.

each evaporating 150,000 lb. per hour and fired with pulverized coal. At 70 per cent full load, a net station heat consumption of 12,988 B.t.u. per kilowatt-hour is expected.[1]

31. Developments in the Mercury-vapor-steam Cycle.—Following the 1923 unit of 3,000-kw. maximum capacity in its Dutch Point Station, the Hartford Electric Light Company in November, 1928, installed a 10,000-kw. mercury turbine in the South Meadow Station. The unit was designed to operate at 720 r.p.m. with mercury vapor at 70 lb. gauge, 884°F., with 28 in. vacuum in the mercury condenser, the mercury vaporized per hour being 1,150,000 lb. At rated output, the unit was to provide for the production of 125,000 lb. of steam per hour at 350 lb. gauge, and 700°F. The year-by-year average performance was 10,812 B.t.u. per kilowatt-hour delivered from 1928 to 1936 and for 1938, 10,000 B.t.u. with an availability of 88 per cent of the time. On many daily runs, the rate dropped to 9,200 B.t.u.[2]

Encouraged by the successful performance of the 10,000-kw. unit, two similar mercury turbine units of 20,000 kw. have been built. The unit at Kearny Station of the Public Service Electric and Gas Company, New Jersey, assumed full load in June, 1934. The outdoor mercury-vapor-steam plant at the General Electric Works at Schenectady (26,000 kw. and 650,000 lb. steam per hour) consists of the following equipment:[3]

One steam boiler, 400 lb. gauge, 750°F., to produce 325,000 lb. per hour.

One mercury boiler to deliver 125 lb. mercury pressure 958°F. at the turbine throttle.

One double-flow five-stage mercury turbine, 900 r.p.m., exhausting at 27.5 in. vacuum and producing 20,000 kw. and 325,000 lb. of steam per hour.

One noncondensing turbine generator operating from 400 to 200 lb. gauge and producing 6,000 kw.

The station is leased to and operated by the New York Power and Light Corporation, the industrial requirements for power and 200 lb. pressure process steam being coordinated with utility power. The unit has been in operation since Dec. 1, 1933.

[1] From *Rept.*, A.I.E.E. Power Generation Committee, 1934.

[2] See *Elec. World*, Oct. 23, 1937, and May 6, 1939.

[3] See *Gen. Elec. Rev.*, July, 1933.

The cost of mercury-vapor installation is estimated at $70 per kilowatt.[1] Unfortunately for investment, it has required 5 to 7 lb. of mercury per kilowatt for mercury and steam (300,000 lb. in the Kearny boiler) and the price has risen from about 80 cts. a pound in 1933 to around $2 in 1940.

Figuring upon a total capacity of 50,000 kw., mercury and steam, a turbine efficiency ratio of 75 per cent, and a terminal

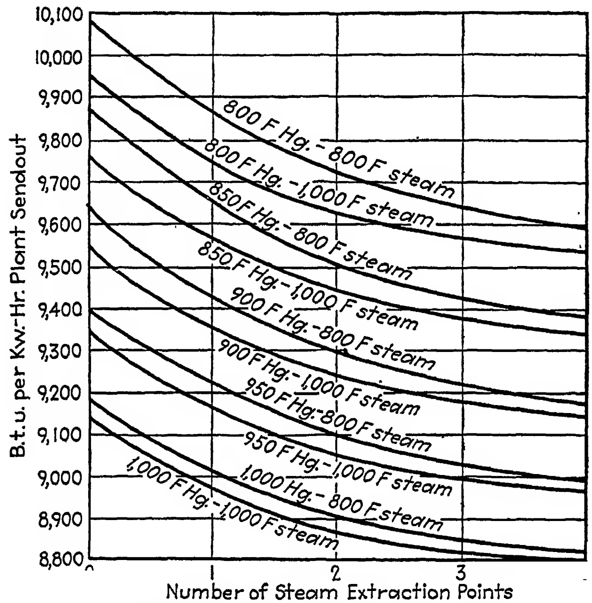


FIG. 37.—Plant performance mercury-steam cycles. Mercury vapor saturated at throttle. Mercury turbine back pressure 4 in. Hg abs. Steam turbine throttle pressure 500 lb. abs. (*Dr. G. A. Gaffert, Trans. A.S.M.E., October*, 1934.)

difference of 30°F. for the mercury-steam heat condenser, Dr. Gaffert has selected mercury throttle pressures of 85, 130, and 200 lb. abs. The Hartford unit operates at 85 lb. abs., the General Electric unit operates at 130 lb. abs., and the 200 lb. is for the future. The expected cycle efficiencies are shown in Fig. 37, the cycles progressing by even increments of temperature from 800°F. initial mercury temperature to 1000°F. mercury temperature. Thus with mercury at 200 p.s.i., 1020°F., there is a possibility of 8600 B.t.u. per kilowatt-hour under the assumed

[1] See *Power Plant Eng.*, December, 1933, p. 512.

conditions as compared with 10,660 B.t.u. for a steam plant at 1,200 p.s.i., 1000°F., and 1 in. vacuum.

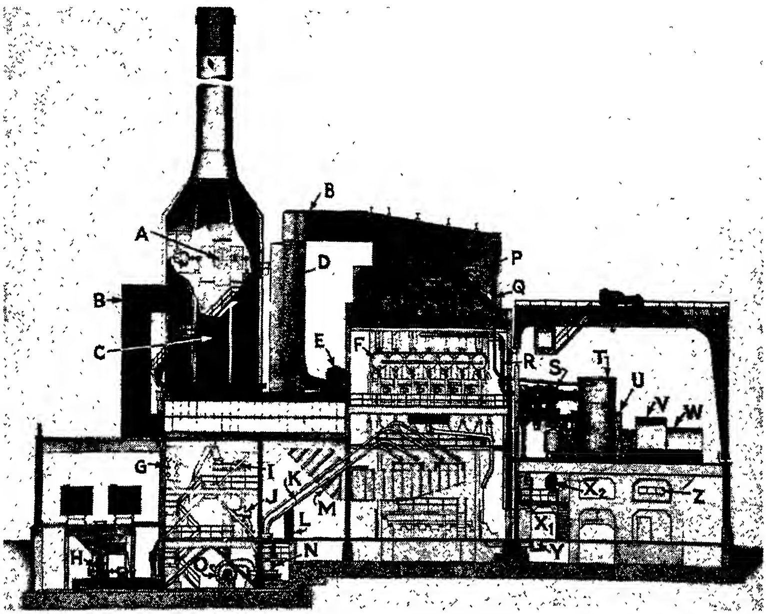


FIG. 38.—Cross section 20,000-kw. mercury-steam-electric plant, Schenectady. (*Courtesy of General Electric Co.*)

A = induced draft fan.
B = flue.
C = coal bunker.
D = air preheater.
E = forced draft fan.
F = mercury boiler drums.
G = bucket elevator to bunker.
H = coal crusher.
I = weigh scale.
J = coal feeder.
K = coal pipes.
L = boiler control board.
M = combustion air duct.
N = coal pulverizer.
O = primary air fan.
P = steam superheater.
Q = steam pipe to superheater.
R = mercury-vapor pipe from mercury boilers.
S = control valve.
T = steam dome.
U = mercury-condenser boiler.
V = generator.
W = exciter.
X_1 = mercury pump.
X_2 = mercury pump.
Y = ash sluice.
Z = generator air cooler.

Research and development continue on other materials that have the necessary properties for a binary cycle. Such com-

pounds are diphenyl, diphenyloxide, aluminum bromide, and zinc ammonium chloride. In the future, the thermodynamic possibilities of some of these cycles may be found very attractive.

32. Improvements in Turbine Generators.—The years since 1935 have been prolific of change in generating stations and in the development of a very definite trend toward higher pressures and temperatures in the thermal cycle. With the recurrence of a need for material increases in power-plant capacity, superposition marked a new era in turbine practice in 1937. This is a method of cross-compounding by exhausting a high-pressure turbine to the existing low-pressure machines and replacing the old low-pressure boilers with a high-pressure steam generator. This is, therefore, a form of plant rehabilitation and offers a means of adding 4000 B.t.u. per kilowatt-hour capacity to an existing plant at a cost of $80 to $110 per kilowatt.[1] Accordingly, it makes possible also a material improvement in the total fuel economy. Because of the metallurgical advances that had produced steels capable of standing high temperatures, turbines were now available to take steam at 1,250 p.s.i. and 925°F. on a regenerative cycle without reheat. Also in this year, large generators stepped up to 3,600 r.p.m. from the old speed of 1,800 r.p.m. and were available with hydrogen cooling. This came about with the improved physical characteristics of new alloy-steel forgings using nickel, molybdenum, and chromium, and with increased output from given material because of improved stator and rotor ventilation. Units approximating this specification were installed by the Consolidated Edison Company, 53,000 kw.; West Penn. Power Company, 50,000 kw.; Appalachian Electric Power Company, 40,000 kw.; and Cincinnati Gas and Electric Company, 35,000 kw. By 1939, the high-pressure superposition turbine capacity had been increased to 60,000 kw. at 3,600 r.p.m. in the double-shell unit at 1,250 p.s.i., 925°F. for Windsor Station of the Ohio Power Company, and in 1940 to 65,000 kw. for the Consolidated Edison Company. As compared with 62,500 kva., now considered the largest capacity practicable at 3,600 r.p.m. for air cooling, with hydrogen cooling, generators as large as 81,250 kva. at 3,600 r.p.m. are practicable, ventilated with internal fans.

[1] See Superposition, by E. H. Krieg, *Mech. Eng.*, September, 1936.

The advantages of hydrogen cooling, first applied to commercial generators in 1937, are given by Freiburghouse and Snell:[1]

1. Due to lower density of hydrogen, windage losses of a machine operating in hydrogen (at 97% purity) are $\frac{1}{10}$ of their value in air.
2. As a result of the superior cooling properties of hydrogen, about 20% greater output can be obtained with the same amount of active material, for the same temperature rise of the windings. (Hydrogen has a thermal conductivity of 7 times that of air, which greatly reduces the thermal resistance of all heat-flow paths that include gas spaces, such as the insulation and laminations. Also, the rate of surface heat transfer in hydrogen is about 35% greater than in air, which permits increased surface intensity of loss for the same surface temperature rise).
3. The effect of corona on insulation is negligible in hydrogen, and the insulation retains its flexibility longer.
4. As hydrogen will not support combustion, fire hazard is eliminated.

In 1939, the establishment of high pressure and temperature was shown by the 90 steam-generating units for 700 lb. or greater which were completed or on order with a total capacity of 31 million lb. of steam per hour. Thirty-three of the units were for 1,400 to 1,500 lb., 26 were for 800 to 1,000 lb., and 31 were for 700 to 800 lb. Two-thirds of the boilers were for steam temperatures of 900°F. or over.[2]

In order to expedite the production of steam turbogenerators, the National Defense Power Committee in November, 1938, adopted the standards set out in Table 18.

General adoption of the recommended standards would do much to reduce design and manufacturing costs and make possible earlier deliveries.

Following the outstanding performance of earlier 1,200-lb. units, a third 110,000-kw. vertical compound turbine was installed by the Ford Motor Company in 1939 (Fig. 33). The use of hydrogen cooling in the generators saves about 1 per cent at full load and 4 per cent at one-quarter load. A second 80,000-kw. unit to operate at 1,230 p.s.i., 850°F., has been ordered for

[1] See Hydrogen-cooled Generators, *Power*, August, 1938.

[2] See Power Generation Forging Ahead, by E. L. Hopping, *Elec. World*, June 15, 1940.

TABLE 18.—PREFERRED STANDARDS FOR STEAM TURBINE GENERATORS*

CONDENSING TURBINES

General: All sizes: Back pressure, 1 or 1½ in. Hg abs.; short-circuit ratio, 0.9; generator voltage, 13,800; excitation voltage, 250.

Rating, kw	10,000	12,500	15,000	20,000	25,000	35,000	50,000	75,000	100,000
Speed, r.p.m.	3,600	3,600	3,600	3,600	3,600	3,600	3,600	1,800	1,800
Throttle pressure, p.s.i. gauge	650	650	650	850	850	850 or 1,250	850 or 1,250	850 or 1,250	850 or 1,250
Throttle temperature, deg. F	825	825	825	900	900	900	900	900	900
No. of extraction openings	3	3	3	3	3	4	4	4	4
Temp. at extrac. openings 10°F. at rated output	170/225/290	170/225/290	170/225/290	170/225/290	170/225/290	170/225/290/350	170/225/290/350	170/225/290/350	170/225/290/350
Turbine capacity, % of kw. rating	125	125	125	125	125	125	125	125	125
Power factor	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
Generator cooling	Air	Air	Air	Air	Air or hydrogen	Hydrogen	Hydrogen	Hydrogen	Hydrogen

SUPERPOSED TURBINES

General: All sizes: 3,600 r.p.m.; throttle pressure and temperature, 1,250 p.s.i. gauge, 925°F.; back pressure, 200 to 300 p.s.i. gauge; short-circuit ratio, 0.9; generator voltage, 13,800; excitation voltage, 250.

Rating, kw	10,000	12,500	15,000	20,000	25,000	35,000	50,000	60,000
Turbine capacity, % of kw. rating	111	111	111	111	111	111	111	111
Power factor	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8 or 0.9
Generator cooling	Air	Air	Air	Air	Air or hydrogen	Hydrogen	Hydrogen	Hydrogen

* See *Power Plant Eng.*, Chicago, January, 1940.

Port Washington which will have a hydrogen-cooled generator. One 690,000 lb. per hour boiler will supply the new turbine.

Leading on to new cycles and improved economies are the 1,600-lb., 955°F., 50,000-kw. topping unit for the Sherman Creek station of Consolidated Edison Company of New York; the 1,800-lb., 950°F., 25,000-kw. superposed turbogenerator unit for the Montaup Electric Company; and the 2,400-lb., 940°F., 22,500-kw. turbogenerator cross-compounded with a 385-lb., 45,000-kw. unit at the Twin Branch plant of the Indiana and Michigan Electric Company. The result of studies on four cycles under consideration for the latter installation was as follows:[1]

Summary of Relative Efficiencies and High-pressure Capacities for Four Heat Cycles

Cycle No.	Description of cycle	Net station heat rate, per cent	High-pressure capacity for fixed low-pressure capacity, per cent*
1	2,400 lb. abs, 940°F., 900°F., reheat, 1 in. Hg abs.	100	178
2	1,200 lb. abs., 940°F., 850°F., reheat, 1 in. Hg abs.	102.9	135
3	1,200 lb. abs., 940°F., no reheat, 1 in. Hg abs.	107.3	135
4	850 lb. abs., 940°F., no reheat, 1 in. Hg abs.	108.4	100

* All low-pressure units have 400 lb. abs. throttle pressure.

Among the hydroelectric generators, the Boulder Dam 82,500-kva. machines are in several respects the largest in the world.[2] Because of the 270-mile transmission to California, the generators and 287.5-kv. transmission lines are desired to have a high degree of system stability. Therefore each generator has about double normal flywheel effect equivalent to 110,000,000 lb. at a 1-ft.

[1] See 2400 Pounds—a New Power Milestone, Philip Sporn, *Elec. World*, Oct. 9, 1937.

[2] See McClellan, L. N., Boulder Dam Generators, *Mech. Eng.*, July, 1934.

radius and a low transient reactance of only 17.5 per cent. The generators also have about double normal short-circuit ratio, *viz.*, 2.4. They are 40 ft. in diameter, 32 ft. in height above floor level, and each weighs approximately 2,000,000 lb. They will operate at either 50 cycles, 150 r.p.m. and 13,800 volts, or 60 cycles, 180 r.p.m. and 16,500 volts. Figure 39 shows one of the units.

Another development to keep the generator currents to their present size, in spite of the increase in machine capacity, is the use

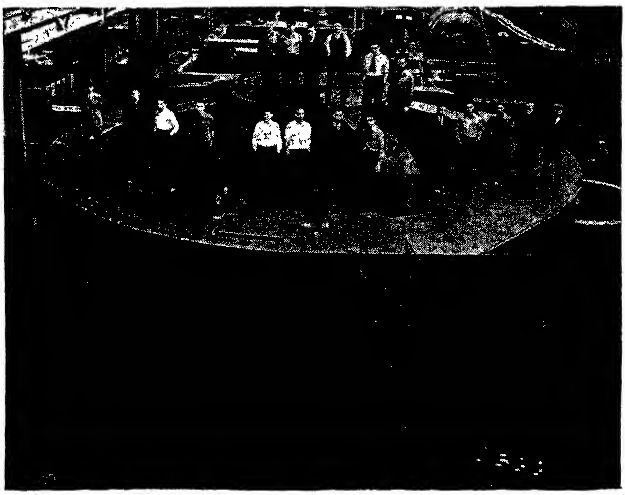


FIG. 39.—Boulder Dam generator, 82,500-kva., three-phase, 16.5-kv., 110,000,000-lb.-ft. flywheel effect, 17.5-per cent inherent reactance, short-circuit ratio 2.4. (*Courtesy of General Electric Co.*)

of divided circuits in the generator. Thus a machine may have two completely independent windings, arranged in alternate slots and so connected under the poles that the terminal voltage of the corresponding phase of each winding is in phase with and numerically equal to that of the other winding. This will permit a bus system in which the series reactance of the two circuits is interposed between bus sections, thereby eliminating the installation of bus reactors. The interrupting duty on the circuit breakers may also be reduced to approximately one-half that for a bus fault on single-winding units with 10 per cent reactors between the sections of a single ring bus. Such a double-winding single-shaft turbogenerator is installed in the Richmond Station

of the Philadelphia Electric Company. The generator is rated at 183,333 kva., 90 per cent power factor, 13,800 volts, 60 cycles at 1,800 r.p.m., and is shown in Fig. 40.

33. Savings in Cost Due to Interconnection.—Within the last decade, any new station of a public-utility power system has come into being not as a single individual power producer by itself, but as a new element joining other established and operating stations on the system. Ties were immediately established between such a new plant and all the others, or between the more important plants of the system, so that power might be transmitted from station to station, or from any station to the transmission network within the same system. In this way, the generating station and the transmission system combine into the single problem of power supply. But in addition to company ties, by the establishment of interconnecting lines from the transmission network of the foregoing power system to that of an adjoining power system or systems, the new station may send or receive power to or from other power companies. Thus the total amount and diversity of the available reserve power to ensure reliability of service are enormously increased, and a sufficient power reservoir may eliminate entirely the installation of spare units in the new station with the consequent burden of their fixed charges. As regards the whole system, each may dispense with some portion of the reserve capacity, which it would other-

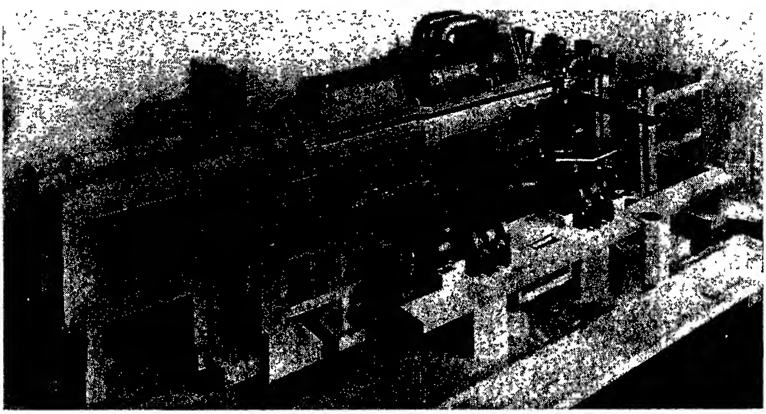


Fig. 40.—Model of installation for the 165,000-kw. turbogenerator for Richmond Station, Philadelphia Electric Company. (*Courtesy of Westinghouse Electric and Manufacturing Company.*)

wise carry, through this pooling of the reserves. Further, with a multiplicity of circuits between the station and the transmission network, the station auxiliaries may be almost entirely motor driven, since the auxiliary power service is backed by such a variety of reserve supply.

For trunk-line interconnections, the associated companies may also coordinate their construction and operating programs in order to secure minimum investment and operating expense for the combined systems. In construction, the individual systems can stagger their expansions, each company postponing building additional power plants until those of the other companies are loaded up. They can also have greater freedom in the selection of their size of unit and can provide large capacities where favorable features of water power or advantageous conditions of fuel and condensing water prevail. If there is any diversity in the daily load curves or the time of annual peak loads, advantage may be taken of such conditions in reduced capacity, since in this case the peak load of the interconnected systems will be less than the sum of the individual peaks. In operation, obviously, such interconnections permit the more efficient generating plants of the combined systems, and those particularly favored as to run of river or surplus water conditions at the time, to be loaded as base plants, the less efficient stations being used as peak plants, with the possibility of realizing important savings in production cost.

These benefits, particularly that of service protection under extreme contingencies, are much greater than the straight cost of capacity or energy output obtainable under normal supply conditions.[1]

The value of interconnections for emergency service and of mutual help was well shown by the experience of Cincinnati in the great flood of 1937, when boilers, turbines, and switch galleries above the 72-ft. level had not been high enough for an 80-ft. stage. The city network was never de-energized, and the peak of 47,900 kw. was carried, after all local generation had failed on Jan. 25, with help transmitted from Dayton and relayed from systems at Indianapolis, Springfield, and Chicago.[2]

[1] See Economics of Modern Generation, by Philip Sporn, *Elec. World*, May 22, 1937.

[2] See *Elec. World*, Feb. 20, 1937.

North America offers so many examples of interconnection that today the interconnected operation of systems totaling several million kilowatts of load is taken for granted.[1] One of the large interconnected systems is that of the Hydro-electric Power Commission of Ontario, with a system peak of 1,564,200 kw. in 1939, and handling nearly nine billion kilowatt-hours per year. Another is the bulk power supply for the Chicago district where some seven companies cooperate, using, in 1939, 305,000 kw. on the peak from outside the city while 767,000 kw. was supplied inside the city. A notable tie line of 160,000-kw. capacity connects the Niagara-Hudson system with the New York Edison system, thereby combining 1,570,000 kw. of generating capacity, half hydroelectric, with 2,566,000 kw. of generating capacity exclusively steam. Since the limiting factor in the rating depends upon the losses in the underground cable section, the two circuits are capable of carrying 220,000 kva. for 3 hr., provided the average loading during the preceding week has not exceeded 100,000 kva. The 220-kv. transmission lines of the Philadelphia Electric system are also of large capacity since they serve a system of some 1,126,000 kw., whereas on the West coast similar transmissions are used by the Pacific Gas and Electric system with its 1,059,800-kw. peak in 1939, and by the Southern California Edison Company with a corresponding peak demand of 684,500 kw.

In England, Scotland, and Wales, the 132-kv. "Grid" illustrates the idea of interconnection expanded to a national basis.[2] In the place of some five hundred separate stations mostly of small size and poor economy, generating at 15 to 20 different frequencies, the Central Electricity Board established 135 large stations, all generating at 50 cycles and tied together and to the distribution systems with 4,000 miles of transmission lines. The system output was more than 15 billion kw.-hr. for the first seven months of 1939.

With the interconnection of large systems, the technical problems of maintaining control over operations, keeping specified tie-line loadings, and limiting the frequency variations to plus or minus one-twentieth of a cycle become of great importance. It is

[1] See Interconnected Electric Power Systems, by Philip Sporn, *Elec. Eng.*, January, 1938.

[2] See *Elec. World*, July 21, 1934.

also difficult to provide satisfactory voltage regulation, to have fast selective relaying and circuit-breaker operation, and to keep system stability when transferring large blocks of power.

In the interconnection of systems reaching at times from Chicago to Tampa, with a total capacity of about 7,000,000 kw., manual control seems adequate for staged tests, but subsequently one system operator inadvertently negatives the corrective efforts of another. Supervised automatic control at all major interconnections has become necessary, and the cost of regulating equipment and telemetering has been justified by the increased effectiveness of the tie lines.[1]

The Federal Power Commission report of May, 1940, on Electric Power Statistics, 1939, lists in Table 17 of the report the movement of electric energy across state lines. Also it records the net energy imported from Canada as 1,914,394,000 kw.-hr. and the net export to Mexico of 14,043,000 kw.-hr.

[1] See *Elec. World*, Feb. 20, 1937.

CHAPTER III

ECONOMIC DECAY

34. General Factors.—The factors that determine the useful life of a power unit are usually classified as physical depreciation and functional depreciation, including under the latter heading the two causes of inadequacy and obsolescence.

35. Physical Depreciation.—This is the result of wear and tear on the unit in service together with the decay and corrosion due to time and the elements. We daily see examples of the latter action in the decay of poles and in the festoons of cotton braid hanging from some distribution conductors after "weatherproof" wire has endured long years of service. All the parts of a normal operating property are kept in reasonable repair by the ordinary running maintenance, *i.e.*, by the replacement of the broken and worn-out parts. But there comes a time when this increasing maintenance cost and the accompanying decreasing efficiency, due to the increasing frequency and seriousness of the ills of the machine, so increase the cost of operation that it would be cheaper to pay the costs on a new machine or piece of apparatus. At such a point, the machine has reached the end of its useful economic life. Such a thing as a transmission line has a small upkeep at the start which grows larger as time goes on up to a certain point where it finally becomes approximately constant. On the other hand, a reciprocating engine starts out its life with a similar increase in upkeep, but it differs from the transmission line in that the maintenance costs do not reach a steady value but keep on increasing. Such costs from the nature of the case are likely to be large and to occur at irregular intervals.

An illustration of this situation is strikingly shown in Fig. 41, Maintenance, Renewal and Replacement Costs for Delray Steam Turbine Plant, for the period 1915 to 1928. The graph is based on the actual costs as shown by the books of account. In this accounting system, Current Maintenance includes such labor and material, necessary to maintain the plant in a state

of operating efficiency, as do not result in a substantial change of identity in any particular unit of property. It includes the cost of minor replacements of small parts commonly called the cost of "repairs," but it does not include the cost of replacing entire structures or units of equipment. "Renewal and Replacement" cost includes the cost of the particular unit replaced plus the cost of dismantling less its salvage value. Examination of the graph shows that on both the maintenance and the renewal and replacement curves for Delray there is a valley during

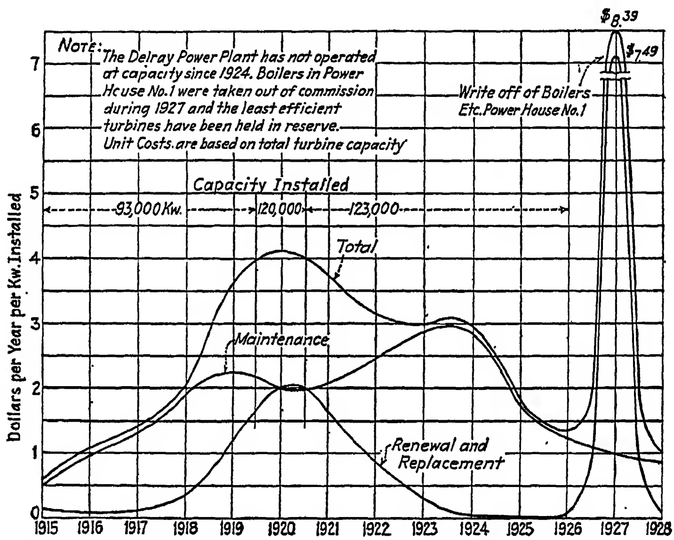


FIG. 41.—Maintenance, renewal, and replacement costs, Delray Steam Turbine Plant of the Detroit Edison Company, begun in 1903.

the years 1917 and 1918, whereas peaks occur in 1919 and 1920. This variation is due to the fact that during the war period and immediately thereafter certain maintenance work had to be deferred, chiefly on account of lack of spare capacity to carry the load. Spare capacity became available in 1920, and then a large amount of work was done. If it had not been for this unusual condition, some work would have been done earlier, which would have resulted in filling up the valley and lowering the peak. Also the period in question was one of great variations in the prices of labor and material and of change in the amount of work performed by labor. If the figures were further adjusted to allow for the abnormal increases in costs of material and

labor and the inefficiency of labor, then the peak of 1920 would be further lowered. After the considerable overhauling in 1920, the plant, of course, was in better operating condition, so the costs for 1921 and 1922 decreased. Also the prices of labor and material had decreased somewhat.

In an effort to eliminate the effect of price changes on the annual costs of maintenance and replacements, the data of Fig. 41 for Delray were rearranged. The figures for maintenance were

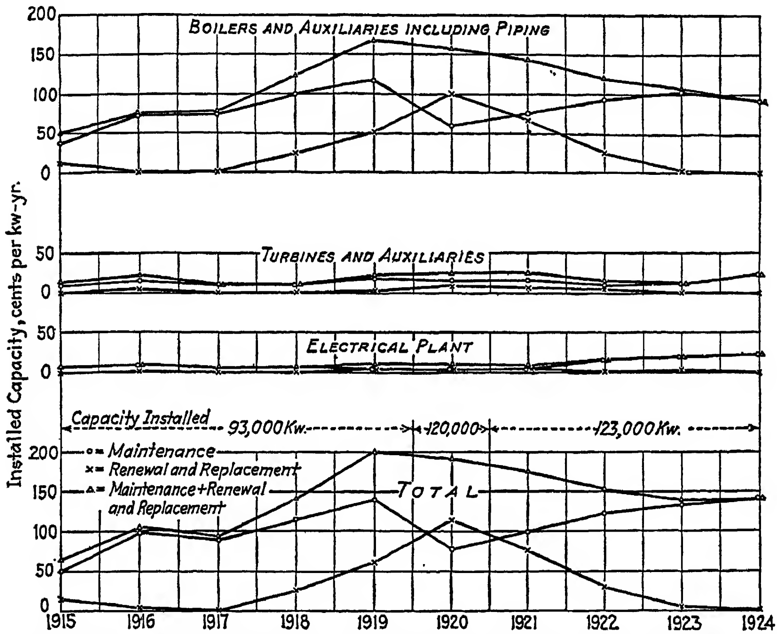


FIG. 42.—Maintenance, renewal, and replacement costs, Delray Steam Turbine Plant, Detroit Edison Company, all costs on basis of 1915 prices.

divided into 50 per cent material, 50 per cent labor, and the figures for renewals and replacements were divided 35 per cent for labor and 65 per cent for materials on turbine and electrical plants, but 20 per cent for labor and 80 per cent for materials on boiler plant. The divided amounts were reduced to a 1915 basis by the application of an index number appropriate to the subdivision for each particular year. The reworked items were then totaled and the cost per kilowatt-year determined. As reconstructed to this uniform basis of price the data are shown in Fig. 42. The remarkable uniformity of the maintenance and

renewal costs for the electrical plant is worthy of note, being approximately 5 cts. per kilowatt-year for 1915 to 1918, then 10 cts. per kilowatt-year for 1919 to 1922, after which the maintenance increases sharply, showing the effect of the greater care necessary to keep the old machines in satisfactory operating condition. The turbine costs are also fairly uniform, rising from 10 cts. per kilowatt-year in 1915 to 25 cts. per kilowatt-year in 1921, and again in 1924. In the boiler plant, however, the costs are not only variable but are many times those of the turbine department. A boiler, for instance, operates with gradually increasing maintenance costs for several years until the settings have to be replaced, at which time it is usually given a thorough overhaul and put into as good shape as is possible. This causes a peak in the cost curve of that individual unit which will be followed by a valley, because after the thorough renovation the maintenance will drop off for a period. In a large plant, of course, units would be in all conditions of upkeep, and the peaks and valleys would hardly be noticeable. The renewal and replacement cost curve on the boilers illustrates this variation quite typically. It starts at 13 cts. in 1915, falls to 2 cts. in 1916, then rises more or less gradually to a maximum of 100 cts. in 1920, and then decreases to zero in 1924. At the time of the peak replacement cost of 100 cts. in 1920, the maintenance cost is at a minimum of 65 cts. per kilowatt-year. The shape of the total cost curve is almost identical with that of the boiler curve, since the latter makes up such a large part of the total cost.

As is noted on Fig. 41, the Delray Power Plant did not operate at full capacity after 1924 but served mainly as stand-by capacity for peak-load periods. For such purpose, the station average load was about half the rated capacity, so that a number of boilers and turbines were in reserve and the maintenance correspondingly reduced.

36. Inadequacy.—Equipment installed today may not meet the needs of the business 10 years from now and may, therefore, have to be replaced though still physically intact and efficient. Such inadequacy is properly a burden of growth, and unless the growth of the business is wholesome enough ultimately to take care of its own necessities, growth should not be encouraged.

W. B. Curtiss has ably explained this form of depreciation in his article, "Depreciation of Property," in the December, 1915, issue of the *General Electric Review*. He says:

In the case of public utility corporations, such as street railways, electric light and power companies, gas companies, etc., inadequacy is frequently the result of public demands for better service. A review of the trolley-system growth in almost any city offers a good example of inadequacy. Some twenty years ago a city's electric transportation service was usually furnished by small single-truck cars running over light weight tracks. The growth of the city and the consequent demand for better transportation facilities made it necessary for the railroad to add to its equipment from time to time. Naturally, the new equipment was of the type that was modern at the time of installation and gradually replaced the older equipment not because the old was worn out but because it had become inadequate for the service demanded. If any of the first cars be in existence at this date, they have outlived their usefulness as passenger cars and have become hopelessly inadequate to take any substantial part in the transportation problem as it exists today.

With regard to electric power supply, the development has been very similar. The steam-electric plant for the service of the buildings at the University of Michigan probably offers a typical example. Constructed in 1894, it was located among the buildings it served and contained eventually two 220-volt direct-current direct-connected engine generator sets. The coal was hauled from the nearest railway yard in wagons and fired by hand to the 300-hp. boilers to produce steam at 100-lb. pressure. The building volume which it supplied with heat, power, and light was 7,291,000 cu. ft. By 1912, the building volume had grown to be 15,685,000 cu. ft., and the constantly increasing load could no longer be fed efficiently by 220-volt direct-current mains. The entire plant, therefore, was abandoned and a new station built at a site off the campus permitting railway-car delivery of coal. In this plant, a cross-compound engine drove a three-phase alternator of 525 kva. to deliver 2,300 volts which was transmitted to the campus and fed step-down transformer banks for the building services. Thus within 18 years the old power station had run its whole course of life. In its turn, the engine set was replaced as an active operating unit by turbogenerators in 1925, and was finally removed in 1930 to make room for a 2,500-kw. turbo set.

In public-utility stations, the action is even more rapid. The Delray power houses of the Detroit Edison Company, shown in Fig. 43, furnish a striking example. The first plant was constructed in 1904, the second in 1908, the third went into com-

mission in 1929, and the high-pressure extension of No. 3 was built in 1939.

The present widespread necessity for large increases in the generating-station capacities has rendered inadequate much of the electrical switching equipment installed with the first units. The increased concentrations of power due to the larger units and

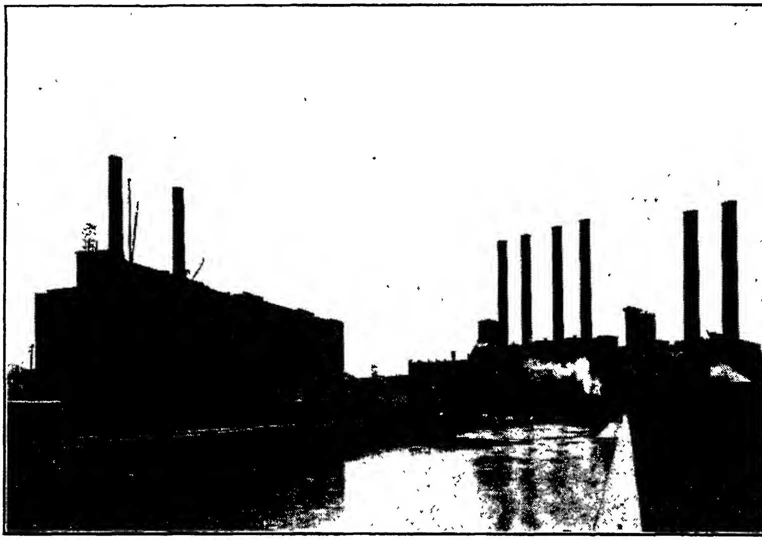


FIG. 43.—A typical example of power-station obsolescence; the Delray Stations of the Detroit Edison Co. Power House No. 1 at right, built 1904, five 3,000-kw. units. In 1910, the 3,000-kw. units were replaced by 9,000-kw., 4.8-kv. units; steam, 200 lb., 550°F.; boilers, 520 hp. Power House No. 2 in center, built 1908, four 14,000-kw., 4.8-kv. units; steam, 200 lb., 550°F.; boilers, 2,365 hp. Extended 1920, one 30,000-kw., 12-kv. unit added. Power House No. 3 at left, built 1929, three 50,000-kw., 12-kv. units; steam, 375 lb., 700°F.; boilers, 2,390 hp.; one special 10,000-kw. unit, 365 lb., 1,000°F. Equipment of Power House No. 2 written off in 1934. No. 3 extended in 1939 for 75,000-kw. units to work at 815 lb., 900°F. (*Courtesy of Detroit Edison Co.*)

to interconnections with other power systems have passed far beyond the capacity of the old devices.

37. Obsolescence.—There are certain types of fixed costs sometimes assumed which should not enter into the engineer's calculations, though they do involve a very real charge against the investment necessary to do business. One of these is due to obsolescence—the state of being old-fashioned or out of style—which may easily enough occur well within the reasonable physical life of apparatus. Obsolescence indicates that as a

result of engineering achievement, the present apparatus, whether it be as one piece or a whole class, has become uneconomical of use. As was pointed out under Developments in Fuel-burning Plants, the field of power generation and distribution has developed so rapidly that larger and more efficient machines and methods are constantly being brought out. Whether the present equipment should be replaced before it is worn out depends upon the amount of the saving in operating expenses which the modern equipment could effect as against the increased fixed charges due to its purchase and the retirement of the old units. It may be the case, for instance, that a steam turbine installed now will in 10 years be too expensive to operate because of advances in turbine design which have made the model of 10 years hence much more economical than that of the present. This has been almost common experience in the electric-power industries. Analyses of the turbogenerator records presented by the Prime Movers Committee of the National Electric Light Association showed that out of 268 units observed in this country only 5 were 17 years old.

Studies by the late Dr. Hirshfeld, chief of the Research Department of the Detroit Edison Company, as announced at the Boston meeting of the N.E.L.A., March, 1926, show an average operating life of about 5½ years for turbines in a plant life of 15 years. This is rather typical of the history of engineering achievement in developing more efficient types of main power-generating units. In the earliest plants, the belt-driven bipolar generator was common from 1890 to 1893. Then the small vertical engine units, direct connected, covered a period from 1894 to 1897. These were followed by the large vertical engine units, direct connected, in the years 1898 to 1904. In the period from 1905 to 1911, the small vertical steam-turbine units were installed, followed by the large vertical turbine units from 1912 to 1918. Next the horizontal turbines came in during the years 1919 to the present with ever increasing pressures and steam temperatures. These cycles together with the accompanying performances in terms of pounds of coal per kilowatt-hour are shown in Fig. 44 as extended from the original study which appeared in the *Electrical World* of April, 17, 1926.

"Superposition," which was mentioned in Sec. 32, has come to connote the rehabilitation of existing plants in an effort to post-

pone obsolescence by increasing the station efficiency. In general, the savings in production costs made possible by lowering the operating costs of the plant to be superposed did not carry the investment with sufficient margin to warrant the adventure until new capacity was needed.[1] In discussing the problem, Philip Sporn reports "that where properly applied it has been possible to install superpositions which were self-liquidating from the savings over the old units and to obtain the increment of

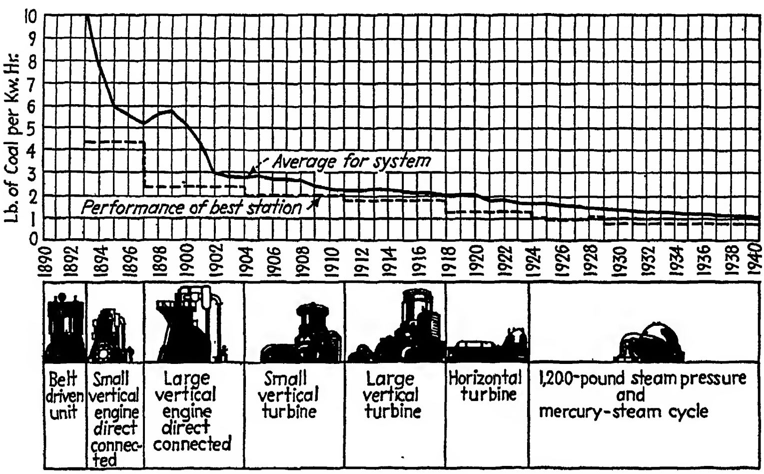


FIG. 44.—Prime mover development and improvements in production efficiency, 1894–1940.

capacity as a practically clear gain."[2] He points out the following weaknesses of superposition:

1. In many cases, there results only a comparatively small increment of capacity, and economically it can be done only by going to a limited number of boilers. This involves the disadvantages of a single boiler feeding a multiplicity of units which, although it may be economical at normal full load, is not so at fractional loads.

2. Many superpositions are sound as to thermal efficiency and economic return only when given a load factor of 65 to 90 per cent.

3. The entire plant has to be fitted to the load curve as obsolescence occurs rather than individual units.

[1] See Superposition, by E. H. Krieg, *Mech. Eng.*, September, 1936.

[2] See Economics of Modern Generation, *Elec. World*, May 22, 1937.

4. The plant is susceptible to stability difficulties when the high-pressure unit goes off the line for any reason.

The following installations are among the notable superpositions:

Waterside station of Consolidated Edison Company, New York, two 50,000-kw. units, 1,300 lb., 1937 and 1938.

Fisk Street Station of Commonwealth Edison Company, Chicago, one 30,000-kw. unit, 1,200 lb., 900°F., 1937.

Logan plant of Appalachian Electric Power Company, one 40,000-kw. unit, 1,250 lb., 925°F., 1937.

Rivesville plant of Monongahela West Penn Public Service Company, one 25,000-kw. unit, 1,250 lb., which reduced the station heat rate for 1939 to 13,965 B.t.u. from the 20,730 of 1935.[1]

Springdale Station of West Penn System, one 50,000 kw. unit, 1,250 lb., 1937.

Montaup Electric Company, one 25,000-kw. unit, 1,800 lb., 950°F.

West Reading station of Metropolitan Edison Company, one 20,000-kw. unit, 1,250 lb., 900°F.

L. Street station of Boston Edison Company, one 25,000-kw. unit, 1,250 lb., 900°F., 1938.

Miller's Ford plant of Dayton Power & Light Company, one 42,000-hp. unit, 1,200 lb., 900°F., 1937.

So far as concerns the engineer's design, he should compute investment costs as if the present model were the last word that would ever be spoken on turbine design; his assumption being that 10 years from now he will not take out the 10-year-old turbine and replace it with a better one, unless the saving to be made at that time will justify the new investment on top of the then unretired portion of the old one.

There should be no misunderstanding of the fact that obsolescence must be provided for. It should be collected out of the rates if the product is sold, or a fund should be provided in case the power produced by a private plant is used by the owner in manufacturing or other commercial pursuits. The reason is evident if one considers that 10 years from now a competitor coming into the field might be able to drive the promoter of the present out of business, through the advancement in engineering of which the competitor would be able to take advantage, while the original promoter would be saddled with his old investment. This would be true whether, in the case of a public-service enterprise, the competition took the form of new developments

[1] See Rivesville Topping, by W. V. Drake, *Elec. World*, Feb. 24, 1940.

in small, highly efficient, and inexpensive private plants or of a competing public utility whether privately or governmentally owned; or in the case of a manufacturer through the entry into the field of a competitor who, through the use of a more modern plant, could save enough in the processes of production to undersell the older manufacturer. If the pioneer has in hand a distinct obsolescence fund, or has a depreciation fund augmented by obsolescence considerations, he can draw down such funds, operating at what would otherwise have been a loss until the old equipment has worn itself out and paid for itself. Or he can, if the change is justified on strictly engineering merits, install the more modern and efficient equipment without increasing his capitalization, and then being free from further obsolescence accumulations on the old equipment can make his rates to meet the competition.

The obsolescence charge, then, is a perfectly legitimate charge against the business as a whole or, in the case of a power-plant auxiliary to some other business, against the power plant as an integer, but for the purposes of the engineer in proportioning the plant has manifestly no place. However, in view of the history of central-station performance,[1] it would be well for him, in so far as he can without prejudice to the present, to design rather generously with respect to ground area, floor area, cubical contents, and clearances so that the greatest flexibility will be available in case of rehabilitation or replacement.

What the obsolescence charge does is to burden the easy earnings of early youth with the needs of a less productive old age. The morality of this action is sometimes questioned, but the expediency and, indeed, the imperativeness of it are evident. One would scarcely care to invest money in an enterprise unless the earlier years could provide against the contingency of a large part of the investment being swept away through shrinkage of value when competition of lusty youngsters comes into the field.

The obsolescence charge, of course, should be determined for each class of detail by deciding just when complete obsolescence of that class is likely. But such a decision is exceedingly difficult because of the variable and limited history of any class of

[1] See Hirshfeld, C. F., Rehabilitation of Steam Power Plants, *Elec. World*, July 6, 1929.

equipment as we know it and the total lack of knowledge of the future. We may achieve the desired result, however, by estimating the depreciation rate based on physical decay, on which history and experience are more complete, and then augmenting this by a factor to cover the chance of obsolescence. This would naturally give a total depreciation cost in excess of that based on physical life, and such excess could be considered as a separate obsolescence accumulation if such were desired. It would not be at all disadvantageous and, indeed, the practical effort involved is so small that one might carry out three classes of obsolescence computations, one for individual types of equipment, a second one for the power plant as an integer, and the third one for the business to which the power plant is an auxiliary. For any individual type of apparatus, the shortest period of the three should be taken. As an example, a property devoted to the manufacture of munitions during a time of war might readily become obsolete as a whole within the life of one contract, irrespective of the physical life or obsolescence of the power plant for such munitions factory. In a more stable manufacturing enterprise with no presumption of that particular line of manufacture becoming obsolete within a period of, say, 30 years, the engineer might anticipate a supersession of his initial method of generating power at the end of 15 years. In such a case, the power plant as a whole would have to carry a depreciation charge for the purpose of arriving at obsolescence based on a 15-year life, while the rest of the property would enjoy a much smaller obsolescence burden. Indeed, the physical life of the rest of the property, if intermediate between 15 and 30 years, would be the determinant factor in computing depreciation on such property other than power plant. Again, though the power plant might be assumed to have a physical life of 25 years as an integer, and presumptive obsolescence by supersession in approximately 15 years, a steam turbine which might physically last as long as the rest of the plant might properly be assumed to be liable to supersession by more efficient apparatus in, say, 8 years. We would then have depreciation charges against such details as the steam turbine based on an 8-year life, for the power plant exclusive of such details based on a 15-year life, and for the entire business exclusive of power plant based on a 30-year life or on the shorter physical life of property other than power plant.

Although the engineer need have no consideration of obsolescence in proportioning his plant, he has one responsibility to his client or employer in connection with obsolescence and must figure on it in discharging that responsibility. It is his duty to advise his employer whether the power plant as a whole is justified, in view of the business necessity of accumulating a reserve for obsolescence more rapidly than would be indicated by mere physical decay, it being assumed that the employer is less expert in judging power-equipment obsolescence than is the engineer.

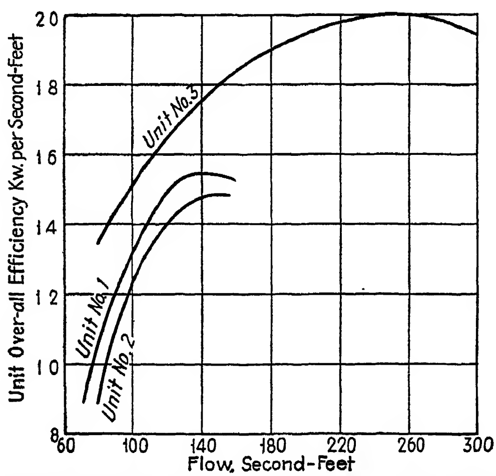


FIG. 45.—Relative efficiencies of three hydro turbines, Borel plant, 260 ft head. Unit 3 of 1932, 1 and 2 of 1904. (*"Plant Rehabilitation,"* by *J. B. Davenport and F. M. Scott, Elec. World, Dec.* 31, 1932.)

In Sec. 30, there is an extended discussion of the development of new thermal cycles, and Figs. 30, 35, and 37 show the increased efficiencies that are possible in the later designs. For the hydraulic plants, although obsolescence has not proceeded at such a rapid rate as in the thermal plants, still there has been a regular improvement in design and increase in size of unit, resulting in the rehabilitation of plants and the replacement of old turbines. The reconstruction of the Borel plant of the Southern California Edison Company on the Kern River in 1932 is a case in point. Two 1,000-kw. horizontal turbine units installed in 1904 were replaced with single Pelton-Francis horizontal turbines in 1910, 1911, and 1912. In the reconstruction, a new vertical Pelton-Francis turbine of 5,000 kw. was installed, since under normal

stream flow this unit of 300 c.f.s. capacity could carry all the load 35 per cent of the year. Two of the older units were kept to regulate the load with the remaining water when available. Figure 45 shows the relative efficiencies of the three units under the 260-ft. head developed. It is notable that the new unit when fully loaded delivered 1,300 kw. more output than could be furnished by the two old units with the same quantity of water. The new turbine efficiency was 91.1 per cent at best points. The rehabilitation and conversion to automatic operation saved 44 per cent in operating costs and gave 24 per cent increase in generation per year.

38. Test for Obsolescence.—Sentimental obsolescence may play quite a part in causing scrapping or rearrangement of apparatus. For example, distribution lines located on poles in the residence district of a city may become very unsightly when compared with the surroundings, and public opinion may carry sufficient weight with the public-service corporation to have the lines placed underground. Some of the utilities have agreements with the cities they serve to thus change a certain portion of the lines year by year from overhead to underground.

Economic obsolescence is not always evident on the face of things, but can be determined as follows:

If we have today a piece of apparatus installed r years ago with a life expectancy of n years and an expected scrap value of S_n; if present scrap value is S_r and operating cost has been constant at O, we may replace the old apparatus now or when it has worn itself out. Let it be assumed that improvements in engineering have produced an equivalent apparatus whose life expectancy is n', depreciable cost $(P' - S'_{n'})$, and constant operating cost O'. It will be convenient to consider the matter with regard to the three periods of time shown in Fig. 46. Period 1 is the r years preceding the present year during which time the old unit has been in operation. Period 2 is the $n - r$ years to follow the present year, being the still unused portion of the life of the old unit. Period 3 is $n' - (n - r)$ years; *i.e.*, if the old unit were replaced at the present year by the newer unit, the third period would be that portion of the life of the new unit left to it after it had passed the end of the estimated life of the old unit. It is evident that any savings during period 3 will accrue whether the newer apparatus is installed during the present year or after

$(n - r)$ years, since at the end of $(n - r)$ years from the present, the depreciation charges will have been completed on the old unit, and the new unit will come into service carrying only its own regular charges. The area in period 3 marked Y is, therefore, not a saving due to early supersession; it could have been

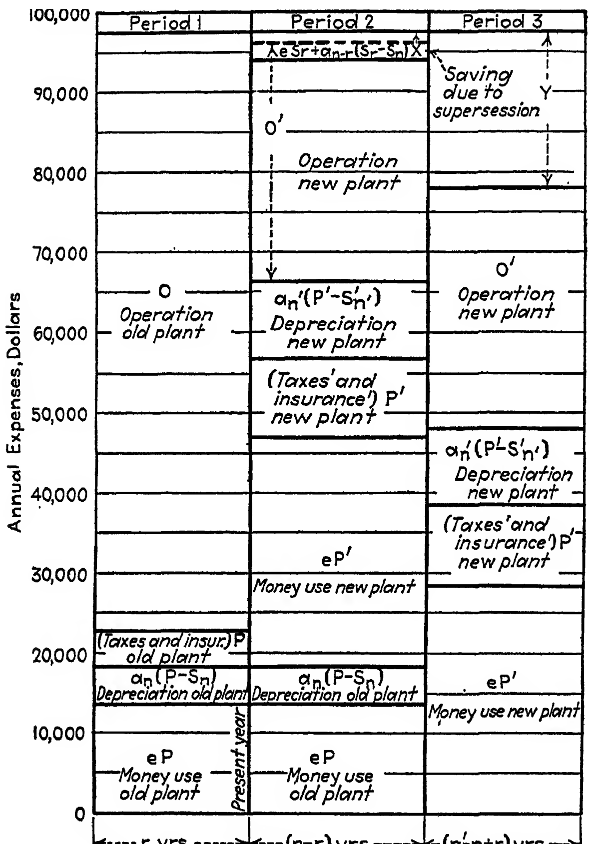


FIG. 46.—Test for supersession, three periods.

realized just as well by installing the new unit at the end of the life of the old apparatus.

Period 2 shows the cost of operating the newer apparatus so as to have it complete the program of which period 1 was the first part. Evidently taxes, insurance, and operating costs of the old apparatus will have been replaced by those of the newer, and the

new plant will have to pay depreciation and money use on its own investments in addition to completing the same for the old plant.

Carrying depreciation on the old plant through period 2 will complete the accumulation of the depreciable amount and will enable the starting of period 3 exactly as if the old plant had been worn out in service. This necessarily involves carrying money use eP until the old plant shall have been depreciated.

As an aid to early supersession, we have in hand at an early date a scrap value S_r, in general, greater than S_n. To complete the initial program—to compare costs under supersession with those attending actual wearing out of the old plant—we may consider the money use eS_r and the wearing down of excess salvage receipts $a_{n-r}(S_r - S_n)$ as sources of income. That is to say, we may have an annual income of the interest earnings on the increased salvage value plus an annuity derived from wearing down the increase in the salvage value to the final salvage value of S_n. The latter amount is all that is required at the end of the life of the plant to add to the depreciation reserve in order to reproduce the principal amount again.

The area X in period 2 thus shows the annual incentive to supersession which takes place if

$$\$X = O - O' + t + \text{ins.} - t' - \text{ins}'. - eP' - a_n'(P' - S'_{n'}) + eS_r + a_{n-r}(S_r - S_n) \geqq 0. \quad (26)$$

Evidently, by applying this criterion the costs during period 2 will be no greater than during period 1, and it is, therefore, improper arbitrarily to charge for obsolescence as a part of the cost of doing business during period 1. Had an obsolescence reserve been accumulated during the past r years, the income from it would be as useful to the old as to the newer apparatus and would neither aid nor hinder supersession.

An alternative scheme consists in turning over to the newer apparatus all the assets and all the liabilities of the superseded machinery. In this event, the new apparatus will have to recover within its lifetime the original investment in the first apparatus and in its successor, less the money in hand at the supersession date—*viz.*, actually recovered therefrom—and less the expected scrap value $S'_{n'}$ of the new apparatus.

Clearly, the money use on R_r and S_r may be used as revenue to deduct from the carrying and operating costs during the life of the new apparatus.

It is evident that if the original investment P in the old apparatus is not recovered until the end of the life of the new, we shall have to pay money use on it until that time.

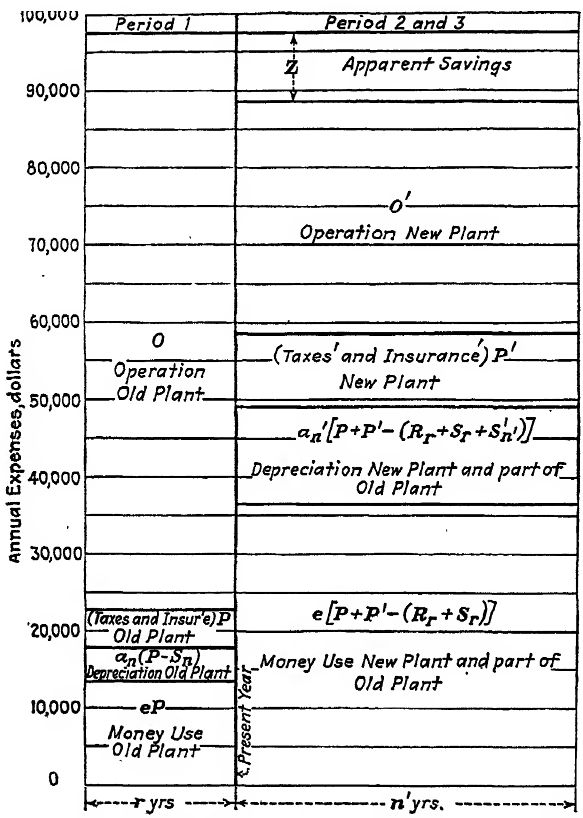


FIG. 47.—Apparent savings due to supersession, two periods

The "apparent" annual saving then due to supersession, as shown in Fig. 47, is

$$Z = O - O' + t + \text{ins.} - (t' + \text{ins.}') + a_n(P - S_n) - a_n'[(P + P') - (R_r + S_r + S'_{n'})] + eP - e[(P + P') - (R_r + S_r)] \quad (27)$$

or

$$Z = O - O' + t + \text{ins.} - (t' + \text{ins.}') + (a_n - a_n')P - a_nS_n \\ - a_n'[P' - (R_r + S_r + S'_{n'})] + e(R_r + S_r - P'). \quad (28)$$

We should note that this apparent annual saving evaluated as of the present date must equal the sum of X, the real saving per year due to supersession, and Y, the saving realized by use of the new plant after the end of the life of the old plant, evaluated as of the same date, since in both treatments the apparent savings have arisen from the same facts. If, further, we note that Y is not a saving due to supersession, it is obvious that not all of Z accrues from supersession. Therefore, if Z is barely zero, supersession is not permissible.

This second treatment should not be used as a test for supersession, but it is very useful as a financing device after this replacement has been determined, since it distributes the savings uniformly over the n' years of life of the new plant and avoids the sharp change in savings at the end of the life of the old plant in the plan outlined in Fig. 46.

39. A Problem in Obsolescence.—A power plant built 12 years ago at a cost of \$260,000, financed from the sale of 35-year 5's at 96, had been estimated to last 25 years and have a final salvage value of \$25,000. Its present salvage value is \$35,000. Taxes and insurance are 1.75 per cent on full value. The operating cost has been approximately constant at \$75,000 per year. Today we can build a new type of plant of 25-year life with a final salvage value estimated at \$70,000 for a cost of \$550,000. The annual operating cost of the new plant will be \$30,000.

1. What will be the annual saving due to supersession?

2. What will be the "apparent" annual saving due to supersession?

The annual expense of owning and operating the old plant will be

Cost of money use, 5.24% on \$260,000	=	\$13,620	eP
Depreciation, a_{25}, 1.98% on \$235,000	=	4,660	$a_n(P - S_n)$
Taxes and insurance, 1.75% on \$260,000	=	4,550	$(t + \text{ins.})P$
Operating expense	=	75,000	O
Total	=	\$97,830	

The annual expense of owning and operating the new plant and completing the program of the old plant within its estimated life, will be

For the new plant:

Cost of money use, 5.24% on \$550,000	=	\$28,820	eP'
Taxes and insurance, 1.75% on \$550,000	=	9,630	$(t' + \text{ins.}')P'$
Depreciation, a_{25}, 1.98% on \$480,000	=	9,500	$a_{n'}(P' - S'_{n'})$
Operating expense	=	30,000	O'
Total	=	\$77,950	

Still carried for the old plant:

Cost of money use, 5.24% on \$260,000	=	\$13,620	eP
Depreciation, a_{25}, 1.98% on 235,000	=	4,660	$a_n(P - S_n)$
Total	=	\$18,280	

Less *income* from the old plant:

Money use on present salvage, 5.24% on \$35,000	=	\$ 1,835	eS_r
Annuity of excess salvage value, a_{13}, 5.47% on \$10,000	=	547	$a_{n-r}(S_r - S_n)$
Total		\$ 2,382	
Net additional cost of old plant	=	\$15,898	
Total for new and old plants	=	\$93,848	

Therefore (1) X per year = \$97,830 − \$93,848 = \$3,982. See Eq. (26). That is, there is a real saving of \$3,982 per year for the next 13 years owing to replacement of the old plant now before it has completed its estimated life. This is shown graphically in Fig. 46 which has been drawn on the basis of the foregoing problem.

Now that the economy of supersession at the present time has been established, the financial plan may be improved upon by spreading X and Y evenly over the years of life of the new plant, as discussed in connection with Z of Eq. (28). Giving the new plant all the liabilities and assets of the old brings the annual charges on the new plant as follows:

Money use,

$$5.24\% \text{ on } [\$260{,}000 + \$550{,}000 - (\$76{,}300 + \$35{,}000)] = \$36{,}700$$

$$e[(P + P') - (R_r + S_r)].$$

$$\left[R_r = \frac{a_n}{a_r}(P - S_n) = \frac{1.98}{6.1} \cdot \$235{,}000\right]$$

Depreciation,

$$1.98\% \text{ on } [\$260{,}000 + \$550{,}000 - (\$76{,}300 + \$35{,}000 + \$70{,}000)] = \$12{,}450$$

$$a_{n'}[P + P' - (R_r + S_r + S'_{n'})]$$

Taxes and insurance on new plant, 1.75% on \$550,000	= \$ 9,630
Operating expense new plant	= \$30,000
Total	= \$88,780

Therefore (2) Z per year = \$97,830 − \$88,780 = \$9,050. See Eq. (27). That is, considering both the real saving X due to supersession and the saving Y which arises from having a more efficient plant follow the old plant at the end of the latter's life, there is an apparent saving of \$9,050 per year. This is shown graphically in Fig. 47 which has been based on this problem.

40. Test for Supersession Due to Physical Decay.—It was pointed out in the previous paragraph on Physical Depreciation that in spite of repairs and renewals of parts the age and physical decay increase so that physical property eventually reaches a stage where it will be more advantageous to abandon the plant than to continue with the repair. With some kinds of property, this depreciation occurs with the passage of time whether the property is in or out of use. For example, the insulated wires and cables carrying current have covering layers of cotton, rubber, and other materials which deteriorate with age and with exposure to the weather. Thus, entirely aside from supersession due to inadequacy or obsolescence as discussed

possibilities of design modification will thus drop out. Then, if we are able to discover the ideal time at which to scrap this machine, we shall have a standard for judging each succeeding machine of identical design. For cases in which the supersession is to be made by a different type of installation, we must use a derivation similar to that employed in the discussion of obsolescence.

In the first place, as our machine continues in use, certain parts will be subject to wear, and will be replaced at an operating cost for maintenance which will increase as the more slowly accumulating repairs pyramid on top of the (smaller) frequent early repairs. Also, since restoration of worn parts will take place

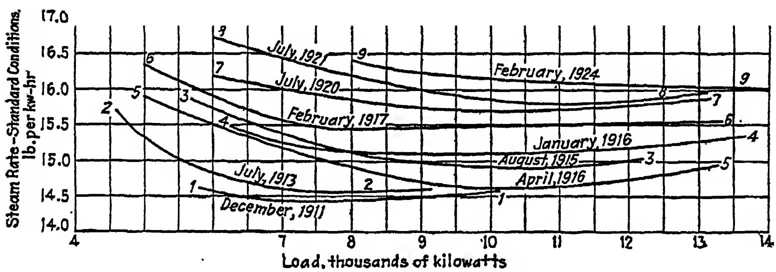


FIG. 48.—Steam rate of a 14,000-kw. vertical Curtis turbogenerator. Effect of use over a period of 13 years. (*By Sanford, Detroit Edison Co., in Power, Mar.* 23, 1926.)

only when they have deteriorated functionally, machine performance will always be lower than when new, and a given output can be secured only at a higher operating cost than when the machine was first installed. These two factors unite in encouraging us to replace the present machine at as early a date as possible.

Figure 48 illustrates this higher operating cost, due to increased clearances and blade corrosion with use, as shown by a 14,000-kw. vertical turboalternator unit. On 10 machines studied by Sanford, the average annual increase in steam rate was 0.16 lb. of steam per kilowatt-hour over an average period of 5.7 years. This corresponds to an average annual increase of 1.2 per cent of the guaranteed steam rate. The average annual increase in steam rate for the individual machines ranged from 0.11 to 0.25 lb. per kilowatt-hour. These values were encountered in properly maintained power plants and apply to machines in normal

operating service. All the turbines were of the Curtis type, two of them being horizontal and the remainder being vertical machines. The average increase in steam rate for the horizontal units was slightly lower than for the vertical units. The annual increase in steam rate for the unit whose performance curves are given in Fig. 48 was slightly less than the average for all the machines.

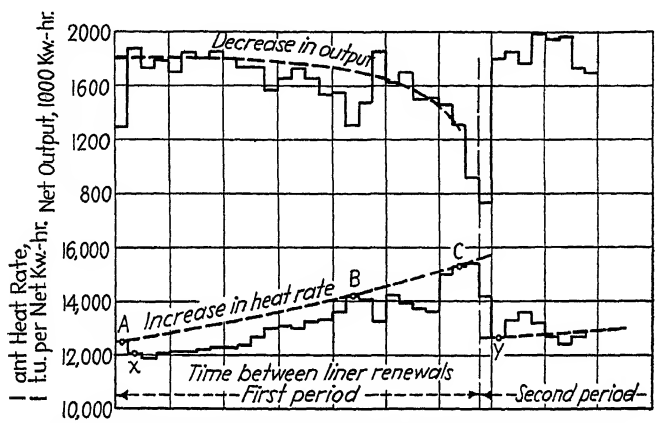


FIG. 49.—Increase in diesel-plant heat rate caused by wear.

In Fig. 49, Lee Schneitter gives data[1] from a plant containing slow-speed generating units covering the period from the initial starting of the plant. He reports as follows:

Point A shows that this plant started with a heat rate of 12,500 Btu per net kw-hr for an output of 1,300,000 kw-hr. At point B when this same output was produced, the station heat rate had increased to 14,200 Btu and at a still later date (point C), when liners were at the end of their normal life, the heat rate increased to 15,300 Btu for practically this same output. Comparison of points X and Y show that the heat rate after replacement of the worn liners was about 4.5 per cent greater than the value when the plant was new. The increase in heat rate from the time the plant was initially started to the time liners were replaced is 22 per cent; in other words, the average increase in heat rate between liner renewals should have been figured at about 11 per cent.

Another illustration is drawn from the cost of chassis repairs in a study of over 75 trucks used by a public-utility company.

[1] See Diesel Engine Maintenance, *Mech. Eng.*, February, 1937.

These repairs are usually the most important item of truck-operating cost and in this case amount to 15 to 30 per cent of the total operating cost per year. Figure 50 shows the variation of the annual cost with the actual age of the truck in years.[1]

On the other hand, if supersession is postponed, there will be an increasingly longer time in which to collect the depreciable part of the investment, and the yearly cost of this accumulation will be less and less.

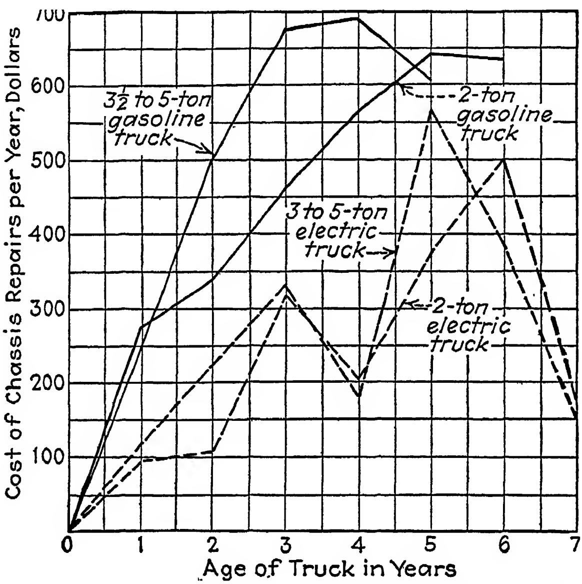


FIG. 50.—Increase of repair cost with age of trucks in years.

Lastly, there will be the fixed charges of taxes, insurance, and money use. Since we have already made our original investment, and since we are not at this time considering any changes in the design of our machinery, these three costs may be considered as constant, and will not affect our judgment in either direction.

To determine the economically ideal time for supersession, due to economic decay alone, we may write a cost equation that involves all these factors.

$$\$\text{ per year} = O + (\text{taxes} + \text{insurance} + e)P + a_n(P - S_n) \quad (29)$$

[1] See RASMUSSEN, C. F., Lower Operating Costs for Distribution Trucks, *Elec. World.*, Feb. 17, 1934.

where $\$$ = total annual costs.

O = equivalent uniform annual operating expense.

eP = cost of money use on the investment.

$a_n(P - S_n)$ = annual accumulation to the depreciation reserve to care for the depreciable part of the investment.

As here used, a_n is the annual accumulation rate based on the "actual" life of the machine, not on the estimated life.

By differentiating with respect to time and equating to zero, we shall find the point of ideal cost, *i.e.*, the point at which supersession should occur. The second term of the right-hand member being a constant will drop out, and we have

$$\frac{d\$}{dt} = \frac{dO}{dt} + a_n \frac{d}{dt}(P - S_n) + (P - S_n)\frac{d}{dt}a_n \tag{30}$$

$$= \frac{dO}{dt} - a_n \frac{d}{dt}S_n + (P - S_n)\frac{d}{dt}a_n = \text{zero for a minimum cost.} \tag{31}$$

Simplifying the third term, since by Eq. (4)

$$a_n = \frac{e}{\left(1 + \frac{e}{2}\right)^{2t} - 1},$$

where t is the time in years,

$$\frac{da_n}{dt} = \frac{-e\left(1 + \frac{e}{2}\right)^{2t} \cdot 2 \cdot \log\left(1 + \frac{e}{2}\right)}{\left[\left(1 + \frac{e}{2}\right)^{2t} - 1\right]^2} \tag{32}$$

Rearrange this in parts, then

$$\frac{da_n}{dt} = \left[\frac{-e}{\left(1 + \frac{e}{2}\right)^{2t} - 1}\right] \cdot \left[\frac{\left(1 + \frac{e}{2}\right)^{2t} - 1 + 1}{\left(1 + \frac{e}{2}\right)^{2t} - 1}\right] \cdot \left[2\log\left(1 + \frac{e}{2}\right)\right] \tag{33}$$

$$= -a_n \cdot \left(1 + \frac{a_n}{e}\right) \cdot 2\log\left(1 + \frac{e}{2}\right). \tag{34}$$

Expand the last term by Maclaurin's series, and

$$\frac{da_n}{dt} = -a_n\left(1 + \frac{a_n}{e}\right)\cdot e\left[1 - \frac{e}{4} + \frac{e^2}{12} - \frac{e^3}{32} + \cdots\right] \tag{35}$$

or since the value of e is generally in the neighborhood of 0.06, the bracketed terms become, respectively,

$$\left(1 - \frac{0.06}{4} + \frac{0.0036}{12} - \frac{0.000216}{32} + \cdots\right)$$

which are

$$(1 - 0.015 + 0.0003 - 0.000007 + \cdots),$$

and

$$\frac{da_n}{dt} = -a_n(e + a_n), \text{ approximately.} \tag{36}$$

It is also noted that dS_n/dt will, in general, be negative, since the salvage value decreases as time goes on. Substituting the value of da_n/dt from Eq. (36) in Eq. (31) gives

$$\frac{d\$}{dt} = \frac{dO}{dt} - a_n\left[(P - S_n)(e + a_n) + \frac{dS_n}{dt}\right] \tag{37}$$

in which the sum indicated within the brackets will be an arithmetical difference. If the value of the equation is zero, then the ideal time for supersession has just been reached; if the value exceeds zero, the ideal time has passed; and if the value is less than zero, the ideal time has not yet been reached.

The application of this criterion to any particular instant of time is a comparatively easy matter. The first term may be evaluated directly for n, P and S_n are known for that date. The term dS_n/dt may be conveniently arrived at by plotting the different salvage values against the times at which they occur and graphically finding the slope of the tangent to the curve at the point in question. In the determination of dO/dt, however, another method should be used.

It will be remembered that we originally used the symbol O to designate operating cost *per year*. When it comes to plotting this quantity as a function of time, the question immediately arises as to how this shall be done. The operating cost as taken from the ledger accounts is the total cost for the whole year and cannot, therefore, be accurately plotted as occurring at a date

corresponding to the end of that year. An approximation possibly will be given if it is assigned to an abscissa corresponding to the middle of the year.

We may also plot a curve between time and *total* expenditure for operating costs based upon steady prices for fuel, supplies, labor, etc., and on a constant load up to each period of time. The first derivatives of this curve obtained by reading the tangents to the curve at various points will then give values of O at any particular date, and if we plot as a second curve, the values of O so determined, against t, we may evaluate dO/dt as we did dS_n/dt.

However, such graphical methods are subject to the overemphasis of some sudden and startling increase in the operating expense for a period, which is probably not followed by similar increases in the next periods. This will show a prohibitive value for the derivative and falsely indicate supersession at that period. When examined in the light of later experience and balanced with succeeding periods of lesser rates of increase, it will be found that the economic life is prolonged.

In order to prevent this undue stress then, in the discussion in this section, the symbol O is used to designate an equivalent uniform annual operating cost spread over the n years of operation, not the actual operating cost during the nth year. The following method of analysis[1] may be used to obtain this value. The actual operating cost for each period is transferred, by an appropriate factor, to a present value at the beginning of the life of the unit under examination. These zero-time values are added serially to obtain the zero-time amount of the total operating cost throughout the period under consideration. An equivalent uniform operating cost is then determined from this total which would be effective for each period of the total time the unit has actually run. This equivalent uniform operating cost for the period, added to the depreciation allotment for the same time, gives the total variable cost for that particular period. This total will decrease at first with the lowering of the depreciation, owing to the actual use over more years of life, but will increase later on owing to the increased cost of operating the unit in its older years of life. Where the total variable cost reaches a minimum is the significant point for our analysis.

[1] Developed by Assistant Professor Archer, Department of Electrical Engineering, University of Illinois.

As an example of this type of problem, consider the case of owning and operating an automobile, a power unit with a relatively short life. The original cost is a matter of record, and the resale or salvage value in the open market is readily determined.

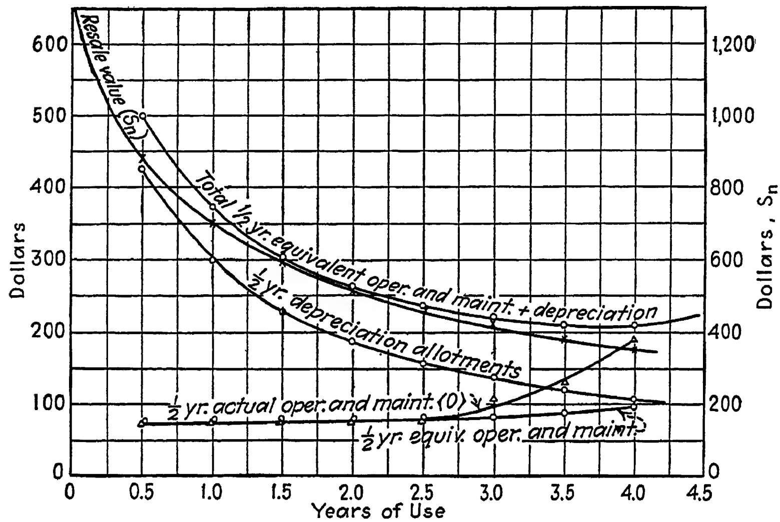


FIG. 51.—Study of economic life of automobile by half-year periods.

The owner can very easily determine the record of operating expense from check stubs or garage bills.

The following problem is based on a car purchased at a cost of $1,305. The resale values and expenses for operation, including supplies, renewals, and repairs for each 6 months' period, are listed below.

Time, years	Resale value	Operation and maintenance	License and insurance
0.5	$880	$ 74	$41
1.0	700	74	10
1.5	600	80	14
2.0	525	80	19
2.5	470	82	14
3.0	420	110	24
3.5	380	130	14
4.0	350	190	24

Money cost 6 per cent.

The successive steps in the solution; based on a half-year period, are as follows:

Time, years	Actual operating and maintenance cost	0-time value factor	0-time value operating and maintenance cost	0-time value Σ operating and maintenance cost	Equivalent uniform operating and maintenance cost	$\frac{a_n}{2}$	$(P - S_n)$	$\frac{a_n}{2}(P - S_n)$	Total ½-year operating and maintenance + depreciation
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0.5	74	0.971	71.85	71.85	74.00	1.00	425	425	499.00
1.0	74	0.943	69.85	141.70	74.00	0.492	605	298	372.00
1.5	80	0.915	73.20	214.90	76.10	0.323	705	228	304.10
2.0	80	0.888	71.04	285.94	77.00	0.239	780	186.50	263.50
2.5	82	0.863	70.70	356.64	77.80	0.188	835	157	234.80
3.0	110	0.838	92.18	448.82	83.00	0.155	885	137	220.00
3.5	130	0.813	105.80	554.62	89.20	0.131	925	121.20	210.40
4.0	190	0.7894	150.00	704.62	100.80	0.113	955	108.00	208.80

The saving due to the longer accumulation period available in which to cover the depreciation, less the loss in salvage value, gives a greater net saving than the increase in operating cost for each 6-month period up to the end of 4 years. After this time, it seems evident that the operating cost will increase so that it will overbalance the decrease in fixed cost. Therefore, the car was economically worn out at this time. The data of columns 2, 6, 9, and 10 and the values of S_n are plotted in Fig. 51.

41. Problems.

1. A piece of apparatus was installed 10 years ago at a cost of $100,000 with the expectation of a scrap value of $20,000 after 15 years. Its scrap value today is $30,000.

Its operating cost has been constant at $70,000 a year.

Money was raised from the sale of 50-year 6's at 95 and can today be raised on the same basis.

For $125,000, we can replace the old apparatus today with improved machinery estimated to last 20 years and to have $50,000 salvage value. It is estimated that the new apparatus can be operated at $60,000 a year.

By permitting the new apparatus to take over all assets and all liabilities of the old apparatus and to distribute them over the life of the new machinery,

a. What "apparent" annual saving will the new machinery effect throughout the 20 years?

b. Reduce this to an equivalent lump sum at the present date. Taxes and insurances are 3 per cent.

2. A machine was purchased 15 years ago at a cost of $50,000 with an expected life of 18 years and estimated scrap value of $6,000. Money was raised by the sale of 25-year 6½ per cent bonds, which were sold at that time at 106. The operating cost for the last 5 years has been practically constant at $140,000 a year.

A breakage has just occurred. Repairs costing $10,000 will put the machine in running condition again and allow the completion of the original program. The accident, however, has reduced the immediate scrap value to $6,500. Will it be cheaper to repair the machine, or to replace it?

3. Five-kilovolt-ampere distribution transformers 2,300-$^{230}/_{115}$ volts 60 cycles, last patent date 1896, were discarded from the service mains of the Detroit Edison Company. The average core loss per transformer is 150 watts. Assume their purchase price was $45 each, final salvage value $5, at the end of a 20-year life, after removal.

A modern 5-kva. distribution transformer would have a core loss of 40 watts and cost $75, with salvage $5 after removal. Money costs 6 per cent, taxes and insurance are 3 per cent, estimated life is 20 years. At time the old transformers were replaced, they were 10 years old; assume their junk value removed at $10 each. It costs $25 to install a transformer. Production cost of energy is 1 ct. per kilowatt-hour. The plant, transmission, and distribution capacity represents a cost of $250 per kilowatt with fixed

charges at 14 per cent. System losses (exclusive of core losses) are 10 per cent.

What was the economic justification for junking one of the old transformers?

4. The scrap value of a machine can be expressed as $\$S_n = \dfrac{\$100{,}000}{t \text{ yr.} + 5}$, and the annual operating cost as $O \dfrac{\$}{\text{year}} = 1{,}000t + 50t^2$. If money costs 6 per cent with semiannual accumulation, find whether at the end of 5 years we have approached, reached, or passed the proper scrapping time—and why.

5. A machine purchased for $1,200 depreciates at a uniform rate to total loss in 10 years. However, at the end of 6 years; it is shown that the machine has just reached the end of its economic life. If money costs 6 per cent and if the operating expenses, which were $100 for the first year, have risen at a uniform rate, what are the operating expenses for the sixth year?

6. A machine costs $1,750 new with salvage value, if retired from service at any time, of $250. Operating costs are $500 for the first year + $100 for each following year. Money earns 6 per cent annually. What is the economic life of this machine? (See J. H. Haynes, Depreciation and Amortization, *Elec. World*, June 22, 1929.)

7. A 12,500-kva. three-phase 11.5-kv. 60-cycle waterwheel generator has had its coils break down so that it will have to be rewound. If new silicon iron is installed now, it will cost $11,000. Generator runs 8,000 hr. per year, and energy costs 7.7 mills. Fixed charges are 12 per cent. Total repairs will cost $30,000 and performance is compared as follows:[1]

Losses	Original machine	New coils, new iron
Friction and windage	30 kw.	30 kw.
Core	190 kw.	110 kw.
Armature I^2R	123 kw.	98 kw.
Load	100 kw.	90 kw.
Field I^2R	85 kw.	85 kw.
Total	528 kw.	413 kw.
Armature temperature basis 40°C. ambient	140°C.	110°C.

What repair investment could be substantiated? Assume 20-year life and cost of money 6 per cent.

[1] See Brown, G. C., Machinery Rehabilitation, *Elec. World*, Apr. 21, 1934.

8. A normal wood pole in residential area carrying several circuits has a weak butt, but the top is good for eight more years. A new creosoted 11-ft. stub banded to the old butt will cost installed \$8.50. To replace the entire pole will cost \$30. Interest, 6 per cent; depreciation, 5 per cent; taxes and insurance, 2 per cent. What is the annual margin in favor of stubbing?[1]

[1] See Silver, A. E., Stubbing and Periodic Treating, *Elec. World*, May 12, 1934.

CHAPTER IV

POWER-PLANT LOAD CURVES

42. Estimates of Load.—In addition to the points noted in Chap. I under the heading The Preliminary Report and Estimate, it must be borne in mind that the design of a plant or system must not only take care of the present load, but must also be capable of expanding efficiently and economically to take care of the future demand. Most of the public-utility stations are connected to rapidly growing districts with an ever-increasing demand for power. Hence the engineer should foresee the conditions which the plant must meet so that a proper unit will be selected and adequate expansion be possible. To this end, studies of the growth of the system will be made, each plant being a particular problem in itself. The important thing is to endeavor to predict with a fair degree of accuracy the load curve of the plant, say for the next 10 years. In connection with this determination, a decision must be reached as to the character of the daily load, how it will change during the hours of the day, what the maximum and minimum values will be, how the daily curves will change for the various months of the year, when the peak loads of the year will occur, and what will be the average yearly load on the plant. From these data, the plant factors, which have such an important bearing on the operating cost of the plant, may be determined.

Considerable care to develop accuracy and skill in these forecasts will be entirely justified since an incorrect estimate of the load growth may involve a much more serious effect than merely postponing the time of installation of an additional unit. Suppose the pessimistic estimate of the increase of load called for a 35,000-kw. unit, whereas the optimistic estimate demanded a 55,000-kw. machine. This variation may represent a fundamental difference in the type of plant to be designed, in that the 55,000-kw. turbogenerator may use 1,200 lb. pressure and call for unit transmission lines, whereas the 35,000-kw. size would

normally take moderate steam pressure and use common transmission lines.

43. Cost of Power as Affected by Plant Factor.—Obviously, the fixed charges accumulate steadily on the total installed capacity in the station. If it were possible to utilize each kilowatt of capacity for all the 8,760 hr. of the year, each kilowatt-hour would carry only 1/8,760th part of the fixed charges on the kilowatt, but if the utilization covers only 4,380 hr. of the year, then each kilowatt-hour must carry twice the former charge, therefore, the cost varies inversely as the plant factor. It should be noted that the plant factor, *i.e.*, the ratio of the average annual load to the rated capacity, is the proper one to use in this connection, not the annual load factor.

The load factor for the plant or system is the ratio of the average power to the peak power for a certain period of time. The peak power is generally taken as that prevailing for a half-hour period, and the average load may be that in a period of one day, one month, or one year. The ratio would then be the half-hour daily, half-hour monthly, or half-hour annual load factor depending upon the period of time covered. Any one of these may be the basis of distribution of the operating costs depending upon the study in hand. Of course, the fixed charges run uniformly throughout the year.

If the maximum load corresponds exactly to the plant rating, then the load factor will be identical with the plant factor.

The utilization factor defined (A.S.A.) as the ratio of the maximum demand of a system, or part of a system, to the rated capacity of the system, or part of the system, under consideration is another important item in power operation.

In the operating items, fuel and labor costs increase per unit of energy with decrease of plant factor, although operating repairs are probably independent of the plant factor. At light load, the fuel used must not only be sufficient to supply that load, generally at a poor efficiency, but also auxiliaries of a capacity for much greater load must be run and fires must be banked under sufficient additional boilers to provide a capacity for the peak load of the day. As was pointed out in Sec. 25, the labor efficiency is low for a small plant and there would be but little change in labor for change in load. With increase in number of units, there will come a closer relation.

These variations of the cost of energy per kilowatt-hour with the plant factor for steam stations are shown in Table 19.

TABLE 19.—COST OF STEAM-ELECTRIC GENERATION FOR VARIOUS PLANT FACTORS*

Fixed charges, 12.5 per cent. Cost per kw. complete, $100, no reserve.
Yearly fixed charges per kw. installed $12.50.

$$\text{B.t.u. per kw.-hr.} = 10{,}500 + \frac{30{,}000}{\%\ \text{plant factor}}.$$

Coal = 14,000 B.t.u. per lb.

Plant factor, percentage	100	80	60	40	30	20	10
Hours of use	8,760	7,008	5,256	3,504	2,628	1,752	876
Fuel per kw.-hr., lb	0.773	0.777	0.786	0.804	0.821	0.857	0.964
Charges, cts. per kw.-hr.:							
Fixed	0.143	0.178	0.238	0.357	0.475	0.714	1.429
Other, except fuel	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Total, all charges except fuel	0.223	0.258	0.318	0.437	0.555	0.794	1.509
Cost of coal, cts. per kw.-hr.:							
Coal at $1 per long ton	0.0345	0.0347	0.035	0.0358	0.0366	0.0382	0.043
Coal at $3 per long ton	0.1035	0.1041	0.105	0.1074	0.1098	0.1146	0.129
Coal at $5 per long ton	0.1725	0.1735	0.175	0.1790	0.1830	0.1910	0.215
Total cost, cts. per kw.-hr.:							
Coal at $1 per long ton	0.2575	0.2927	0.353	0.4728	0.5916	0.8322	1.552
Coal at $3 per long ton	0.3265	0.3621	0.423	0.5444	0.6648	0.9086	1.638
Coal at $5 per long ton	0.3955	0.4315	0.493	0.616	0.738	0.985	1.724

* From Progress in the Generation of Energy by Heat Engines, by George A. Orrok, *Trans. A.S.C.E.*, 1939.

In hydroelectric generation, as contrasted with steam electric, a larger part of the investment for a given site is practically a constant, consisting of the cost of the dams, tunnels, reservoirs, riparian rights, and purchase of land. Thus, for the initial development of 378,000 hp. at the Conowingo plant, the estimated cost of $52,200,000 was subdivided approximately as shown in the table on page 146.[1]

Because the underlying cost of the development is such a large part of the total, there is the urge to install the greatest machine capacity that can be justified in order to take full advantage of the low "incremental" cost of installation which may vary, in

[1] See *Elec. World*, Aug. 13, 1927.

general, only from $55 to $70 per kilowatt for plants from 3,000 to 200,000 kw.[1] Since much of this construction is in heavy earthwork and reinforced concrete, the depreciation may be spread over 40 years or so and the insurance risk is slight. Also since the developments are generally in rural districts far from centers of population, the taxes may be lower. The fixed charges, then, may be taken fairly at 10 per cent (perhaps even 8 per cent)[2] rather than at the 12.5 per cent for steam plants.

	Per Cent
Constant items:	
Interest during construction, preliminary, engineering, legal and administrative expenses, contingencies	15.0
Road approaches	0.9
Water rights, land for reservoir and railroad	9.8
Railroad relocation	10.5
Dam, surveys, borings, highway bridge	16.8
Railroad connection, machine-shop equipment quarters	2.04
Intake, tailrace, and reservoir clearing	4.8
	59.8
Variable items with capacity:	
Power-house building, foundations, and station yard	16.8
Hydraulic machinery	5.9
Generators, exciters, and transformers	7.4
Switch gear and wiring	4.8
Transmission system	5.3
	40.2

Operation and maintenance costs at hydro plants will vary with the capacity, size, and number of units, Fig. 52 showing the values for typical modern plants.[1] However, it is in the determination of the relative value of the hydro installation in terms of replaced steam capacity that it is most necessary to consider each hydro project as an individual problem. For the particular system which requires the added capacity and under the special conditions that prevail, it must be decided whether the addition of steam or hydro power will result in the least total cost for the power supply as a whole. Steam capacity installed is available

[1] See Irwin and Justin, Economic Balance between Steam and Hydro, *Elec. World*, Aug. 20, 1932.

[2] See E. W. Kramer's discussion on Economics of Energy Generation, *Trans. A.S.C.E.*, 1939, p. 1032.

up to full rating any time when fired and maintained in repair. Hydraulic capacity, on the other hand, is available only in the amount and for the hours per day that the river flow of the occasion, as modified by pondage, can supply it. Outside of the large river flows of the Niagara and the St. Lawrence, enormous storage works are necessary to provide anything approximating constant power capacity on our variable-flow rivers. As a case

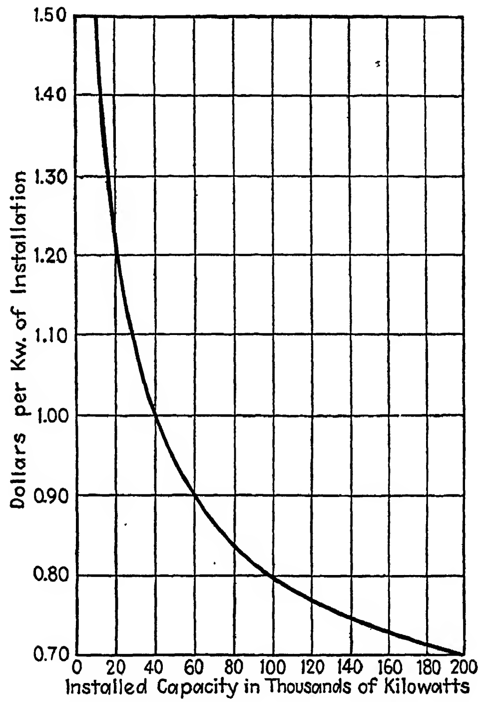


FIG. 52.—Operation plus maintenance cost at typical modern hydro plants. Based on records of over 30 plants. (*"Economic Balance between Steam and Hydro," Irwin and Justin, Elec. World, Aug.* 20, 1932.)

in point, Fig. 53 shows the power plotted against percentage of time for the discharge of the Tennessee River at Wilson Dam. The primary power thought available 100 per cent of the time was only 66,000 kw., and about 76,000 kw. for the average year. The secondary power was about 48,000 kw. for 80 per cent of the time, 100,000 kw. for 65 per cent of the time, 164,000 kw. for 50 per cent of the time, 300,000 kw. for 30 per cent of the time, and 368,000 kw. for 24 per cent of the time, all for the

average year.[1] For greater availability of the ranges above the prime power, therefore, regulation must be obtained to augment the minimum flow, or other reserve power (generally steam) must be available to supplement that of the hydro plant. Full realization of the potential output of Wilson Dam accordingly is dependent upon use of the storage reservoirs at Wheeler, Norris,

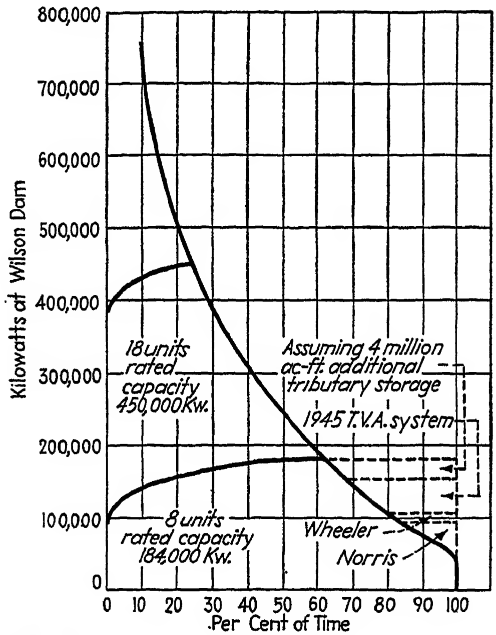


FIG. 53.—Average power available at Wilson Dam 100 per cent of time, estimated upon critical dry periods in 1925. (*Courtesy of Tennessee Valley Authority.*)

and other upstream dams together with interconnection to a large thermal system that can absorb vast amounts of peak power.[2]

As was noted above, the fixed charges on the incremental cost of greater machine capacity will still be a moderate part of the total development expense, and the operating cost for hydro plants will be almost constant irrespective of plant factors.

[1] See TYLER, M. C., Wilson Dam, *Elec. World*, Oct. 10, 1925.

[2] For power-duration curve of Columbia River at Bonneville, see Kaplan Turbines at Bonneville, by Heslop and Gessop, *Trans. A.S.M.E.*, February, 1939.

Large hydro plants on variable rivers with ample pondage for weekly regulation will be attractive, therefore, when their associated steam-plant system can absorb all the power developed. The hydro plant can then be used to carry a large peak load with its poor plant factor at probably a lower cost than it can be

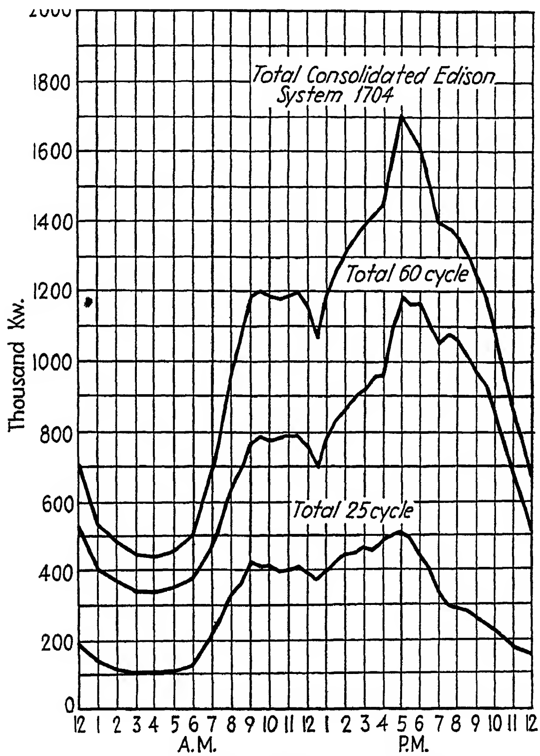


FIG. 54.—Load curve of Consolidated Edison Co. of New York for Wednesday, Dec. 13, 1939. Weather cloudy, rain in afternoon. Load factor 59.7.

provided by steam generation. Thus steam and hydro capacity under modern load conditions generally give the best system economy when used as complementary sources of power.

Figure 97 illustrates such a use of hydro capacity to carry the 5½-day peak loads of a utility system with both steam and hydro stations.

A most interesting analysis of the cost of power for any given load curve at the various hours of the day may then be made by applying data similar to that of Table 19 to the loads obtaining

at the different hours. As an example, we shall consider Fig. 54, which shows the system peak winter curve. The average load for the day is 60 per cent of the peak load, but for almost 10 hr. of the day the load is below this average value, and for nearly 5 hr. of the day the load is only 30 per cent of the maximum value. Hence, according to the relative values per kilowatt-hour shown in Table 19, the cost of energy per unit will be very much higher for the light loads of the morning hours than for the more heavily loaded part of the day. For this reason, the manager of a commercial power company should make a special effort to find customers who can use energy in the valley periods and so build up the load factor and decrease the cost of production.

44. Estimate of the Plant Load.—If the plant being laid out by the engineer is for a very distinct and definite purpose, such as the power supply for an office building, the problem of plant size is a reasonably simple one, since he will have no problem of probable future growth with which to contend. At the other extreme from this is the determination of plant size for a public-service plant, where the ultimate demands cannot be at all definitely determined.

In the first case, the major portion of the load will be for lighting purposes, and it is pretty sure that the development in illuminating engineering will supply units of sufficiently greater efficiency as the years go by to take care of any increase in the standard of illumination. Plant size in such a case is determined by the maximum number of lights likely to be turned on at any time, with a reasonable leeway representing a factor of ignorance to cover the possible use of signs, floodlighting, aero beacons, or other special features. No exact rules can be laid down for the predetermination of the plant's size, even in an office building, but it will not be far from right to take the architect's layout of lamps—or the illuminating engineer's if any such exist—and assume that 80 per cent of all the connected load in the offices will be simultaneously in use, adding to this the connected load of corridors and other general service departments.

For the elevators and other power-using equipment, a small office-building plant will have to provide, in addition to the lighting capacity, very close to 100 per cent of the maximum demand made by all such machinery, since with relatively few such units there is no certainty of any reasonable diversity in their use.

If, however, the office building is of very large size with six or more elevators, the diversity in use can be taken into account, using an increasing diversity as a higher number of such power-using units is installed. In a plant of such a nature, the extreme evening peak is likely to be highly accentuated and of very short duration, so that, proper voltage regulation in the plant itself granted, it is permissible and indeed desirable so to select the size of the units that they will carry approximately 25 per cent overload for the short interval of evening peak during the few worst winter months. In other words, the nominal plant capacity should be made approximately 80 per cent of the maximum load to be carried.

In this, however, due attention must be paid to the character of the community in which such an office building is located. In the larger cities, it is usual for offices to close earlier than in the smaller ones, and the relation between solar time and clock time materially influences the evening peak. For example, an office building in a town habitually terminating business at 5 P.M. would have a much less accentuated peak than a similar building in a town where 6 P.M. was the ordinary termination of the business day. The evening peak would be less accentuated in a town where standard time is ahead of sun time than in a town where the clock points to 5 when the sun says it is 5:30. There will not be so much opportunity to utilize overload capacity in a plant whose peak load is not accentuated as in a plant where the peak is of short duration and relatively high magnitude. The character of tenancy will very largely determine the nature of the power load. If the building is occupied almost exclusively by lawyers, the evening peak will be considerably lower than in the case of a building occupied by insurance men, machinery agents, etc.

Adequate prediction of the demand to be expected and of the size of the plant necessary to supply it can be made only by an engineer who has had considerable experience in the community, and who is familiar with the requirements of buildings of the general type under contemplation.

In making provision for the plant for such a building, the engineer must survey the possibilities of adjacent buildings coming under the same ownership, or of extensions being made on the building as initially planned, and his plant layout should be so

designed as to permit of growth to meet such contingency if at all within the range of even remote probability.

The selection of plant size for a building of a miscellaneous character, such as a combination of office and club rooms, restaurants and offices, with hotels and theaters in close conjunction, can be based only on a minute study of each detail in the demand of each of the users of the service from the prospective plant. In considering a plant for these services as for industrial plants in general, the designer will have to provide for a varied steam supply for process work, space heating, sterilizing, cooking, hot water, etc., along with electrical service, and will be particularly interested in the time relations between the requirement for steam and that for electric power.[1] If the building volume is large, the capacity of the boiler plant may be far beyond that corresponding to the size of the generators. Hence for public-utility supply the customer will need both electric service and central steam supply, or if he must operate a boiler plant, will be interested in an analysis of a noncondensing or bleeder-type turbine power plant or perhaps uniflow engine units. Figure 55 shows the winter and summer curves of electric and steam generation for the University of Michigan noncondensing steam turbine plant. At the early morning hours in winter when it is necessary to bring the buildings up to comfortable temperatures, there is but little electrical load available at the University so the plant sells its spare output to the Detroit Edison system, purchasing in its turn later in the day and in summer months when its electrical load exceeds its steam requirements.

The determination of plant size for an interurban traction proposition is very easy, since the initial project of the railway will have carried with it a complete analysis of the business to be transacted, a very definite prediction of train schedules, and a forecast of the probable growth of the road.

Similarly, plant studies for steam-railway electrification carry with them minute analyses of train movements and hence of power demands.

In the case of urban traction and, indeed, of city power service in general, the problem of load prediction is very much more

[1] For combined electric power and steam supply, see Deepwater Station of American Gas and Electric Co., *Elec. World*, Dec. 6, 1930, etc.

complex, depending as it does on a great many unpredictable factors which may be determined by current vogue or public whim. A public improvement may open up a tract of marginal territory which will develop into home sites calling for greatly increased transportation facilities. Housing sentiment, which in the late nineties turned toward apartments, may shift to the

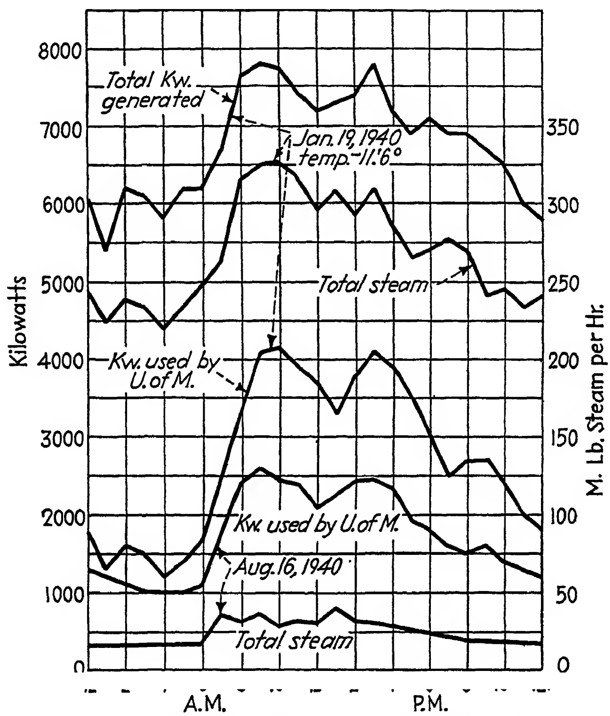


FIG. 55.—Winter and summer curves for electric and steam generation at University of Michigan power plant.

detached house, thereby considerably increasing the demand for carrying service. On the other hand, one recalls the vogue for bicycling and yet later the advent of the cheap automobile as material factors in influencing railway growth, and hence plant size. One recalls again the "jitney" development of 1914 and 1915, so that it is evident that the proportioning of a power plant for city trolley service is more or less a matter of individual guess.

Examination of the electric street-railway statistics shows that there has been a steady decrease in miles of track operated

from 43,106 in 1924 to 22,984 in 1937, and in most of the large cities there has been a marked reduction in the "riding habit" of the public, owing to the use of private automobiles. The latter have more than taken away from the traction companies the increased business that should have resulted from the growth

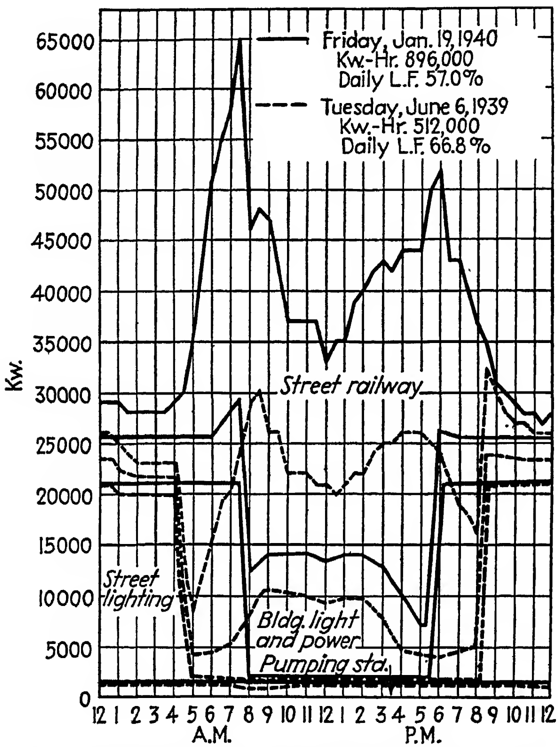


FIG. 56.—Winter and summer load curves, Municipal Power Plant, Detroit. (*Courtesy of Public Lighting Commission.*)

in the population. Additions to service are being made largely by the installation of gas and electric trolley busses to supplement the existing trackage and, in fact, quite a number of cities have entirely replaced the electric street railway with a total bus service.

Figure 56 shows the winter and summer week-day load curves for the municipal plant of the city of Detroit.

The determination of the size of power plant for general city service including domestic lighting, commercial lighting, indus-

trial power, railway and street lighting is perhaps less difficult since the rate of growth of such load may be determined from the previous years' records in the given community and reasonable extrapolation made for a period one decade ahead.

45. Demand Factors and Load Factors.—As a guide in the estimation of what proportion of the total connected load will actually come on the power plant at one time, the demand factors listed in Table 20 should be consulted. Here the demand factor is the ratio of the actual maximum demand made by the load to the total rating of the connected load. The load factor given is the ratio of the average power to the maximum demand for a 730-hr. month.

TABLE 20.—SURVEY FACTORS OF VARIOUS INDUSTRIAL PLANTS, DETROIT[1]

Type of plant	Area, thousand sq. ft.	Demand factor, per cent		Monthly load factor 730 hr., per cent	
		Power	Light	Power	Light
Automobile body........	1,120	48	80	53	35
Refrigerating............	450	39	35	55	41
Airplane................	25	6	80	19	22
Steel windows...........	44	26	34	62	19
Cigar manufacturing.....	24	64	52	38	21
Cigar manufacturing.....	40	58	47	12	19
Printers................	30	30	60	23	12
Printers................	10	39	57	25	44
Printers................	15	55	58	26	19
Stereotype..............	8	31	86	23	9
Print shop..............	6	68	41	52	90
Motor manufacturing....	10	63	54	20	12
Electric fixtures.........	12	82	76	45	40
Salvage.................	10	49	48	51	21
Machine shop...........	10	44	75	42	62
Machine shop...........	11	35	99	37	18
Machine shop...........	9	91	87	27	21
Machine................	1	48	93	16	32
Knitting mills...........	7	63	97	24	18
Envelopes..............	4	60	62	20	27

[1] COOK AND WARD, Detroit Edison Co., *Elec. World*, Nov. 16, 1929.

The reader is referred also to the "Standard Handbook for Electrical Engineers," sixth edition, Secs. 13, 15, and 16. The

Electrical World of Apr. 10, 1937, page 58, gives the load characteristics of retail commercial customers in Chicago.

TABLE 21.—GROUP DEMAND FACTORS[1]

	Per Cent
Heavy business districts	80
Light business districts	90
Factory districts—lighting	90
Factory districts—power (Dependent upon the characteristics of each individual group)	
Residential districts—more than 20 customers	33
Residential districts—less than 20 customers	50

[1] From SHAPIRO, L., How System Losses Were Reduced, *Elec. World*, July 16, 1932, p. 78.

The values for Group Demand Factors are given as the result of tests on the system of the Central Illinois Public Service Company (Table 21).

TABLE 22.—DIVERSITY FACTORS[1]

	Residence light	Commercial light	General power	Large users
Between consumers	2.5	1.46	1.44	
Between transformers	1.30	1.30	1.35	1.15
Between feeders	1.15	1.15	1.15	1.15
Between substations	1.10	1.10	1.10	1.10
Consumer to transformer	2.5	1.46	1.44	
Consumer to feeder	3.25	1.90	1.95	1.15
Consumer to substation	3.74	2.19	2.24	1.32
Consumer to generator	4.11	2.41	2.46	1.45

[1] "Standard Handbook for Electrical Engineers," 6th ed., Sec. 15. See also JORGENSEN and MATTESON, Residence Load Characteristics, *Elec. World*, Nov. 19, 1932, p. 688; and Residential Loads, Diversities, by Arridson and Cronin, *Elec. World*, Oct. 21, 1939, p. 43.

46. Diversity Factor.—After deciding as to the maximum demand of each part of the load on the plant, the engineer must consider how these individual maximum demands will occur in point of time. Generally, when one element of the load, say the lighting, is at its maximum point, at about 8:30 P.M. of a summer day, the traction and factory power loads will have passed their peak points by some 3 or 4 hr. This is considered in the so-called "diversity factor," which is the ratio of the sum of the maximum power demands of the elements of the load to the maximum demand of the whole load. Table 22 gives the diversity factors

for a large system. It is to be noted that the consumer's actual maximum demand divided by the diversity factor of the consumer will determine his effective demand at the transformer. Then there will be diversity in the time of the maximum demands of the transformers on a feeder, similarly as regards the feeders to a substation and as regards the substations and the genera-

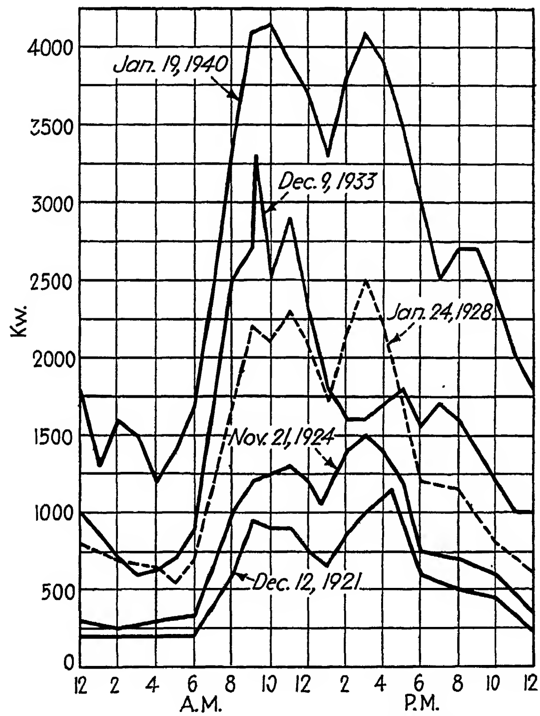


FIG. 57.—Peak-day load curves, University of Michigan power load.

tors. Thus to determine the effective demand of a consumer on the generators, multiply his connected load by the demand factor and divide the product by the diversity factor for consumer to generator.

47. Study of Growth, Small Power Plant.—As an example of the methods of analysis applicable to a small plant, the study of the growth of the load for the power station at the University of Michigan may be typical. This station supplies most of the electric power used in the buildings for lighting, ventilating, elevators, refrigeration drive, shops of the engineering and

grounds departments, and laboratory motors. A typical curve for the day of the peak load is shown in Fig. 57. The peak occurs on a winter day when the heavy lighting loads of the buildings come on at such a time as to combine with the motor loads of shops, laboratories, and ventilating fans.

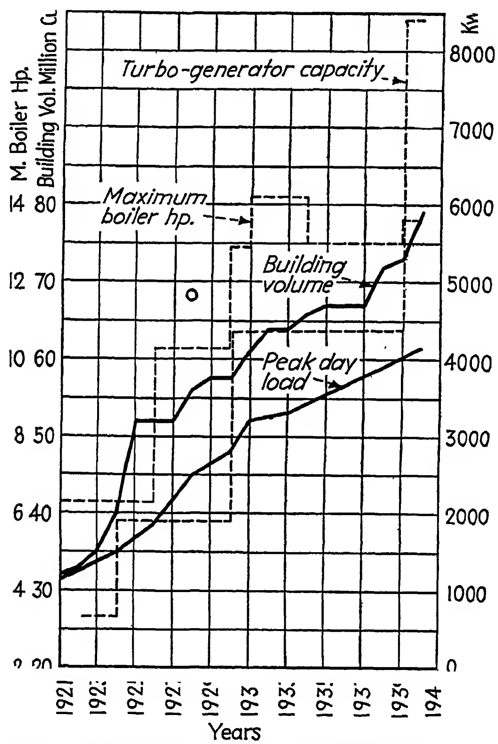


FIG. 58.—University of Michigan growth of building volume and load for power plant.

In order to care for the growth of the University, a power survey of each of the more modern of the buildings was made with graphic wattmeters to determine the maximum demand for each building on the power station, and the relation of such maximum to the connected load of the building. Taken in relation to the volume of the building metered, these demand factors gave a unit of demand per cubic foot for that type of building which, when modified for increase of lighting and more power use, could be applied to the volume of a new building

of the same type. Figure 58 is the time chart showing when the added constructions came onto the power-house load and the resulting peak loads occurring at the power station.

48. Study of Growth, Medium Plant.—In his paper[1] on "Steam Power Plants for Small Utilities, Municipalities and Universities," Dr. Gaffert places the annual costs as about 60 per cent for fixed charges and 40 per cent for production costs.

In line with keeping down investment, therefore, small utility systems stay fairly well within the limits of tried and proven cycles and equipment. If we consider that a small utility is limited to turbine unit sizes below 10,000-kw capacity and that unit sizes are more frequently of the order of 2,000 to 5,000-kw capacity, our field is fairly well defined. Machines up to 7,500-kw capacity will have an initial pressure range 0–400 lbs. ga. and 750°F while machines of 10,000 kw and above will extend the range from 0–450 lbs. Small plants will conform more nearly to 400 lb. throttle pressure in order not to enter the expensive pressure design of the 451–600 lbs. class machines and any departure from this standard has to be justified by an economic study. Fortunately, the operating temperature affects only the superheater in the boiler, the main steam pipe lines, and the high pressure end of the main turbine. Engineering requirements do not call for the use of alloy steel at 750°F because of stress or creep limitations. However, initial temperatures up to 825°F are being considered in certain cases where the unit size is 5,000 kw or above and where fuel costs are high. A study of these comparative conditions for which plant heat rates are shown in Fig. 59, indicates that it takes ten years to pay for the high pressure and temperature conditions with 5,000-kw units, $3 per ton coal, and with a 50% load factor. A unit of 10,000-kw capacity is of sufficient size to warrant consideration of 600 lb. and 825°F design conditions, particularly when future station growth is considered. . . . Fig. 59 shows the variation of turbine efficiency and cost with unit size. In this low capacity range, efficiency improves much more rapidly per 1,000 kw than in the larger sizes, and definitely indicates the advisability of selecting as large a unit as can economically be fitted to the load curve. The cost of steam generating units suitable for turbines of this capacity range will vary somewhat as shown in the same figure. Another curve shows approximate full load plant heat rates, obtainable for stations up to 10,000-kw capacity using modern turbines and steam generating units equipped with heat recovery devices. Considerable thought should be given to selecting the appropriate turbine size, since

[1] Mid-west Power Conference, April, 1939.

35 to 40% of the complete plant cost is thereby established at once in a unit plan.

The results of a study made for a municipal lighting plant are shown in Fig. 60. This plant carried the arc-lighting load of a

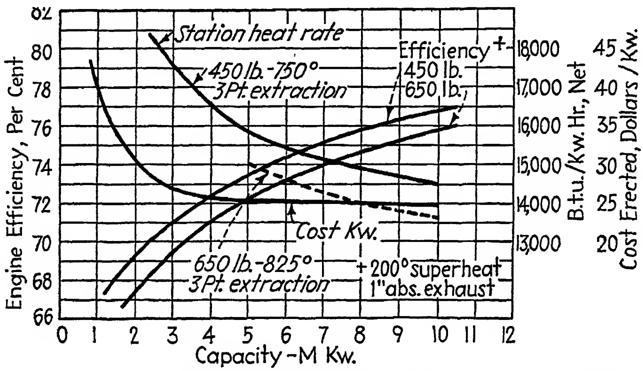


FIG. 59a.—Plant heat rates, efficiencies, and costs of small turbogenerators. (*Courtesy of Gaffert, Sargent, and Lundy.*)

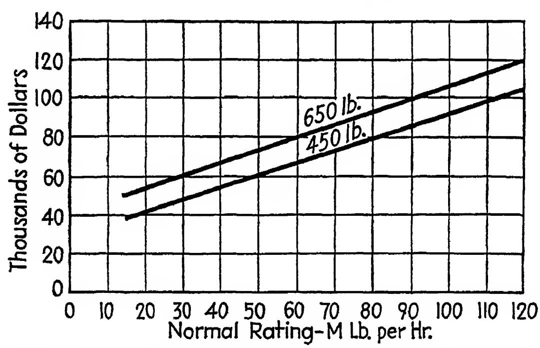


FIG. 59b.—Cost of pulverized-fuel-fired boilers with air heaters, fans, pulverizers, delivered and erected. (*Courtesy of Gaffert, Sargent and Lundy.*)

large city, along with the incandescent-lamp and motor load of the municipal buildings, giving the peculiar load curves of Fig. 61. This graph shows the maximum, average, and minimum loads on the plant for the month of January. The arc-lamp load for the future was determined from a study of the acreage of the city since 1896 and the number of arc lamps at end of each year. The load thus determined, when added to the estimated peaks for the incandescent and motor load, gave the plant peak loads shown in Fig. 60, which reached an estimate of 15,000 kw. in

1940. In addition to these assured items of load for the plant, there was a possibility of the addition of a load of 23,000 kw. for high-pressure fire pumps and of approximately 30,000 kw. for the electric railway system, in case the city took over the power supply for these services.

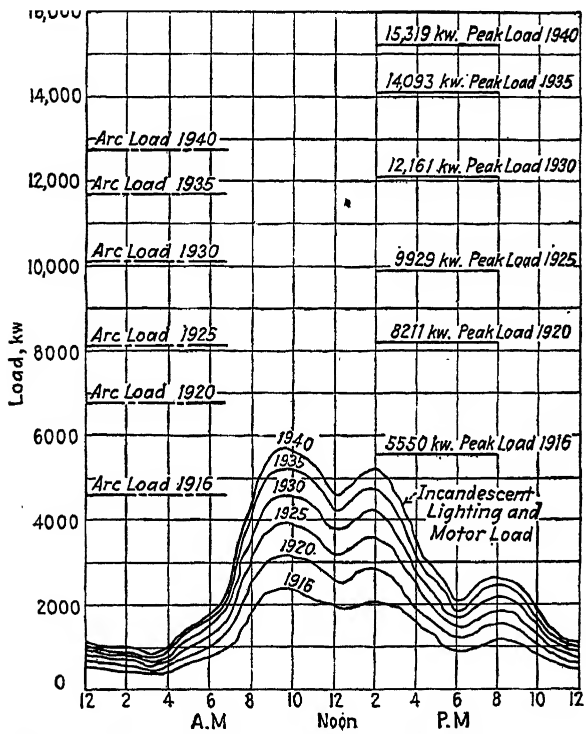


FIG. 60.—Study of growth of load, municipal lighting plant.

49. Study of Growth, Large Plant.—The same methods as applied to the medium plant will serve in the estimation of each element of the large plant, only there will be a greater variety of elements in the load. In general, there will be very complete and abundant records of each load element so that a study can be made covering a long period. Figure 62 shows the relation between the growth in population and the increase in energy used by the various classes of load for a metropolitan area, and Fig. 63 gives a typical day-load diagram of the same district for a winter peak day. Figure 64, Main Components of the System

Load, of the Detroit Edison Company shows the essential elements of their load. Figure 65 shows the peak-day load curves of the same company for 1907 to 1939.

The problem of probable load even for a general central station is, however, a local one, dependent on the industrial character of the community served and having certain critical points dependent on the size of the community. The annual supplements to the *Electrical World*, issued about the first week in May

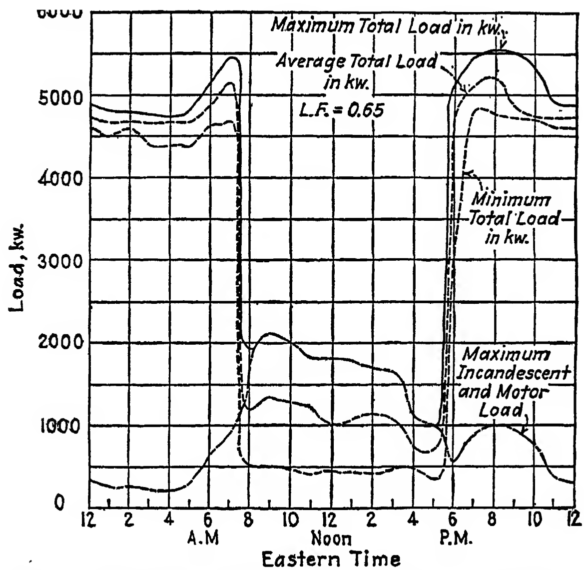


FIG. 61.—Municipal lighting plant, load curve for January.

each year, show the generator rating, output, number of customers, and the distribution of energy sold by the public-utility operating companies of North America. The distribution of output tabulates the energy for the following uses: domestic service, commercial, small light and power (retail), commercial, large light and power (wholesale), municipal service, electric-railway operation, other public utilities, used by the company itself, and system losses. These data will give much information from cities of the size under consideration.

Here again, however, a great many individual local factors come into play. If the standard of street lighting in the town is not up to the average of cities of its size, a commercially signifi-

cant growth of demand from this source may be expected and the plant plans should provide for extension to carry that additional load when secured. After a town acquires a radius of 1 mile to 1½ miles, measured from the business center, a marked increase in the traction load may be anticipated, especially in the

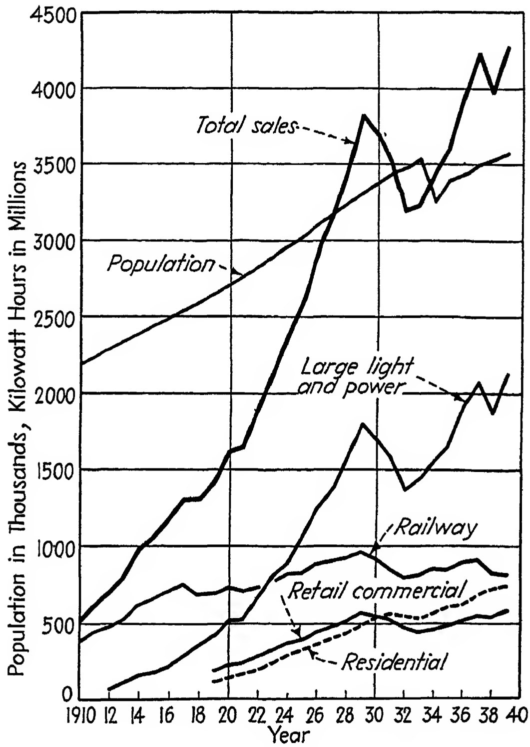


FIG. 62.—Population growth and sales of electric energy in a metropolitan area.

morning and evening, and the noon traction load may be expected to increase rather rapidly with a further increase in radius.

The power load thrown onto the central station may be capable of very considerable increase, and plant size should be provided for in the light of the character of the existing commercial-power sales organization. If this organization is not highly perfected and if the town is not a "saturated" town, the reasonable growth of load may be determined by a survey of the isolated plants in the town, taking into account the possibility of securing these as customers in the light of local competitive conditions such as

fuel cost, cost of labor, and significance of power cost in proportion to total manufacturing cost.

This sort of prediction is rather outside the province of the power-plant engineer and can best be made by a power sales expert, but the prediction of ultimate demand is essential to the determination of plant size. Judging by the experience of the

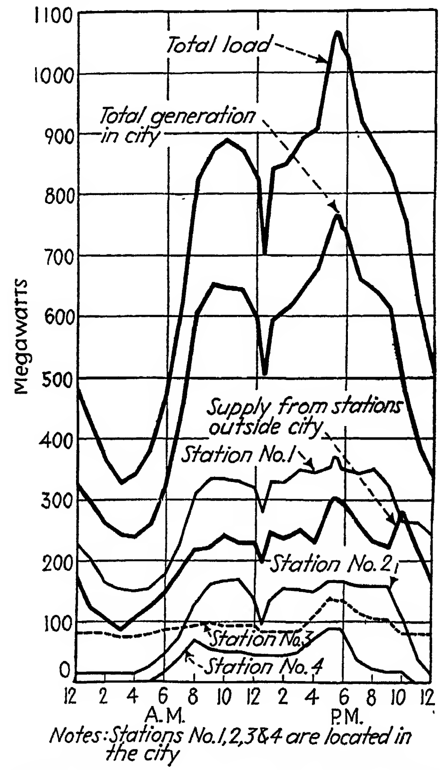


Fig. 63.—Load and generated supply of a metropolitan utility for a maximum winter day.

past, the engineer will do well so to design his plant that his initial determination of probable ultimate size of plant is capable of adjustment as future needs may dictate.

In the earlier days of the central-station industry, and indeed pretty well into the present century, central-station projects were undertaken without much foresight as to the future growth of the individual properties, the whole business being considered as primarily a lighting business. Within the second decade of the

present century, the growth of central-station power demand per capita of population has increased so tremendously as to astonish even those responsible for the development of public-utility enterprises. It is almost certain that, through the development of newer methods of power utilization, the future will witness even greater strides than have taken place in the past, and so the

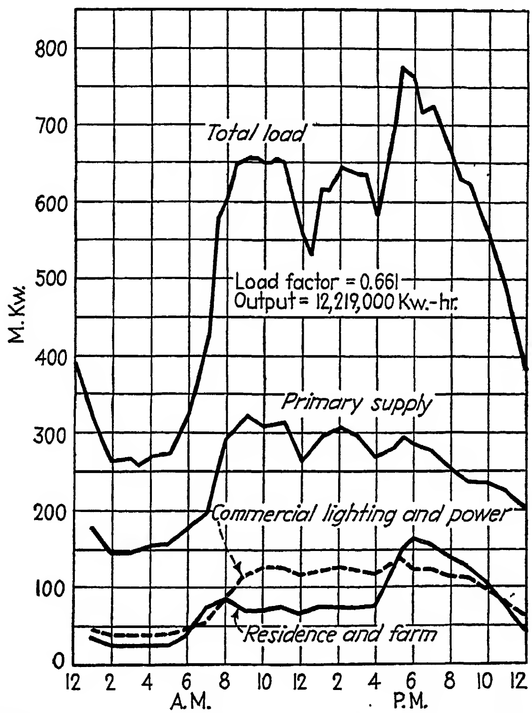


FIG. 64.—Main components of system load of the Detroit Edison Company. Wednesday, Dec. 13, 1939. Cloudy.

engineer should provide enough realty and sufficiently flexible plant layout to permit quadrupling his prediction of plant size made for a period 10 years beyond construction. The cost involved in such flexibility is extremely slight and is excellent insurance against hampering limitations in the future.

Figure 66 shows a study of the energy consumption, the load factor, and the maximum demand of an industrial city of the Middle West and estimates the power requirements up to 1950.

50. Power Plant for a Manufacturing Enterprise.—The same general statements apply to the small or moderately sized plant

for a manufacturing enterprise although probably not in quite so great degree as in the central-station industry. It has been the case that most of our American industries have developed to dimensions away beyond the original dreams of even their most ardent sponsors, and in almost every case power-plant reconstruction has been necessary simply as a result of inade-

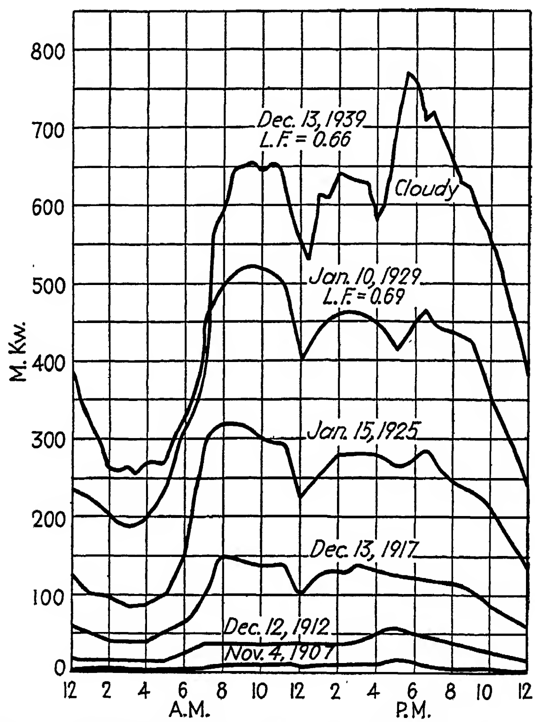


Fig. 65.—Peak-day load curves, Detroit Edison Company.

quacy to meet growing demands. Ample space for future plant extension is the only sane course to pursue.

Aside from the matter of provision for future growth, the power plant for a manufacturing establishment can be predicted on the basis of the manufacturing equipment to be installed at any time, using the proper diversity factors in connection with the individually driven machines if of standard character. In the case of highly specialized machines, an individual study of the probable operating conditions is the best that can be advised. For

example, the load of a paper mill would be based on the extreme demand of all driven machines with no allowance whatever for diversity factor. The same would be true in a plant where large continuous grinding operations are carried on, whereas even a very large plant using automatic screw machines or individually driven machine tools would not vary widely in character from other plants using the same type of machinery but for different purposes.

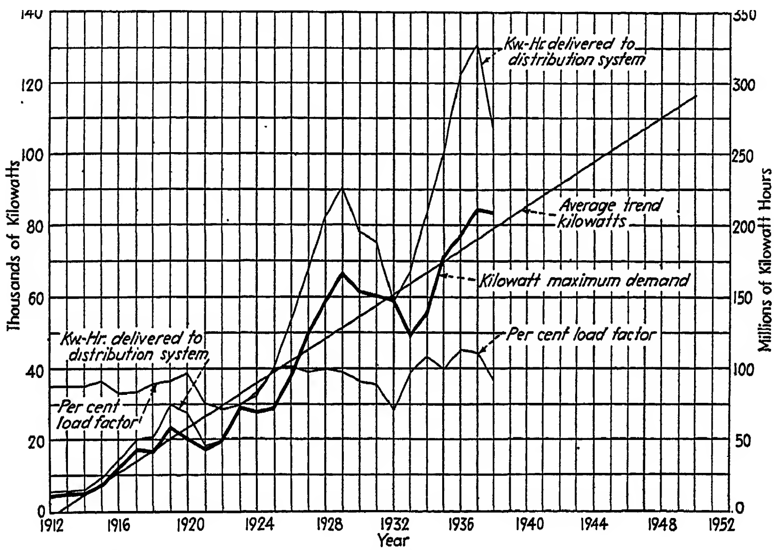


FIG. 66.—Study and estimate of power requirements, for an industrial city, up to 1950.

51. General Scheme—Power Costs.—Before proceeding to the discussion of the actual elements of plant design, we shall do well to have in mind a general scheme of power costs in the light of the facts discussed in the preceding chapters. It should be remembered that power costs discussed in this work are entirely different from those involved in questions of rate making, the fundamental differentiation being that power costs for the purpose of rate making must be uniform for all customers similarly metered, while power costs for the purpose of proportioning the individual parts of a plant design may be arrived at in each individual case on the merits of that case. Moreover, the power costs used in proportioning the plant design are differential or marginal costs

and need not concern themselves with the nonproductive and certain other general costs of doing business.

If we have under consideration the installation of a certain refinement in the plant equipment and are attempting to decide whether the saving to be expected from that refinement will justify the investment, we include in the saving only the costs actually obviated and not the dead losses necessary in order to get the concern started at all, which will exist, and will exist unchanged whether or not the refinement is installed. We are here dealing with our differential or marginal power costs. On the other hand, in the determination of costs for purposes of rate making, every cost entering into the business must be paid by the customers or the concern will go bankrupt, and therefore some or all of the customers, in general the latter. will have to take each his share of such nonproductive costs.

Unfortunately, we have no one word that properly expresses what is commonly understood by the expression "power." As we shall have to use the word power in its technical sense of "the time rate of expenditure of energy" in order to differentiate it from energy, the former being measured in kilowatts or horsepower, the latter in kilowatt-hours or horsepower-hours, we shall in this treatise use the word "service" to include the combination of power and energy, and in using the words "power" and "energy" shall expect them to be understood as differentiating the time rate of the expenditure of energy from the time integral of power.

52. Equations of Cost and Performance—Apparatus.—Analysis of a great mass of technical data shows that many costs may be expressed by a first-degree equation between cost and the dimensions of those factors influencing the cost. For example, the purchase price of a turboalternator may be expressed almost rigorously by the following equation:

$$\$ = A + B \text{ kw.} \tag{38}$$

One would naturally expect something of this sort, since the larger the unit, the more material will have to enter into its construction, and the cost of the material would, therefore, be proportional to the kilowatt capacity of the unit, whereas, irrespective of the size of the unit, drawings would have to be prepared and some labor expended in layout in the shop. All those

elements of cost which are independent of the size of the unit are included in the expression A. From this, it is perfectly clear that it is injudicious, if not absurd, to talk about the cost of a turbine "per kilowatt" or the cost of a boiler "per horsepower." As the unit costs vary inversely with the size of the machine, there is no such thing as a unit cost.

Figure 67 shows the data and cost equations of a line of turbo-alternators. The expression 6,500 + 39.2 kw. covers the

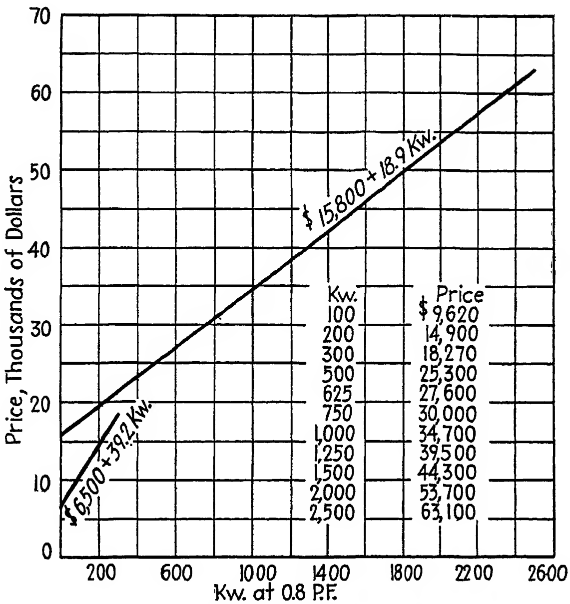


FIG. 67.—General prices (1940) for condensing turboalternators, 400–450 lb., 750°F., 3600 r.p.m. with direct-connected exciters. Plotted in form $\$A + B$ kw.

sizes up to 300 kw., and the expression 15,800 + 18.9 kw. represents the cost for sizes from 500 to 2,500 kw. Similarly, Fig. 68 gives the four first-degree elements necessary to express the cost of a line of transformers. See also Fig. 59 for cost of boilers.

A similar relation will be found to apply almost rigorously to the operating characteristics of machines over a very large range. With modern, well-designed machines of sustained efficiency, one can formulate a very nearly straight-line input-output relationship: for example, it may be found that the pounds of steam per

hour used by a turbine of given size are 10,000 lb. plus 12.4 lb. per kilowatt of electric load. In other words, we may write the equation in the form of the "Willans line"

$$\text{Lb. steam per hr.} = 10{,}000 + 12.4 \text{ kw.} \tag{39}$$

Typical steam-consumption curves for turbo units of 10,000 kw., 20,000 kw., 22,200 kva., and 27,700 kva. capacity, under

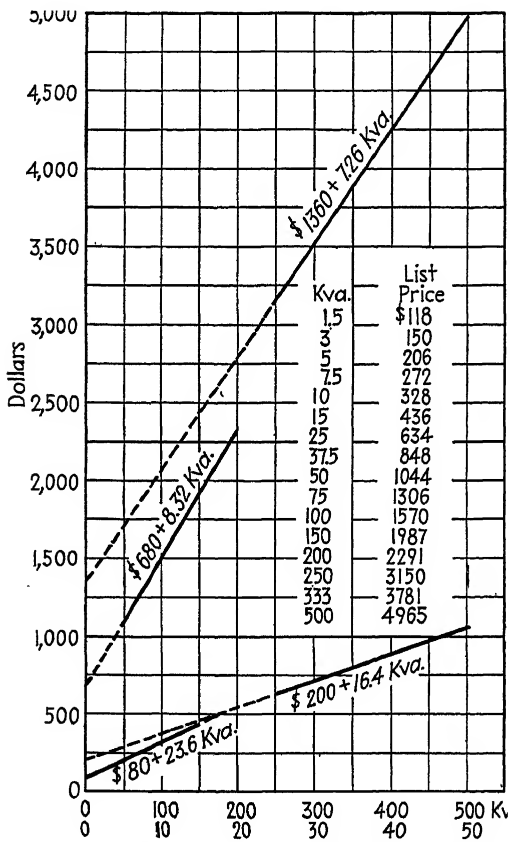


FIG. 68.—List price (August 1940) of distribution transformers, type H, single-phase, self-cooled, 60 cycles, 2400/4160Y to 120/240 volts, continuous rating for 55 C rise. For ordinary use apply 65 per cent discount.

particular conditions of steam and vacuum, are shown in Figs. 69 and 70 in the foregoing straight-line form as well as the customary form of pounds per kilowatt-hour.

A similar expression can be formulated with due cognizance of the variables for almost any type of machinery, even though the losses are of higher or lower degree than the first power of the

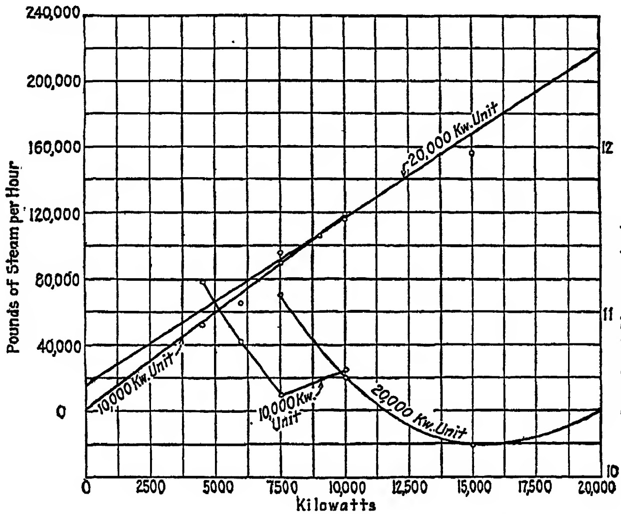


FIG. 69.—Steam consumption for 10,000-kw. and 20,000-kw. turboalternator units working at 250 lb., 250° superheat, 29-in. vacuum.

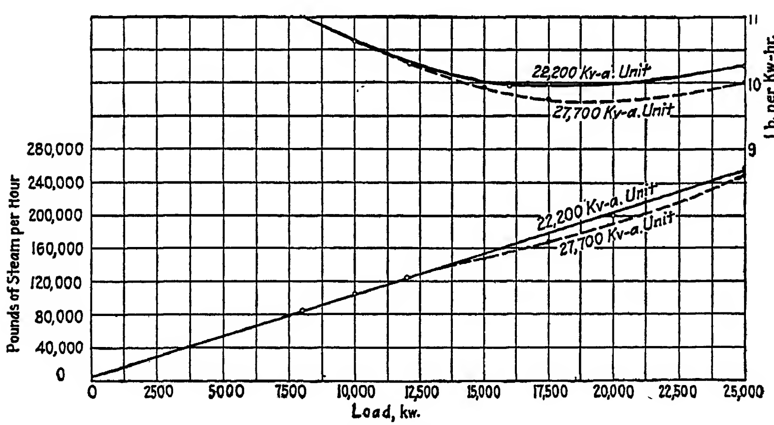


FIG. 70.—Steam consumption for 22,200-kva. and 27,700-kva. turboalternator units working at 325 lb., 225° superheat, 29-in. vacuum.

output, provided the straight-line expression is used only for such load range as permits the approximation, and provided

inferences are not drawn beyond the limits for which the approximation is good.

It is recognized that in a transformer the input will be greater than the output by the fixed iron losses and by the variable copper losses which are a second-degree function of the load. In such a case, if extreme refinement were desired, we should have to resort to a second-degree expression for the cost of secondary service, knowing the cost of service taken at the primary, but no substantial harm would be done in the use of our relationships for plant-design purposes, were an accurate input-output curve to be constructed and a clearly approximating straight line passed through this curve from 75 per cent load to 25 per cent overload. It will be noted that in an approximation of this character, which is general for electric translating and converting devices, the fixed element of service cost may actually become negative owing to our substitution of a first-degree approximation for second-degree rigor. In other words, the apparatus would have been debited for very heavy marginal costs and correspondingly credited on the fixed items. The copper losses in electrical machinery are so small that the inelegance of this approximation is more than outweighed by the facility it gives in design.

53. Equation of Cost, Service.—With these merely suggestive notations as to the reasons for such a method of formulation of costs, and its limitations, it may be said that at any point in the process of electric-service production the cost of such service may for design purposes be expressed as (1) a fixed element independent of the amount of power or energy, (2) an element proportional to the power, (3) an element proportional to the energy, and (4) an element dependent on the number of service hours of each generating unit called into commission. Except in rare cases, this fourth element may be left out of account, and we may then place the cost of service at any point in the process of production as

$$\$ \text{ per year} = A + B \text{ kw.} + C \text{ kw.-hr.} \tag{40}$$

As an example, the estimated cost per year of the two 10,000-kw. units plant to care for the loads of Fig. 60 was \$33,200 + \$6.93 kw. + \$0.00556 kw.-hr. This was determined from the detailed annual costs of the plant operation at the various times, as follows:

ESTIMATE OF DESIGN, 1920
Municipal Power Station

Item	Year				
	1920	1925	1930	1935	1940
Kw. maximum demand	8,200	13,900	16,200	22,100	23,300
Million kw.-hr	31.3	37.8	46.5	53.8	58.3
Investment	$1,202,253	$1,655,429	$1,655,429	$2,655,429	$2,655,429
Fixed charges, 7.5 per cent	90,142	124,157	124,157	199,157	199,157
Labor	31,300	38,480	40,600	49,200	53,200
Repairs, 1.49 per cent	17,870	19,900	19,900	31,850	31,850
Fuel	127,500	156,000	188,000	229,000	241,000
Annual cost	$ 266,812	$ 338,537	$ 372,657	$ 509,207	$ 525,207

Or

$$\text{Cost of service per year} = \$33{,}200 + \$6.93 \text{ per kw.} + \$0.00556 \text{ per kw.-hr.}$$

The total funded cost, including contractor's profit, interest during construction and engineering, was estimated to be $1,655,429. The estimate was made January, 1920.

Electric service has been sold on a similar basis by the Southern California Edison Company, the Los Angeles Gas and Electric Corporation, and the Southern Sierras Power Company to the contractors of the Los Angeles-Colorado Aqueduct. The maximum demand is estimated at 23,000 kw. at three-phase, 66 kv., delivered at Colton, Calif. The rate is $1,500 per month, plus $0.95 per month per kilowatt of maximum demand, plus $0.005 per kilowatt-hour.[1]

The propriety of such a formular expression becomes apparent from a very rough and merely indicative discussion—subject to many qualifications. To deliver service at the terminals of a machine, it is necessary in the first place to expend a certain amount of supervision and labor on the machine whether it is large or small. The cost of this is represented by A in the foregoing expression. If the demand for service is large, the machine will be large, if the demand for service is small, the machine will be small, and the carrying costs on the investment will therefore

[1] See *Elec. World*, Oct. 13, 1934, p. 616.

depend on the power taken whether for a very short time or continuously. This gives us the excuse for the existence of the *B* factor. Now it remains to take energy from the machine. Irrespective of the fact that labor has been paid for and irrespective of whether the power used is large or small and of the use of coal, oil, gasoline, or gas in the case of a heat-engine-driven plant, or water in the case of an hydraulic plant, there will have to be a certain number of units of these commodities entering into the production of each unit of energy output. The cost of energy-producing commodities is represented by factor *C*.

For purposes of plant design, factors *A*, *B*, and *C* must include only those costs or those portions of the costs which enter up to the point at which service cost is being estimated. If we are deciding between the use of steam or electric drive for our auxiliaries, in the case that the latter will take service from the main station bus, obviously it would be improper to include in the cost of the service for such auxiliaries, the investment in feeder switches or the cost of labor in connection with conversion apparatus used for a neighboring electric railway. It would be equally improper to assess against such service for auxiliary drive the cost of energy losses in converting apparatus not utilized for the service of these auxiliaries.

The rough general indication of the *A*, *B*, and *C* cost as being, respectively, due to labor, investment, and fuel is, we have said, subject to very marked modifications. Not all the labor employed in a given station is necessary merely to start the station and keep it operating. Indeed, in a very large station, most of the labor cost is due simply to the fact that the station is large and is properly allocable to power and energy. Not all the fuel burned in a steam plant is chargeable to energy, since though not one kilowatt-hour were delivered from the station, a certain portion of fuel would be required to keep the fires under the boilers and more would be required to keep the turbines turning over in readiness to meet demand. Clearly then, a part of the fuel is chargeable to demand. Certainly not all the investment is chargeable to demand. We have seen that the price of a turboalternator depends on a fixed quantity plus a cost per kilowatt. The fixed quantity evidently is a part of the price of having a station at all. As a matter of fact, a great deal of what is ordinarily termed investment cost is energy cost, since if a

plant were to be operated for but a few hours in the course of a year we would be likely to make it a very simple, initially cheap, and inefficient plant.

Now the most practical method of arriving at the value of the various constants A, B, and C, given in the foregoing expression, is to analyze the influence on the total plant cost of any service use made at any point in the plant. If, as in our illustration of station auxiliary service, we are investigating the desirability of using electric drive taken from the busses, we simply determine how much additional expense will be entailed by the 199 or 200 kw. of load that will be placed on the station bus. Evidently in a given plant the inclusion or exclusion of electric drive for the small auxiliaries is not going to increase any details of investment or operating cost aside from the boiler house, main generating units, and the active portion of the labor—the nonsupervisory labor. Also such auxiliaries will entail only such additional investment as is represented by adding 200 kw. onto the capacity of one machine, a few horsepower onto the capacity of each of the existing boilers, and the few cheaply bought additional cubic feet onto the already demanded investment in buildings. Every cost, then, entering into the supply of such service is merely a small increment cost, and in actual practice we concern ourselves with these costs alone for an existing plant, disregarding our A term entirely except as the mere fact of taking service involves expense.

Such an arrangement is perfectly permissible in the case of a single-plant development of a nonstagnant nature, but is not permissible in the case of a stagnant development or in the case of a general service to be supplied by multiple plants. The differentiation will be made clear by a discussion of the two exceptions.

54. Service Demand on a Stagnant Plant.—Let us consider, first, the case of a stagnant plant which might be exemplified by an existing power plant having ample overload capacity and supplying a business neither growing nor capable of growth. The addition of a service demand at any part of such a plant, provided it kept within the capacity of the existing units, would involve absolutely no increase in the cost of housing, labor, plant investment, or anything else, except the mere fuel supply and a very small portion of the minor supplies and repairs. The cost of such additional service would not include even a pro-rata fuel and

general supply cost. It quite evidently would have to bear no part of the cost of fuel for supplying the friction losses in engines, the radiation from pipe, or the underlying fuel expense represented in the fixed boiler losses. Every cent chargeable against such incidental service consumption would be represented in the coal burned for supplying the last and best kilowatt-hour of service utilization. In such a case, our *B* charge for marginal service is zero and our *C* charge very much smaller than in the case of the growing individual plant which we may take as the typical case.

55. Service Demand on a Growing Plant.—Returning now to our growing individual plant, it is evident that the addition of a service demand of 5 per cent on the busses would scarcely call for the installation of additional main generating units and that so long as the units remained as they were no additional labor would be called for. The same reasoning could be extended to the boiler room and all over the plant. However, if enough 5 per cent individual demands for auxiliary service were made, it is evident that in the aggregate they might be sufficient, were the plant not already built, to occasion the purchase of larger machines than would be planned for without this auxiliary service demand, and to occasion a slight addition to the housing cost of the machines. So far, our auxiliary service demands would have no responsibility for anything other than the low marginal energy and power costs represented in an increment of load on an already planned but not already purchased machine.

If now we consider the case where our plant is already in existence and it is proposed to utilize a considerable amount of auxiliary service, it may be necessary actually to install a new main generating unit, in which case the investment cost occasioned by the auxiliary service includes both the *A* and the *B* factors in the price of such unit. Now if our plant is merely projected and not already built, demands for auxiliary service have potentially, if not in fact, exactly the same effect as if the plant were a growing one, since every increase in plant demands brings nearer, by just so much, the day when an extra unit with its underlying and per-kilowatt charges will have to be installed.

A specific illustration should make this clear. We have already in existence a 50,000-kw. plant consisting of five main generating units of 10,000 kw. each. Our load is increasing at the

rate of 5,000 kw. a year and is at the present time 40,000 kw., giving 10,000 kw. of idle capacity which can apparently be used without further investment cost for the supply of station auxiliaries. The installation of such auxiliaries, however, would leave us only 5,000 kw. to take care of the growing load, so that at the end of one year the station would be loaded to full capacity. Had these auxiliaries not been installed, the station could have run two years without reaching full load. Here then the utilization of idle capacity for auxiliary drive has not in fact increased our investment of the present year, but has shortened by one year the date at which new investment will have to be made. This is exactly the same thing, to all intents and purposes, and therefore in such a typical growing, individual plant the auxiliary equipment must bear its proper share of the A factor in the investment and operating stand-by costs of an individual unit. Precisely identical reasoning applies with reference to boiler-house units, etc. However, there are certain elements in the cost of the plant that will not ever be increased by increment of load through the inclusion of electrically driven auxiliaries. One traveling crane will serve the plant indefinitely, one plant superintendent will serve the plant indefinitely. Only the marginal or increment cost of buildings will be chargeable to such auxiliary service and not the underlying A cost. Here then we are adhering pretty strictly to the use of a differential rather than an average cost.

56. Service Demand on a Metropolitan Plant.—In the case of very large metropolitan service, however, the time will come when it is not good judgment to carry all the eggs in one basket. The growth of the service will be so great that it will ultimately become the part of sanity to sacrifice something in our operating economy and build a second plant so as to ensure continuity of operation. If the engineer is sure, as he may reasonably be in communities of 100,000 or less, that the time is very remote when he will have to go to a multiple-plant arrangement, then he is safe in figuring only differential cost for service used internally by his own plant. If, however, he anticipates the time when more than one plant will have to be installed, he must then figure that every kilowatt of electric or other service taken for use inside the plant hastens by just so much the day when a new plant will have to be built. This new plant will have to carry its own plant

superintendent, its own initial expense for housing, its own machine shop, and all those other things not included within the differential costs of a single-plant development. Here then, the cost of auxiliary service, long before the second plant is started, carries potentially an increment of all underlying investment and nondifferential operating costs. The cost of such

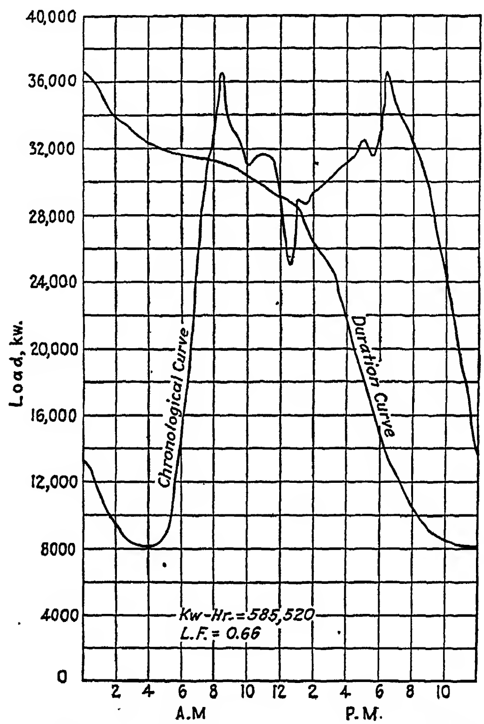


FIG. 71.—Assumed station load curve for average day of year.

auxiliary service must accordingly carry its pro-rata share of these underlying costs, *i.e.*, be a true average cost.

57. Typical Load Curve.—As a result of the engineer's estimate of the load, let it be assumed that the average daily initial load for the plant is to be as shown in Fig. 71. This shows the characteristic morning peak due to an industrial load, as well as the evening lighting load peak. The chronological curve is plotted by placing the ordinates (kilowatts) in their proper time sequence. The "load-duration" curve represents the same data

but the ordinates are rearranged in magnitude sequence, *i.e.*, with the greatest load at the left, lesser loads toward the right, and the least load at the extreme right. Corresponding to any kilowatt ordinate of the duration curve, the abscissa will give the number of hours per day during which that number of kilowatts of load have been on the plant. By averaging the mean daily curves for the months of April and October, an approximate average curve for the year may be obtained, which may be used as a basis for design studies and for determination of annual costs, etc. The area of either the chronological or the duration curve gives the total energy output in kilowatt-hours for the typical day (585,520 kw.-hr.) and hence establishes the total for the year.

58. Number and Size of Units, General Considerations.—After the expected initial load curve has been drawn up and the predictions made as to the extent and manner of growth of the plant load, the engineer is ready to proceed with the determination of the number and size of units to be installed.

In the case of a steam-power plant, as distinguished from an hydraulic plant or from one using internal-combustion engines, our selection of units will have to go back of the main generating units, if we are concerned in the complete plant design, and will have to take up the determination of the size and number of boilers to be used. This, however, is a problem to be settled in the design of the boiler house, as distinguished from the engine house, if the steam for all the main units is taken from a common header. With a given plant load and with anything like a reasonable range in the size of main generating units, no great difference in the steam utilized will be made by any probable variation in the size of main generating units. Therefore, the boiler-house problem will be simply one of producing a certain aggregate amount of steam demanded by the aggregate electric supply put out by the station, which is evidently independent of what happens between the steam header and the electric bus bars. This statement is true only when a common steam header is used. If it has been decided to build a steam station with boilers and turbines operated as an integral unit, the decision as to the size of the turboalternators may involve a corresponding effect on the size, investment, and operating cost in the boiler house, and therefore in such case the turboalternator and its section of

the boiler house will have to be considered as a unit. In our initial study, we shall assume a common steam header.

With a given initial load to be carried by the plant, it is evident that unless the load is excessive one main unit would be, so far as concerns investment, the cheapest arrangement, and were the plant to be operated at full load 8,760 hr. in the year, the one unit plant would be the cheapest to operate. Unfortunately, from the point of view of best plant economy, the load curves we have so far considered show that very few plants operate continuously, the load ever fluctuating from the maximum down to a small minimum. A unit capable of carrying the maximum load would be a somewhat less economical unit at the minimum load, so that if the fluctuations are very wide it may be desirable to install two units, each of half-peak capacity. When the load falls below one-half the maximum, we should then shut down one unit with its auxiliary apparatus and operate the remaining unit. In such case, investment would evidently be higher, owing to the relatively higher cost of the same capacity split up into two units, with a similar but more significant increase in the cost of auxiliaries and with the increased expense of housing, because of the fact that two units of 1,000-kw. capacity each cannot be got into the same space as one unit of 2,000 kw.

The matter of efficiency is of less significance in the large plant than in the small one, owing to the fact that the sustained efficiency of a large unit is considerably better than that of a small unit, other things being equal (see Figs. 30, 69, and 70). Our 2,000-kw. turbine would be likely to develop as high or possibly higher steam economy at 1,000-kw. load than would the 1,000-kw. unit, whereas the 1,000-kw. unit, if operated at 500-kw. load, would be likely to show relatively very poor efficiency. Similar considerations run through the whole range of operating and investment costs, so that we may expect to find the best justification for a multiplicity of small units in a plant of poor load factor and of small size, the least justification for a multiplicity of units in a plant of large size and high load factor.

59. Stand-by Units.—The question of the number of units to be installed is further influenced by the need for "stand-by" units and by the method of operation deemed necessary for these reserve units. A plant consisting of one main generating unit would be very risky were continuity of service imperative, since

the slightest difficulty might put this one unit out of commission and interrupt the whole power service at a time when interruption could not be tolerated. A serious breakdown, mechanical or electrical, might interrupt the service for a period of several days or weeks. Now if continuity of service is not imperative, one may be willing to take a chance of avoiding such accident on the assumption that an interruption, if it does occasionally occur, will not be so serious as to justify the investment necessary to maintain one idle unit simply waiting to fill the gap occasioned by the loss of an active unit. In general, the significance of continuity of service will be less in small unimportant plants than in large ones, and in a privately owned plant stand-by or emergency service may be procurable from a public-utility company at a cost less than the carrying charges on a spare unit. In the case of a large system of high economic importance, continuity of service is absolutely imperative and therefore at least one stand-by unit as large as the largest active unit must be maintained. If there is only one active unit, this will involve double investment in main generating equipment with a relatively heavy though not quite a 100 per cent increase in housing cost. If there are two active units, the stand-by investment need be only one-half of that in active units, although the aggregate investment in active units will be greater than in the former case. With three active units, the additional investment in spares will be only one-third of the aggregate investment in active units, so that the necessity for at least one spare unit goes a long way toward offsetting, and indeed may much more than offset, the excess investment that would otherwise be experienced in the use of a multiplicity of units. This retains the economic advantages accruing to the use of a number of units at all times nearly fully loaded as the station load varies from large to small.

As was noted in Sec. 33, Savings in Cost Due to Interconnection, the engineer has an option of securing the necessary reserve capacity, in whole or in part, from other stations of the same system, or from other power systems.

It is noted in Sec. 69 that the average service-demand availability factor is 99 per cent for hydraulic units, 96 per cent for large steam turbo units, and 92 per cent for diesel-engine units. It must be remembered that these are average values and not the experience of one typical station. The machines with

high service-hour factors are not the machines that show a high total outage. Figure 72 shows the outage factors for steam turbo units of varying years of operation. The necessity of rather frequent stops in the first year's operation of large steam-turbine units for tuning up and correcting defects of design and manufacture should be kept in mind.

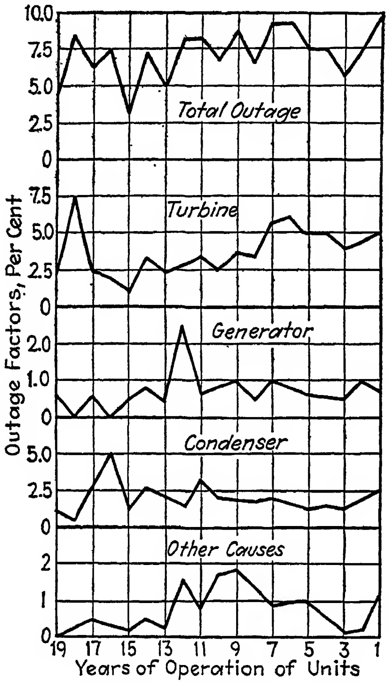


FIG. 72.—Steam turbo outage factors for units of varying years of operation. (*E.E.I. Bull. A-3, Turbines, August,* 1933.)

In the case of steam-turbine units, there is a differentiation between the large and the small plant. With five active units in service and no spare unit whatever, the loss of one unit could be supplied over the short peak-load period by carrying 25 per cent overload on each of the remaining four units. In this case, the overload capacity of the main units takes the place of an actual spare. Large steam-generating units, internal-combustion units, and water turbines, in general, have no overload capacity, since their best economy occurs nearly at the point of full load, beyond which point an hydraulic turbine or internal-combustion engine will stall. In general, the manufacturers of steam turbines of large size have abandoned the practice of equipping them for overload capacity at the expense of efficiency.

At its Summer Convention, June, 1932, the American Institute of Electrical Engineers held a special symposium on "Combined Reliability and Economy in Operation of Large Electric Systems," at which papers were presented on the power supplies of Detroit, Boston, Philadelphia, and Chicago. These discussions[1] brought out the individual solutions for the systems named,

[1] See *A.I.E.E. Papers* 32–70, 32–71, 32–73, 32–76.

considering energy interchange, load schedules, and reserve capacity.

60. Carrying Two or More Spare Units.—The practice of the Detroit Edison Company concerning firm capacity and continuity of service is as follows:[1]

The firm capacity of an individual plant is equal to the sum of the ratings of all main generators in that plant plus the available aid from tie lines, minus the rated output of the two largest units in the plant, modified if necessary by boiler capacity. This definition recognizes the need for routine repair of each main unit in turn together with the fact that there may be an emergency outage of another machine while the first unit is still under repair. Although periodic overhauling is scheduled for the light-load season of the year whenever possible, nevertheless the coincidence of a heavy industrial load and a daytime lighting load due to thunder storms is apt, at any time of year, to produce a short-time load equal to the seasonal peak. This concept of the firm rating of an individual plant (or its corresponding load area) is of special interest in the design of the electrical system and in the operation of an individual plant, and it is fundamental to the construction and use of properly proportioned ties between the various plants or load areas.

The firm capacity of the system as a whole is quite another matter and is of chief interest to the staff responsible for the operation of the entire system. System firm capacity is not merely the summation of the firm ratings of all the plants in the system as just defined. It is assumed, rather, to be the sum of the ratings of all main generators in the system, plus the available aid from outside sources, less the capacity of three machines which are not necessarily the largest in the system. These three machines will have, however, an aggregate capacity appreciably greater than that of the two largest. The capacity at any plant must if necessary be modified by the boiler capacity there available. For instance, at the present time, with its 23 main turbine units aggregating 945,000-kw. name-plate rating, together with 50,000 kw. from interconnections, the Detroit Edison system is assumed to have a firm capacity of 840,000 kw.

On a system of larger rating, the Consolidated Edison Company maintains reserve generating capacity in the New York area equal to the three largest units.

61. Large or Small Units.—The problem of size and number of units is very readily solved when we have to do with merely the initial year of plant operation. However, the load on our

[1] See *Power Plant Eng.*, March, 1939.

station may increase every year and certainly will in the case of a general power supply for a community. In such a case, it may be that that selection of unit size which was initially made will within a very few years prove to be distinctly an unwise one. For example, on a 10,000-kw. peak load, we might have installed two 10,000-kw. units, one active and one spare; or we might have installed five 2,500-kw. units, four active and one reserve. The latter choice may have been perfectly wise at the time the plant was built, but if in the course of 5 years the demand on the plant has doubled, this would involve nine units of 2,500 kw. each, one being a stand-by unit, whereas a 10,000-kw. unit would have given us one spare and two active units to handle the 20,000-kw. load. Were this ratio of increase of total load to continue during the next 5 years, the selection of the 2,500-kw. unit would be manifestly absurd.

Now the power-plant engineer, with his load predictions in hand for a period of 10 or 15 years following the date of design, must decide whether the ultimate possible economy of the plant will justify the present installation and operation of a relatively high investment in large units, or whether the initial economy of using small units will justify mortgaging the future. If the rate of growth of the property is bound to be small, the latter course will be the one to pursue, since it would be unwise to operate —for a great many years much of the time at low load—an expensive investment in large units, simply because somewhere in the distant future larger units may be justified. With a rapid rate of growth, however, the installation of one large unit with a duplication of much of the investment in a spare will be amply justified, even in the face of bad operating economy at fractional loads.

For a large metropolitan central station with heavy load, the units should be of such size that a unit plan of boiler layout will go with each turbogenerator, and the design of boiler house and turbine room will be affected by a change in the size of the generating units. The recent installation of many units, in sizes between 100,000 and 200,000 kva., where the load duration and the daily load cycle permit their efficient use, indicates that they are an important factor in giving better service at a lower cost. These larger sized units have better efficiency, a lower installed cost per kilowatt, require less operating labor and

reduce the required area in turbine room and land.[1] On the other hand, they involve more expense to overhaul, though the frequency of such maintenance is probably no greater than for smaller units. A larger unit also represents a greater capital charge for reserve capacity in the case of a single station. This feature is less important for a system that has more than one

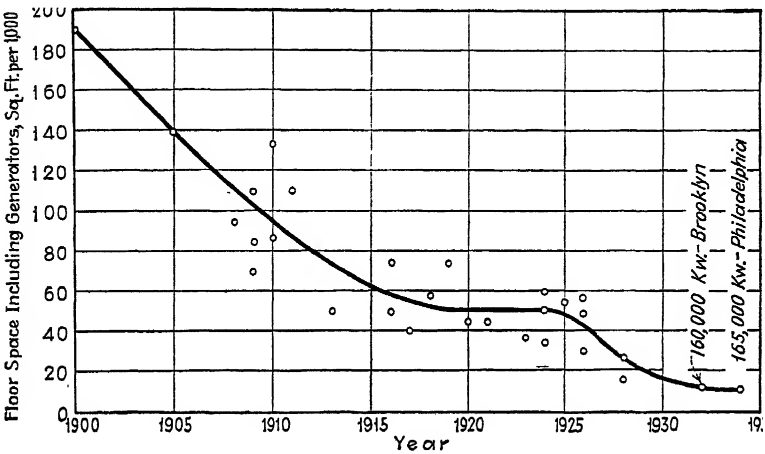


FIG. 73.—Progress in floor-space requirements of central-station turbine units, including turbines, main generators, and governing equipment, but no house generators or exciters.

power station or one that does not use single-barreled machines. Figure 73[2] shows the great saving in floor-space requirements of main turbo units accomplished by the newer designs. Correspondingly, the increases in rating and the newer welded-frame constructions have materially reduced the weights of the machines. Figure 74,[2] showing the pounds per kilowatt for main turbogenerator units, illustrates this very clearly. The rise in the weight per kilowatt shown for 1920 to 1923 is probably due to a speed change from 1,800 to 1,200 r.p.m. for some large units, but with the return to 1,800 r.p.m. the weight per kilowatt again decreased rapidly for the units indicated.

A further qualifying influence on the analysis may be had from a consideration of whether our spare units are to be idle. In the

[1] See ZIMMERMAN, C. D., Power Station Cost Control as Affected by Design, *Elec. World*, Nov. 19, 1932, p. 699.

[2] From GILT, CARL M., Some Reasons for Large Generators, paper before the Power Group, New York Section, A.I.E.E., October, 1929.

case of extremely important service, one spare unit will be kept in operation at all times, either floating on the line—*i.e.*, in parallel with the station bus, but delivering no load—or preferably carrying its share of the load, all the units operating at something less than full capacity. In such case, accident or deliberate intent may drop one active unit from the bus without embarrassment to the system. Were the spare unit kept slowly turning over or absolutely dead and not kept with a full operating crew,

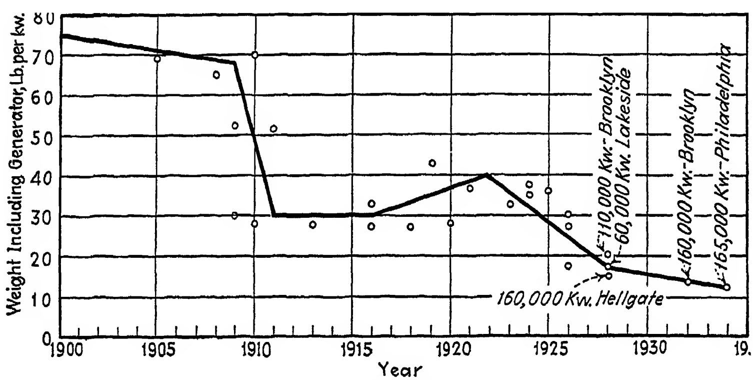


Fig. 74.—Weights of central-station turbine units. Main turbine and generator equipment only.

there would be an embarrassing period after the loss of one active unit during which time some of the station load would have to be dropped in order to prevent "dumping" the whole load, and the spare unit would have to be brought up to speed and synchronized after getting the operating crew to the unit. Such a method of operation, however, offers the chance of dispensing with an extra labor item and avoiding all those operating costs entailed by turning the unit over, together with the cost of operating all its auxiliaries.

If, then, our spare unit is to be kept synchronized at all times or actually carrying a load, it will be able to pick up load very rapidly in case of failure of other source of power supply. For example, by special operating arrangements, the 1,400-lb. turbines of Station A, Pacific Gas and Electric Company, San Francisco, were able to pick up load from 10,000 to 55,000 kw. in 50 sec.[1] The "spinning" spare will involve not only the

[1] See Estcourt, V. F., Tests Demonstrate Steam Plant Standby Characteristics, *Elec. World*, Jan. 13, 1934, p. 86.

investment cost but every underlying cost of a unit of such size, and in that very process places a greater cost burden on service production because of the size of such spare unit. This indicates the additional necessity for the use of a multiplicity of rather smaller units in case the spare is to be instantly available than in the case where it is to remain idle, subject to use only after the transference of labor from a disabled unit and after the spare has been brought up to speed, synchronized, and loaded. Such considerations will have to be taken into account and decision made more or less independently of the analysis of the best number and size of units to use, basing such decision on the more or less intangible importance of extreme continuity of service. However, such a question has its influence on the number and size of units, and hence the total cost of operating the plant. It also has a reactive effect on the cost of its own adoption. It may be that the expense involved in this manner of handling floating spares would be so great that it would not be worth while. Or it might be, on the other hand, that analytic studies would show the expense of such operation to be relatively small in proportion to the value of the greater continuity. The engineer and the commercial officers of a public-service corporation, or the manufacturing force in an industrial plant, would decide whether the extra cost of such operation was warranted, but could do so only in the light of an analytic study based on both alternatives.

62. Number and Size of Units for Initial Load.—On the basis of a load as represented by Fig. 71, let it be assumed that for the generation of steam power for this load, the cost of owning and operating possible units may be expressed, as was discussed under Sec. 52, Equations of Cost, by the following costs per year:

10,000-kw. units = $50,000 + $3.0 hr. + $0.006 kw.-hr.
15,000-kw. units = $67,500 + $3.5 hr. + $0.0055 kw.-hr.
20,000-kw. units = $80,000 + $4.0 hr. + $0.005 kw.-hr.

For the 10,000-kw. units (running time only):

Unit 1,	0 to 10,000 kw.,	operates 24.0 hr. per day
Unit 2,	10,000 to 20,000 kw.,	operates 20.2 hr. per day
Unit 3,	20,000 to 30,000 kw.,	operates 16.4 hr. per day
Unit 4,	30,000 to 40,000 kw.,	operates 10.6 hr. per day
	Operation of all units	= 71.2 hr. per day

Basic cost, 4 units at $50,000	= $ 200,000
Operation cost 71.2 hr. × 365 × $3	= 77,964
Energy cost, 585,520 kw.-hr. × 365 × $0.006	= 1,282,289
Total annual cost	= $1,560,253

For the 15,000-kw. units (running time only):

Unit 1,	0 to 15,000 kw., operates	24.0 hr. per day
Unit 2,	15,000 to 30,000 kw., operates	18.0 hr. per day
Unit 3,	30,000 to 45,000 kw., operates	10.5 hr. per day
	Operation of all units =	52.5 hr. per day

Basic cost, 3 units at $67,500	= $ 202,500
Operation cost, 52.5 hr. × 365 × $3.5	= 67,069
Energy cost, 585,520 kw.-hr. × 365 × $0.0055	= 1,175,431
Total annual cost	= $1,445,000

For the 20,000-kw. units (running time only):

Unit 1,	0 to 20,000 kw., operates	24.0 hr. per day
Unit 2,	20,000 to 40,000 kw., operates	16.5 hr. per day
	Operation of all units =	40.5 hr. per day

Basic cost, 2 units at $80,000	= $ 160,000
Operation cost, 40.5 hr. × 365 × $4	= 59,130
Energy cost, 585,520 kw.-hr. × 365 × $0.005	= 1,068,574
Total annual cost	= $1,287,704

Therefore in standardizing on one size of unit, the 20,000-kw. units would be the cheapest to operate on the load curve.

For a discussion of how to keep the number of extra machine hours to a minimum for the operation of local reserve units, the reader is referred to the article "Scheduling Units for Maximum Over-all Economy," by C. C. Baltzly.[1]

63. Load Division between Two Units.—When an actual load is to be carried by two identical units, the load being in excess of the capacity of one, we desire so to divide the load that the total operating cost is a minimum. Let the load be divided equally between the two units. Then place an increment load on one of them, correspondingly relieving the other. The operating cost will be increased for the first unit, decreased for the second. If cost increase is in excess of cost decrease, we return to the equal load division, otherwise we continue the "unbalancing" process.

In Fig. 75, $-\Delta\$ \lesseqqgtr +\Delta\$$ for cases I, II, III, respectively, and unbalancing the load is undesirable, desirable, and a matter of indifference in the respective cases.

[1] See *Elec. World*, Mar. 4, 1933, p. 284.

In general, if we have two machines, whether similar or not, we observe that if $d\$/d$ kw. is the same for both we have negligible change in total cost if we vary the loading by an extremely small

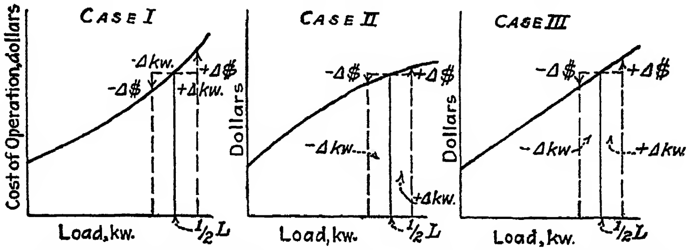


Fig. 75.—Study of load division between two units, change of cost with load.

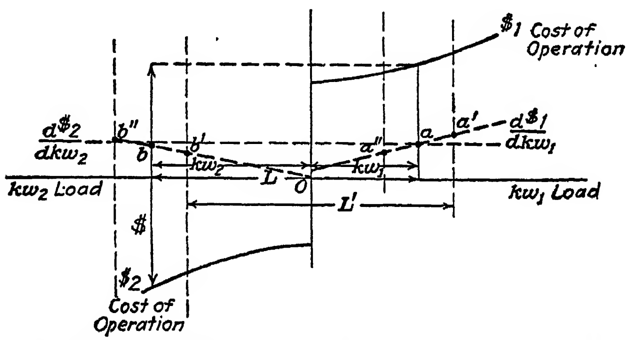


Fig. 76.—Division of load between two units. Cost curves concave away from axis.

amount—either the cost is a maximum or it is a minimum. We may determine which by considering $d^2\$/d\ \text{kw}^2$

$$\left(\begin{array}{l} f(x) \text{ is a maximum if } f'(x) = 0 \text{ and } f''(x) = \text{a negative number} \\ f(x) \text{ is a minimum if } f'(x) = 0 \text{ and } f''(x) = \text{a positive number} \end{array}\right)$$

or by noting the effect of a finite and appreciable departure from the condition

$$\frac{d\$_1}{d\ \text{kw}_1} = \frac{d\$_2}{d\ \text{kw}_2} \tag{41}$$

In Figs. 76, 77, and 78 are shown the load-cost curves of dissimilar machines, that for machine 1 being plotted in the usual manner, that for machine 2 being plotted in the *diametrically* opposite quadrant.

We have assumed no reversal of curvature in any case and only *for convenience*, in comparison, have shown identical plant loads L occurring with identical values of $\$_1$ and of $\$_2$ in all three cases. These identical values are not essential to the discussion. In Fig. 76 and again in Fig. 77, the curvatures of the graphs for the two machines are of the same sense; in Fig. 78, the curvatures are of opposite sense.

The first derivative curves are shown by broken lines. We have seen that a possible minimum total cost *may* be found when the first derivative values for the two machines are equal. We have shown the load (L) so distributed as to meet this condition —note line *ab*.

If, in Fig. 76, the load is so shifted as to increase slightly that on machine 1—see L'—inspection would seem to indicate that our total cost will increase, since additional kilowatts imposed on machine 1 become more and more expensive and hence cost more than the cost of a marginal kilowatt at a ($d\$_1/d\text{ kw}_1$); whereas the kilowatts removed from machine 2 become less and less expensive, and hence their removal saves less than the cost of the marginal kilowatt at b, where $d\$_2/d\text{ kw}_2 = d\$_1/d\text{ kw}_1$. If the curve $\$_1$ to the right of a and the curve $\$_2$ to the right of b were to straighten out, it would be a matter of indifference whether we carried the load distribution L or imposed more load on machine 1. We may then say that if our curves are concave away from the load axis, it is necessary and sufficient to ideal load distribution that the derivative of cost with respect to load be equal on the two machines.

Mathematically for the case of Fig. 76 we have

$$\frac{d^2\$_1}{d\text{ kw}_1^2} > 0; \quad \frac{d^2\$_2}{d\text{ kw}_2^2} > 0 \tag{42}$$

$$\text{kw}_2 = L - \text{kw}_1; \quad \$ = \$_1 + \$_2 \tag{43}$$

$$\frac{d\$}{d\text{ kw}_1} = \frac{d\$_1}{d\text{ kw}_1} + \frac{d\$_2}{d\text{ kw}_1} \tag{44}$$

but

$$d\text{ kw}_2 = -d\text{ kw}_1 \tag{45}$$

therefore,

$$\frac{d\$}{d\text{ kw}_1} = \frac{d\$_1}{d\text{ kw}_1} - \frac{d\$_2}{d\text{ kw}_2}. \tag{46}$$

For a maximum or minimum, $d\$/d\,\mathrm{kw}_1$ should $= 0$, *i.e.*,

$$\frac{d\$_1}{d\,\mathrm{kw}_1} = \frac{d\$_2}{d\,\mathrm{kw}_2}. \tag{47}$$

If departure from this condition causes $d\$/d\,\mathrm{kw}_1$ to increase *above* zero, then we have an optimum condition. This will be the case if $d^2\$/d\,\mathrm{kw}_1^2 > 0$, for then a finite increment of load $\Delta\,\mathrm{kw}_1$ will give

$$\Delta\,\mathrm{kw}_1\left(\frac{\partial}{d\,\mathrm{kw}_1}\frac{d\$}{d\,\mathrm{kw}_1}\right) = \Delta\,\frac{(d\$)}{(d\,\mathrm{kw}_1)} > 0. \tag{48}$$

But from Eqs. (45) and (46)

$$\frac{d^2\$}{d\,\mathrm{kw}_1^2} = \frac{d^2\$_1}{d\,\mathrm{kw}_1^2} + \frac{d^2\$_2}{d\,\mathrm{kw}_1^2} = \frac{d^2\$_1}{d\,\mathrm{kw}_1^2} + \frac{d^2\$_2}{d\,\mathrm{kw}_2^2} \text{ both of which } > 0. \tag{49}$$

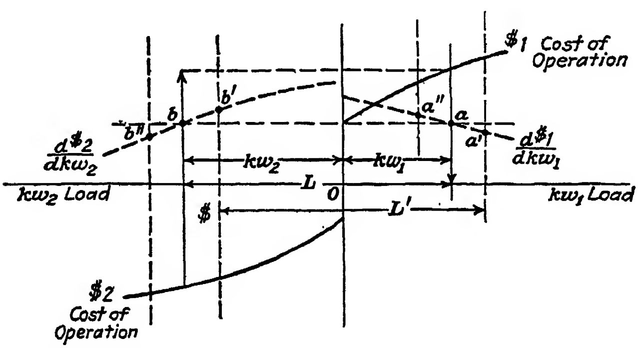


FIG. 77.—Division of load between two units. Cost curve concave toward the axis.

In Fig. 77, inspection shows that redistributing the load as at L' effects savings on $\$_2$ greater than the marginal cost while $\$_1$ increases at a rate less than the marginal cost. In this redistribution of load, we have by Eq. (45)

$$\frac{d\$_1}{d\,\mathrm{kw}_1} < \frac{d\$_2}{d\,\mathrm{kw}_2} = -\frac{d\$_2}{d\,\mathrm{kw}_1} \tag{50}$$

unless both cost curves have become straight at a and b, respectively. We shall have no increase in cost then if we load up machine 1 to the limit of its capacity and may have a saving.

On the other hand, we shall have a saving if the distribution is changed by loading machine 2 to its limit. The question as to which of these savings will be the greater is practically answered by trying both arrangements of the load.

It is apparent that machines whose load-cost curves are concave toward the load axis admit of no load distribution other than extreme loading of one or the other, except as both curves may develop straight segments of equal slope.

Mathematical discussion in this case would prove quite analogous to that of Fig. 76.

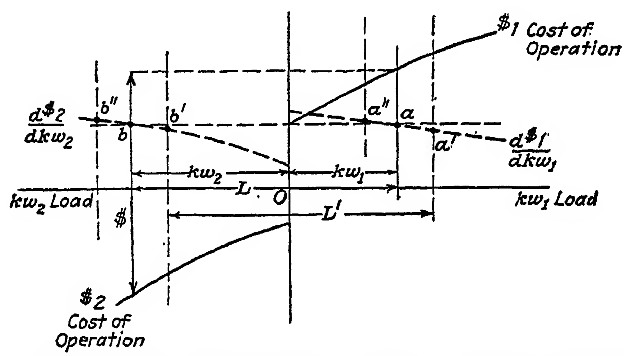


FIG. 78.—Division of load between two units. Cost curves dissimilar.

In Fig. 78, it is not at first apparent that departure from the distribution of load L (where $d\$_1/d\ \text{kw}_1 = d\$_2/d\ \text{kw}_2$) to such a distribution as shown at L' would effect either a saving or a loss. Loading machine 1 more heavily will result in progressively cheaper kilowatts from it, but will take off of 2 progressively cheaper kilowatts. Which effect is the more significant? Here we must actually *use* our derivature curves.

If these are of the same shape as drawn, *i.e.*, if $d\$_1/d\ \text{kw}_1 = d\$_2/d\ \text{kw}_2$ for $\text{kw}_1 + \text{kw}_2 = L$ through all values of kw_1 within any range, then within that range our load distribution is a matter of indifference. This is the highly special condition in which the load-cost curves are of identical shape and requires a unique value of L. For greater loads than this unique value, there will be no possible distribution in which the marginal costs are equal. The entire capacity of the machine whose cost curve is concave toward the load axis should be utilized since any slight loading in excess of the unique value L is progressively

cheaper when placed on the machine whose marginal costs become progressively less as it is loaded.

Were the actual load to be never-so-little less than the unique value L, it would be most economical to load completely the machine whose curve concaves away from the load axis, conversely to the foregoing.

The general form shown in Fig. 79 may be most conveniently studied by a construction in which our curves for the two machines are plotted with their origins displaced by the amount of the plant load; thus L indicates the total load, distributed to the two supply sources as indicated by L_1 and L_2, the defining point P being taken where the first derivatives of cost are equal.

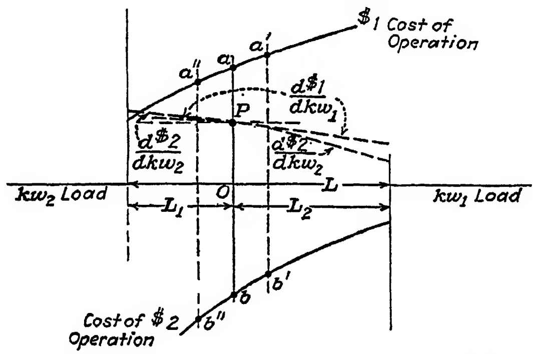


FIG. 79.—Division of load between two units by measuring cost ordinate.

It will be seen that variation of the load distribution, increasing L_2 and equally decreasing L_1, has the effect of throwing more load onto source 2 at a higher and higher expense *per kilowatt* and taking the same amount off source 1 at a less saving *per kilowatt.* The opposite change of distribution—reduction of L_2—would have the effect of saving expense at source 2 at a rate initially the same as the expense increase of source 1, but at a rate that immediately becomes smaller. In this case, we can easily determine the best loading by measuring the total cost ordinates $a - b$, $a' - b'$, $a'' - b''$, etc., between the curves $\$_1$ and $\$_2$. Maximum economy will obtain when we have the minimum ordinate between the two cost curves.

Again, it is to be noted that a load distribution involving only a straight-line relationship between output and cost—the condition assumed for preliminary design—is of very easy solution. The maximum possible load would be thrown onto the source

having the least slope; the next source to be utilized to capacity would be that having the next larger slope, etc.

When more machines or sources are available than are needed, an obvious extension of the foregoing is necessary in order to determine which shall lie idle.

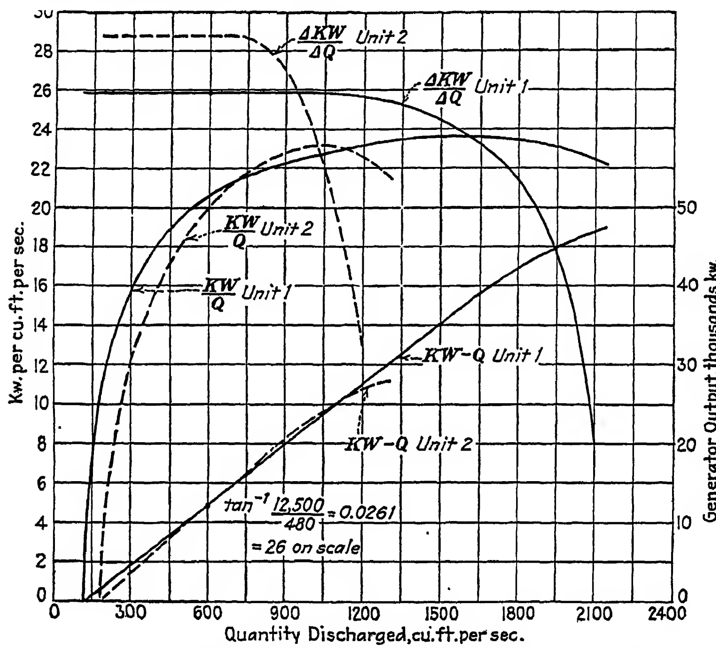


FIG. 80.—Economical load division, two water turbine units.

64. Example of Load Division, Two Water Turbine Units.—In Fig. 80 are shown the power discharge (KW. $-$ Q) curves of two water turbogenerator units: No. 1 of 44,000-kw. capacity with low-speed type runner; No. 2 of 26,000-kw. capacity with higher speed type runner.[1] From these, the kilowatts per cubic foot were derived and plotted (KW./Q), and by taking the graphical tangents of the KW. $-$ Q curves, the first derivative curves ΔKW./ΔQ were obtained and plotted to the same scale as the KW./Q curves. Examination of the KW.-Q curves shows that unit 1 gives more output per cubic foot for loads zero to 14,500 kw.; that unit 2 is better from 14,500 to 25,700 kw.; then that unit 1 is better from 25,700 to 45,000 kw.

[1] From ROGERS, F. H., Acceptance Tests for Hydro-electric Plants, *A.I.E.E. Trans.*, April, 1924.

After the load is such that the two units are necessary to carry it, the respective loadings of each machine may be determined from a series of horizontal lines drawn on the figure giving intercepts of equal value with the derivative curves. Thus for first derivatives of a value of 24, the load for unit 2 would be 23,000 kw., and for unit 1 would be 37,000 kw., making a total load of 60,000 kw. The correct divisions of the load are then given as follows:

Total kilowatts	Unit 1	Unit 2
49,900	27,700	22,200
52,700	30,200	22,500
55,400	32,700	22,700
61,200	37,700	23,500
66,600	42,100	24,500
72,300	45,700	26,600

For a discussion of curves and tables showing the best operating procedure for any condition of river stage, and for any station

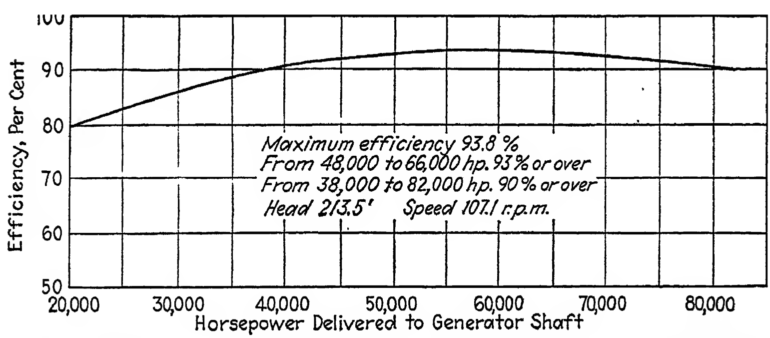


FIG. 81.—Efficiency curve of 70,000-hp. I. P. Morris water turbine at Niagara Falls Power Company. (*Elec. World, Sept.* 26, 1925.)

load, the student should consult the articles "How We Raise Hydro Efficiencies," by E. B. Strowger,[1] and "Load and Plant Factors," by S. L. Kerr.[2]

Figure 81 shows the efficiency curve for one of the 70,000-hp. water turbines of the Niagara Falls Power Company. This installation is one of the finest of the modern hydro developments.

[1] See *Elec. World,* Apr. 14, 1934, p. 535.

[2] See *Power Plant Eng.,* Feb. 15, 1931, p. 258.

65. Example of Load Division, Six Units.—On Fig. 82 are shown the water-rate curves for various combinations of the six turboalternator units in Connor's Creek Power Station. Thus for any given load on the plant, the most economical arrangement of units is readily determined. Three units are 30,000 kw., and three are 60,000 kw., all at 600 p.s.i., gauge and 825°F. There are 11 boilers each of 330,000 lb. per hour normal full-load operation.

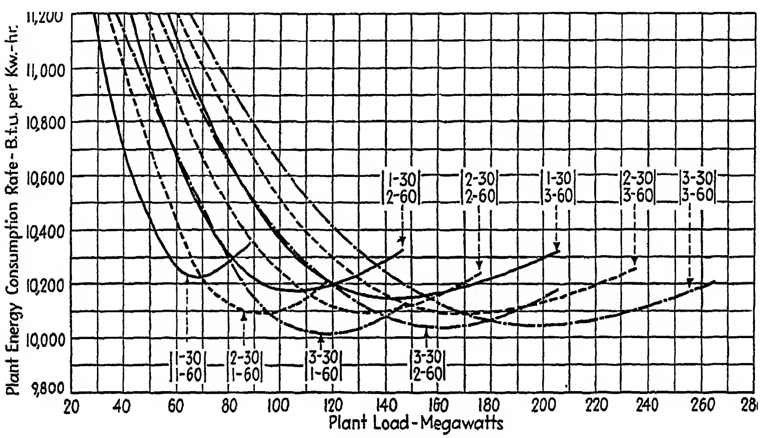


FIG. 82.—Plant energy consumption rate, Connor's Creek Power House, based on 100 per cent boiler room efficiency (1939 average = 86.8 per cent), 33 to 44 F. circulating water temperature. (*Courtesy Detroit Edison Company.*)

66A. Load Division for Steam Stations in Interconnections.[1]—Since the rapidly expanding systems of the present day group together many stations with quite different characteristics, accurate load division between the stations must be made to secure the best operating economy. The accuracy of such determinations will, of course, depend upon how carefully the associated groups of boilers and turbines have been tested and on the present high development of operation and control of boilers which permits dependable station heat-rate characteristics to be duplicated day after day. In an interconnected group, the individual stations do not follow, in general, the characteristic

[1] STAHL, E. C. M., Economic Loading of Generating Stations, *Elec. Eng.*, September, 1931, and Load Division in Interconnections, *Elec. World*, Mar. 1, 1930. See also STEINBERG and SMITH, Incremental Loading of Generating Stations, *Elec. Eng.*, October, 1933, taking into account transmission-line losses.

load curves of the system demand. Some of the stations take base loads, and others are assigned to the peak sections as in Fig. 63. Consequently, there is a shifting of load factors and plant factors among the different stations so that the use of average input-output data has lost much of its value for correct load allocation.

The first great requisite is to provide the necessary capacity in operation to ensure continuity of service throughout the sys-

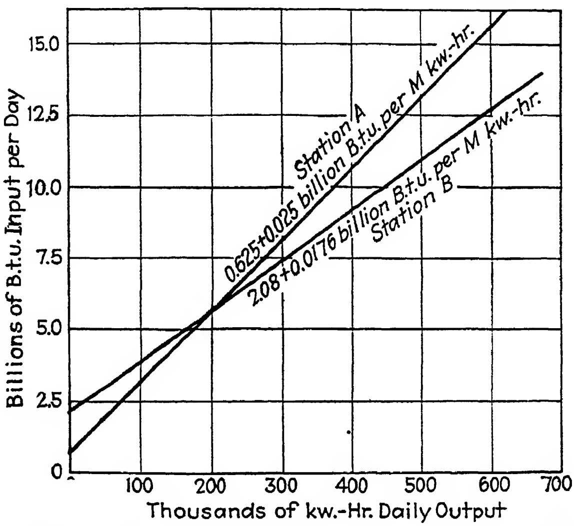


FIG. 83.—Steam station characteristics plotted from daily operating data. (*E. C. M. Stahl, "Economic Loading of Generating Stations," Elec. Eng., September,* 1931.)

tem. This capacity will depend upon the daily load curve, the possibility of local and emergency demands, tie-line capacities, and the experience that has been had with the reliability and stability of the electrical system. Then we select the equipment to carry the load in the order of its effect on the best over-all system economy and divide the load correctly among the equipment that has thus been placed in operation. In Fig. 83 are shown two station characteristics taken from daily operating data, each point representing the integrated 24-hr. output for various station loads plotted against total B.t.u. input. Over a period of time, a large number of values are obtained which give the average economy characteristic trend of the station. The station specifications are as follows:

Station A, 65,000-kw. capacity:

Four 12,500-kw. straight condensing turbines.
One 15,000-kw. one-point bleed turbine.
Vacuums 27.50 to 29.50 in. of mercury.
200 p.s.i., 100 to 125° superheat.
Open feed-water heaters.
Auxiliary exhaust steam for heating.
Eight 760-hp. boilers.
Twenty-four 500-hp. boilers.

Station B, 120,000-kw. capacity:

Two 25,000-kw., one 22,000-kw., one 30,000-kw., one 11,000-kw., one 7,000-kw. straight condensing turbines.
Vacuums 27.50 to 29.50 in. of mercury.
200 p.s.i., 100 to 125° superheat.
Open feed-water heaters.
Auxiliary exhaust steam for heating.
Thirty-two 600-hp. boilers.
Sixteen 680-hp. boilers.

The following methods of load division between the stations of an interconnected system are in common use:

1. The "base-load" method, wherein the best station is loaded to its normal maximum capacity, and then the next best station, etc. Thus in Fig. 83 it is evident that if a daily output of only 100,000 kw.-hr. is needed, Station A will carry the load with less input than Station B; but if the output is to be 400,000 kw.-hr., then Station B will carry it more economically than Station A. If both stations are required to be in operation, then Station A will be operated at its minimum capacity until Station B is fully loaded; then Station A will be loaded. Such a method of loading does not take advantage of the minute-to-minute economy relations between the stations but merely uses the average integrated characteristics. Therefore, this method does not give the best economy.

2. "Best-point" loading; *i.e.*, all stations are loaded successively up to their lowest heat-rate points, beginning with the best station, and then adding the next best, etc., or under certain conditions bringing them all up to their best load concurrently. As is shown in Fig. 30, the heat-rate characteristics of the older

turbines are such that their best economy is obtained just before the opening of the overload valves which by-pass steam into some of the lower stages of the turbine. The points at which these additional valves open are shown clearly in the illustration.

3. The "incremental" method of loading results in the minimum input for any output with any given combination of equipment in operation. The method is based on the principle that after certain required equipment is already in operation and

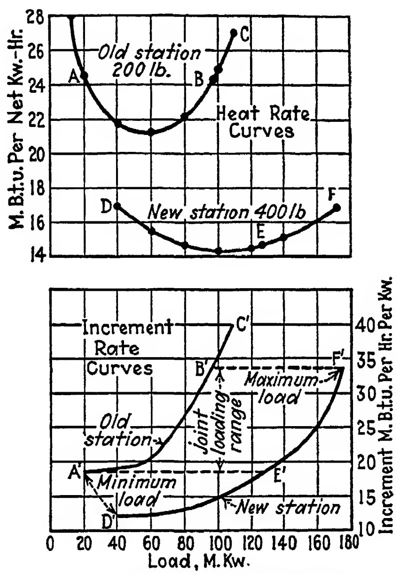


FIG. 84.—Steam-station heat rates. (*E. C. M. Stahl, "Load Division in Interconnections," Elec. World, Mar.* 1, 1931.)

carrying load the next increment of load shall be picked up by that group of boilers, turbines, and auxiliaries which will produce the increment for the least added cost. This keeps all the stations operating progressively at the same increment rate although they may differ widely in their actual heat rate or cost.

Figure 84 shows the heat-rate curves of a typical 200-lb. station and of a typical 400-lb. regenerative station in the upper part of the graph and below them the increment-rate curves derived from these heat-rate curves. The incremental heat rate at a given load already in production is defined as the actual additional heat per hour required to obtain the added load. The points A and A' show the minimum load (20,000 kw.)

to be carried on the old station and D and D' the minimum load (40,000 kw.) to be carried on the new station. From a comparison of the increment rates, it is evident that beginning with a system load of 60,000 kw. the next 1,000 kw. should be taken by the new station at an increment rate of 12,400 B.t.u. instead of 18,400 B.t.u. on the old station. Each additional 1,000 kw. up to a total system load of 145,000 kw. should be taken by the new station. This would bring the new station load to 125,000 kw. at point E on its heat-rate curve and point E' on its increment curve. From here on, according to the increment curves, the load should be increased by joint loading of both stations, keeping the increment rates at the same value throughout.

Let us compare the economies of the various methods, assuming that a total system load of 200,000 kw. is to be carried by the two stations of Fig. 84.

	Kilowatts	British Thermal Units
1. Base-load method:		
New station	170,000 at 16,850 =	2,864,500,000
Old station	30,000 at 22,600 =	678,000,000
System	200,000 at 17,713 =	3,542,500,000
2. Best-point method:		
New station	130,000 at 14,850 =	1,930,500,000
Old station	70,000 at 21,500 =	1,505,000,000
System	200,000 at 17,178 =	3,435,500,000
3. Incremental method:		
New station	140,000 at 15,180 =	2,125,200,000
Old station	60,000 at 21,200 =	1,272,000,000
System	200,000 at 16,986 =	3,397,200,000

The problem of the stand-by and peak-load station has certain definite limitations which must be solved separately before applying the incremental method of load division. In metropolitan service in order that a stand-by station can pick up load thrown on it suddenly, it should be carrying at any time perhaps one-half the load to which it must immediately pick up. If a peak-load station is run over the peak period of only a few hours from no load to full load and down again, its loading in general will not follow the normal incremental loading schedule. The load should be dropped gradually so that the boilers can be reduced in steaming rate slowly enough to prevent blowing off steam and wasting heat.

66B. Load Division between Services.—Where the daily load curve of a power system has a peak demand of large size, it is usually supplied from more than one power station. Thus the 769,900-kw. load of the Detroit Edison Company, shown in Fig. 64, was carried by four steam stations and five small hydraulic plants. The load of the Commonwealth Edison Company is supplied by four steam plants within the city and many interconnections. The 420,986-kw. load of the Consumers Power Company was carried by 52 plants, 9 of which are steam stations and the others hydroelectric plants. Therefore the system load must be economically apportioned to the various plants. Even if all generating stations are steam plants, it is likely that the costs of generation will vary with the different stations, since the equipment may range all the way from an old plant of perhaps 15,000-kw. units at moderate steam pressure and temperature to a new plant of 208,000-kw. units at high steam pressure and temperature. The difference in economy may be appreciated by an examination of the data given in Figs. 30 and 35. When some of the stations are hydroelectric and the remainder are steam plants, it will doubtless be desired to use the water plants to the maximum available flow, but here again conditions and efficiencies will vary in the different hydro stations. For example, the hydraulic plants of the Consumers Power Company are located on the Manistee, the AuSable, the Grand, the Kalamazoo, the Muskegon, and the Shiwassee rivers. It is conceivable that water conditions in the streams may vary materially from day to day, calling for a new apportionment of the load among the plants. The hydro generation on this system for 1939 was 446,075,393 kw.-hr. as against 1,145,473,274 kw.-hr. generated by steam so that 28 per cent of the total generation was from water power.

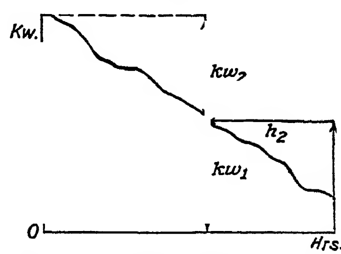


Fig. 85.—Load-duration curve for average day of year.

A load-duration curve, as Fig. 85, enables us to determine the number of *hours per year* (h_1 or h_2) in which the load is greater or less than any given amount (kw_1) and, similarly, the loads

(kw_1 or kw_2) that are carried for more or less than any particular duration (h_1).

The total area of the duration curve shows the total energy output (kw.-hr.), and the upper and lower portions of the curve area show the output corresponding to loads above and below any given amount. The right-hand portion of the area shows the output in excess of any given number of hours duration. The upper left-hand "quadrant" shows the "on-peak" output; *i.e.*, the output having a duration of less than a given number of hours.

In arriving at a decision between two possible sources of power supply of which the annual costs are

$$\$' = A' + B' \text{ kw} + C' \text{ kw.-hr.} \tag{51}$$

and

$$\$'' = A'' + B'' \text{ kw} + C'' \text{ kw.-hr.} \tag{52}$$

where

$$B' > B''$$

and

$$C' < C'',$$

we note that the first is the better source for very long hours duration on account of the saving in energy cost, and vice versa. If used at all, the first source would be used on the base, the second source on the peak.

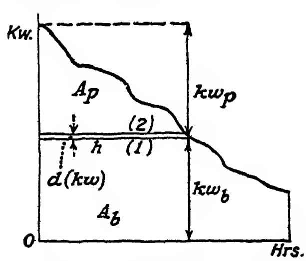


FIG. 86.—Economic division of duration curve for base and peak power sources.

How far may we extend the use of the first source? Note Fig. 86, in which (h, kw_b) is any point on the duration curve. Since line 1 represents a demarkation between the first and second source, A_b is the area below this line, A_p the area above it.

We here have total operating costs of

$$\$_1 = A' + A'' + B' \text{ kw}_b + B'' \text{ kw}_p + A_bC' \text{ (kw.-hr.)} + A_pC'' \text{ (kw.-hr.)}. \tag{53}$$

If now the base power is extended by the amount of d kw to line 2, we shall still have costs A' and A'' but shall have modified our other costs, thus our new total cost is

$$\$_2 = A' + A'' + B'(\mathrm{kw}_b + d\ \mathrm{kw}) + B''(\mathrm{kw}_p - d\ \mathrm{kw}) + (A_b + d\ \mathrm{kw.\text{-}hr.})C' + (A_p - d\ \mathrm{kw.\text{-}hr.})C''. \quad (54)$$

The cost change, then is

$$\$_2 - \$_1 = (B' - B'')d\ \mathrm{kw} + (C' - C'')d\ \mathrm{kw.\text{-}hr.} \quad (55)$$

The optimum condition requires that this change be zero, *i.e.*, that

$$\frac{B' - B''}{C'' - C'} = \frac{d\ \mathrm{kw.\text{-}hr.}}{d\ \mathrm{kw}} = h, \quad (56)$$

where h represents the hours per year and the costs are

$$\text{Base source} = A' + B'\ \mathrm{kw} + C'\ \mathrm{kw.\text{-}hr.}$$
$$\text{Peak source} = A'' + B''\ \mathrm{kw} + C''\ \mathrm{kw.\text{-}hr.}$$

This marks the division line between alternatives *if* two supply sources should be used. Having determined the economic division point on the load curve for the two sources, the total cost of carrying the load so divided can now be determined. A comparison of the total cost of the plan with the cost of carrying the entire load by either power source alone will determine whether there is any economic gain in using both sources of power supply or not.

Noting that there is no economy in using either source *at* the demarking line, but that the source of lower *power* cost does effect a saving for all loads above this line, we see that we can save

$$\$ = (B' - B'')\ \mathrm{kw}_p - (C'' - C')A_p \quad (57)$$

by incurring the underlying expense A''. If

$$(B' - B'')\ \mathrm{kw}_p > (C'' - C')A_p + A'', \quad (58)$$

we are justified in using the peak source, since the saving in the power cost is sufficient to offset the loss in the energy cost and balance the underlying cost of the extra plant. Similarly, our base source is justified when and only when

$$(C'' - C')A_b > (B' - B'')\ \mathrm{kw}_b + A'; \quad (59)$$

for here the saving on the energy cost overbalances the loss in the power cost and covers the underlying cost of the extra plant.

67. Integrated Duration and Mass Curves.—Figure 87 shows an integrated duration curve derived from the average daily load duration curve of Fig. 71, in which the abscissa shows the total number of kilowatt-hours generated at or below any given number of kilowatts; *i.e.*, the abscissa corresponding to each

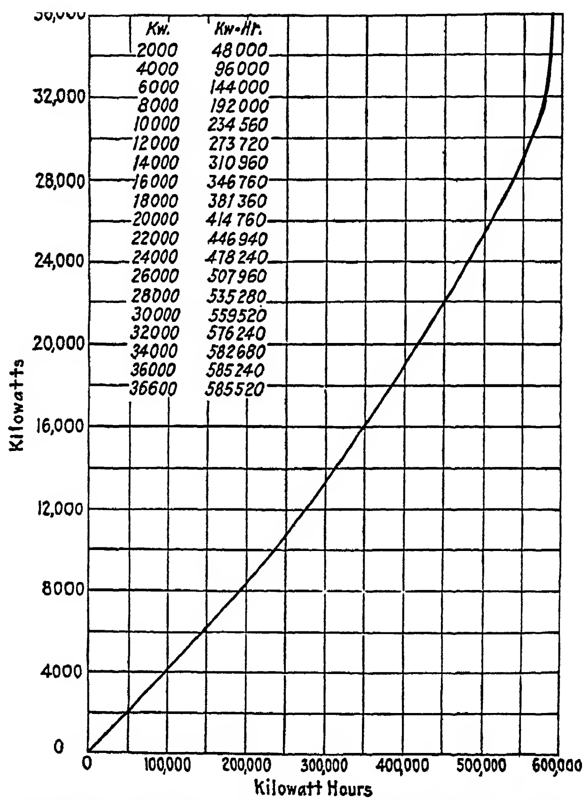


FIG. 87.—Integrated duration curve of the average load-duration curve of Fig. 71

ordinate should equal the area of the duration curve up to the value of that ordinate. Such a curve is useful in that with a given number of kilowatt-hours per day available, say from a river flow, the kilowatts of load that could be carried on the base or peak may be quickly determined. Thus, 300,000 kw.-hr. on the base corresponds to an ordinate of 13,200 kw., or if the base load up to 13,200 kw. were assigned to a plant, there would

be an energy consumption entailed of 300,000 kw.-hr. On the other hand, 300,000 kw.-hr. on the peak would correspond to 585,520 − 300,000 = 285,520 kw.-hr. on the base and an ordinate of 12,600 kw. on the base, or 36,600 − 12,600 = 24,000 kw. on the peak. That is, a plant assigned to carry the peak 24,000 kw. of the duration curve would have to furnish 300,000 kw.-hr. of energy.

Figure 88 shows a mass curve derived from the chronological load curve of Fig. 71 which gives the total energy used by the

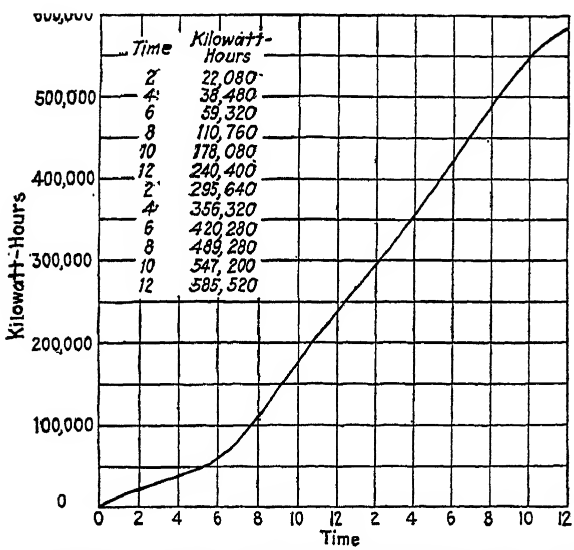


FIG. 88.—Mass curve of the average chronological load curve of Fig. 71.

load up to each hour of the day. Thus up to 10 A.M., 175,000 kw.-hr. had been used; up to 6 P.M., 420,000 kw.-hr.; and at the end of the 24 hr., the total area or 585,520 kw.-hr. had been used by the load. Such a curve enables us to study the variations between the rate of water inflow available for power and the electrical load using such power, and thus make determinations as to the necessary storage called for.

68. Power Supply from Two Sources.—In providing for service from two sources one of which, *e.g.*, a hydraulic plant, has a limited energy supply, we derive an "integrated duration curve" as in *C*, Fig. 89, from the duration curve *B*, which was, in its

turn, derived from the chronological curve A by an arrangement of ordinates in magnitude sequence B rather than in the order of actual occurrence A.

The integrated duration curve is derived by plotting the energy (kw.-hr.) against power (kw.), so that for any base block of power KW_b in A and B we shall have in C the abscissa A_b in proportion to the areas correspondingly indicated in A and B. Any

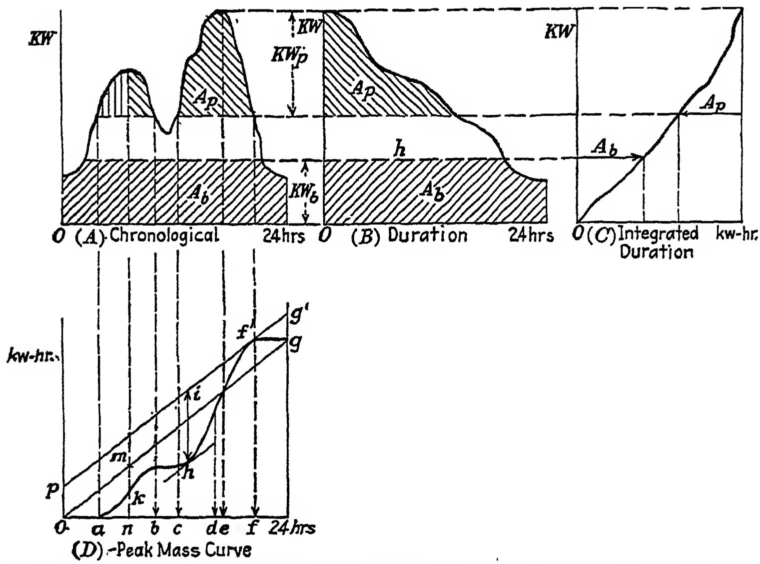


FIG. 89.—Chronological, duration, integrated duration, and peak mass curves of the daily load curve of Fig. 71, as used for load division, two power sources.

abscissa to curve B will thus be the derivative of the corresponding abscissa to C; *i.e.*,

$$h = \frac{dA_b}{d\text{kw}}$$

whether A_b refers to B or to C.

Conversely,

$$\text{Kw.-hr. } C = \int_0^{\text{kw.}} h d\text{kw.}$$

Now letting A_b become a definite value, our limited energy supply, we measure off this value on curve C and find the corresponding kilowatt ordinates on the curves A and B.

The limited energy supply may be studied for peak use by measuring a coabscissa A_p on C and similarly establishing area A_p and KW_p.

Curve D shows a mass utilization curve for the peak area A_p derived from the chronological curve. The ordinates of the mass curve are proportional to the area of A up to the time of the ordinates of D; *i.e.*, nk is a measure of area and

$$\text{Kw.-hr.} = \int_{12\text{ P.M.}}^{t} \text{kw}\,dt \quad \text{or} \quad \text{kw}_a = \frac{d}{dt}\,\text{kw.-hr. } D.$$

Thus up to time a hr. there has been no peak source utilization and the mass curve is flat, then there is slight utilization to time b and the mass curve shows a slight slope, returning to the level $b - c$ during the next nonuse of the peak source. During the absolute peak at time e hr., the mass curve has its greatest slope.

At such a time as n, the peak source will have supplied energy nm. If the peak supply, as by an hydraulic flow, has been steady for the day, giving a straight-line supply mass curve og, we shall have had an excess of supply over utilization km at the time n, and at time e the excess supply, if stored, will have become exhausted. After this time, there will have been an excess of utilization beyond the past supply until at f there will be a maximum deficit.

The tangents to the utilization mass curve, parallel to the supply mass curve, show at h the time of full pond, and at f' the time of empty pond. The ordinates op and gg' give the necessary initial storage, and hi the necessary total storage capacity.

It is seen that, *with steady supply*, the size, and presumably the cost, of storage will be greater with peak utilization of our limited supply than with base utilization. From this, it follows that it might be desirable to develop a limited source for basic utilization even though the power cost from this source were dearer than from the unlimited source. Let the cost of development of service be represented by

$$\begin{aligned} \$_1 &= (\$_p + \$_b) = \\ &\{A_1' + B'\,\text{kw}_1 + C'\,\text{kw.-hr.}\} + \\ &\{A'' + B''\,(\text{KW.-kw}_1) + C''\,[(\text{KW.-HR.}) - (\text{kw.-hr.})]\} \end{aligned} \tag{60}$$

where A_1', B', C' represented the readiness to serve, power, and energy costs of the limited supply when used on the peak, KW. the total power to be developed, KW.-HR. the total energy to be developed, kw_1 the power, and kw.-hr. the energy available from the limited supply. We note that placing the limited supply on the base will, in general, result in a value $A_2' < A_1'$ due to cheaper storage, dam, etc., whereas kw_1 of necessity $> \text{kw}_2$ the power that can be developed from the limited supply when used on the base.

Thus we have an alternative cost

$$\$_2 = A'' + B''\,(\text{KW.} - \text{kw}_2) + C''\,[(\text{KW.-HR.}) - (\text{kw.-hr.})] + A_2' + B'\,\text{kw}_2 + C'\,\text{kw.-hr.} \quad (61)$$

We note that

$$\$_2 - \$_1 = A_2' - A_1' + (B'' - B')\,(\text{kw}_1 - \text{kw}_2). \quad (62)$$

If this quantity is positive, we shall use our limited source on the peak, but this can happen only if the saving due to the development of a large amount of power (kw_1) at the low cost B' is sufficient to justify the larger underlying cost A_1' of the greater storage capacity.

It is quite possible that complete utilization of the limited energy supply on peak or base may push the *marginal duration* h above or below (respectively) the value

$$\frac{B' - B''}{C'' - C'}$$

determined as the economic limit of supplemental supply.

69. Service Conditions in the Parallel Operation of Hydro and Steam Plants.[1]—Owing to the seasonal variations of most river flows, the hydro plant operating in parallel with a steam plant will have two extreme river conditions to meet, *viz.*, high flow and low flow. For the high-flow period, the hydro plant can operate continuously at full capacity with a high-load factor, carrying all possible base load and, by generating a maximum energy output, make a large saving in fuel consumption for the steam plant. For the low-flow period, the hydro plant will carry as much as

[1] See Operating a Combined Hydro- and Steam-electric System, by Harrington and Strawger, *Trans. A.S.M.E.*, February, 1939.

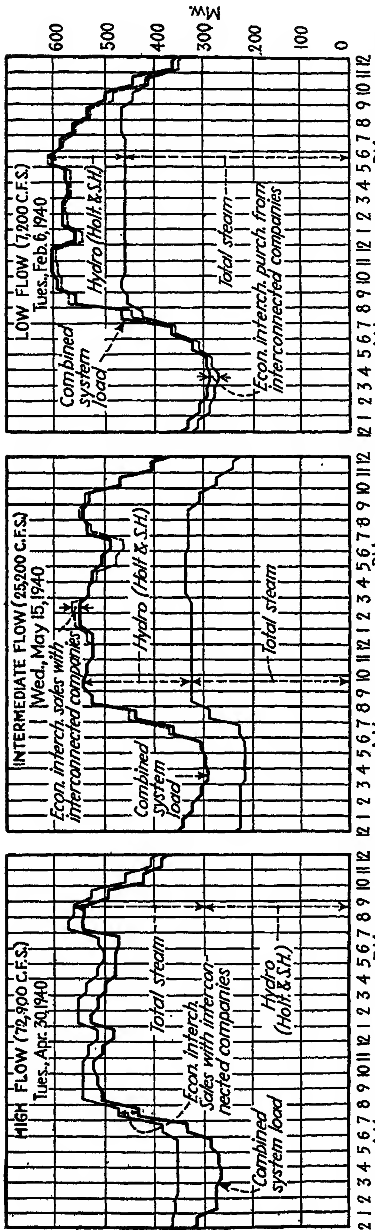


FIG. 90.—Hydro output in relation to total system load for combined Baltimore-Washington-Holtwood-Safe Harbor system during high-, intermediate-, and low-flow conditions of Susquehanna River. (*Courtesy of Pennsylvania Water & Power Company.*)

possible of the peak service, at a poor load factor, leaving the good load-factor operation to the steam plant on the base.

In Fig. 90, F. A. Allner, vice president of the Pennsylvania Water & Power Company, shows the hydro output in relation to the total system load for the combined Baltimore-Washington-Holtwood-Safe Harbor system during typical high, intermediate, and low-flow conditions of the Susquehanna River. The hydro output is the combined generation of the Holtwood and Safe Harbor plants which are operated in conjunction with the Baltimore, Washington, and Holtwood steam plants as a unified system.

On account of the different conditions under which they have to operate, there is a marked difference in performance, and in methods of operation, between water turbines and steam units. For the water turbine, the speed is low and the range of temperature is between 32 and 68°F., with slow seasonal changes. When given oil pressure and field excitation, a unit can be brought from standstill to full load in a few minutes. Reductions in hydro-plant capacity forced by operating circumstances are due chiefly to loss of head in flood times and obstructions of the screen racks by ice and trash. These troubles come gradually at rather definite seasons and if at high-flow period there will be ample steam capacity available as a reserve, but to be available quickly it must be floating on the bus. The service-demand availability factor for hydro units is about 99 per cent.

For the steam turbine, however, the speed is high and the temperature changes rapidly from 68 to perhaps 825°F. in coming into service. A very considerable time must be taken then to allow the turbine to become evenly heated and to be gradually brought up to full speed. Meanwhile, the air and circulating water pumps are being put into operation and the vacuum formed in the turbine's condenser. Altogether probably an hour is necessary for a large unit from standstill to full load. Forced reductions in steam-plant capacity may occur suddenly and without any previous warning, such as turbine-bucket failures or troubles with high-pressure piping or the turbine auxiliaries. Should it be necessary to shut a steam unit down when the parallel hydro plant is working on low-flow condition, the load can be picked up readily by one of the hydro units in reserve, particularly if a unit has been floated as a synchronous con-

denser. For the steam-turbine units, the service-demand availability factor is perhaps 96 per cent.

70. Example of Parallel Operation of Hydro and Steam Plants. Another very interesting example of this combination is the modern Conowingo hydroelectric plant, described in the *Electrical World*, Aug. 13, 1927, built for the Philadelphia Electric Company. This plant will ultimately use the entire normal flow of the Susquehanna River, developing 396,000 kw. in 11 units. The initial installation is 252,000 kw. in seven units, the power being transmitted to Philadelphia over two 220,000-volt circuits

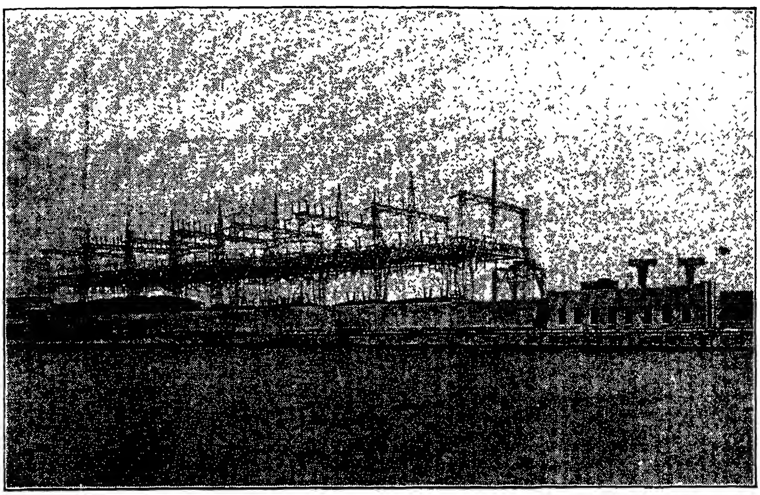


FIG. 91.—Conowingo Hydroelectric Station. View of power house from the up-stream side; 220-kv. bus on station roof; seven turbines of 54,000 hp. each. (*Courtesy of Philadelphia Electric Company.*)

and providing about 1,250,000,000 kw.-hr. annually. As is shown in Fig. 92, the Susquehanna River has an extremely variable runoff from day to day, from week to week, and from season to season. A maximum flow occurred in June, 1889, with 730,000 c.f.s. The minimum recorded flow was in 1909 and measured 2,200 c.f.s. The dam forms a lake which covers nearly 9,000 acres and contains about 14 billion cu. ft. of water. The normal head of water maintained is 89 ft.

On account of the variability of the flow of this river, the Conowingo plant carries either peak loads or base loads, as may be required by the river flow. The graph of Fig. 92 shows the

estimated operation for the week of Sept. 6 to 13, for a low-flow condition during one of the summer months. For that week, the hydrograph shows an average flow of 4,330 c.f.s., which will produce about 4,500,000 kw.-hr. delivered to the Philadelphia load for the week. By drawing a little on the pondage for the

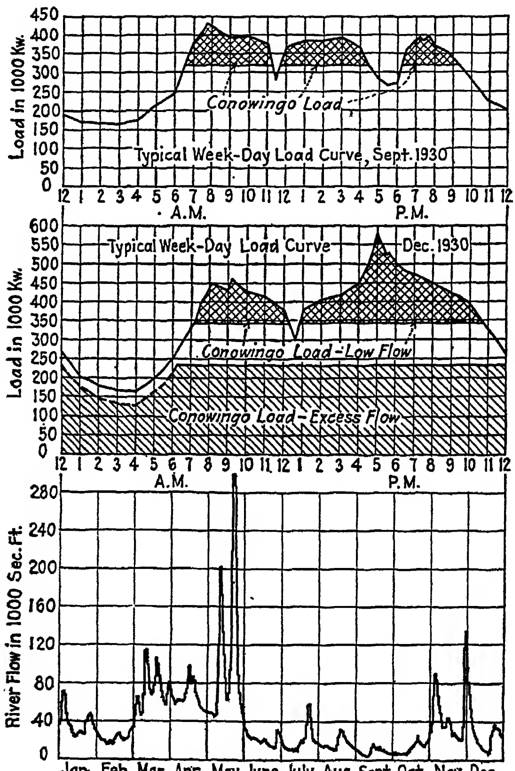


FIG. 92.—Typical load curves for combined hydro and steam, Conowingo, with hydrograph of the Susquehanna River. (*Elec. World, Aug.* 13, 1927.)

week-day peaks and allowing it to refill on off-peak periods and on Saturday and Sunday, it is estimated that Conowingo can deliver each week day to the Philadelphia load a total of about 760,000 kw.-hr., which will be distributed as shown by the shaded area on the typical week-day load curve for September. This will take 112,000 kw. from the peak of the week-day load. In addition, Conowingo will be able, with the flow shown, to take

103,000 kw. from the Saturday peak and 90,000 kw. from the Sunday peak. This will give the Philadelphia steam plants a very good load factor for the entire week.

Second, the week of Nov. 30 to Dec. 6 is taken to show the operation for a high-flow condition during the peak month. Under this condition, when the available flow equals or exceeds the requirements, Conowingo will carry a base load as near as possible to its rated capacity, amounting to approximately 233,000 kw. delivered to the Philadelphia load. It will be necessary, however, to maintain a minimum load at all times of 35,000 kw. on the Philadelphia steam plants as an emergency measure. The distribution of this Conowingo delivered power is shown by the single-hatched area on the typical week-day load curve for December. During high flow, the maximum load possible will be carried by Conowingo on Saturdays and Sundays, as well as on week days. By this method of operation, about 33,400,000 kw.-hr. can be delivered to the Philadelphia load during the week of high flow.

The week of Dec. 14 to 20 is taken to show the operation for a low-flow condition during the peak month. For that week, there is an average flow of 8,500 c.f.s., which will produce about 8,800,000 kw.-hr. delivered to the Philadelphia load for the week. This output will be used to take the greatest share of the peaks for each day. By drawing a little on the pondage for the week-day peaks and allowing it to refill on off-peak periods and on Saturday and Sunday, Conowingo can deliver to the Philadelphia load each week day a total of about 1,326,000 kw.-hr., which will be distributed as shown by the double-hatched area on the typical week-day load curve for December. This will take 233,000 kw., or the rated capacity of the Conowingo development, from the week-day peak. In addition, Conowingo will be able, with the flow shown, to take 166,000 kw. from the Saturday peak and 140,000 kw. from the Sunday peak, giving the Philadelphia steam plants a very good load factor for the entire week.

For further discussion of the plan followed in coordinating Conowingo Station with the Philadelphia and connected systems, the student is referred to the article "How Loads May Be Allocated for Best Station Economies," by Estrada and Finlaw in the *Electrical World*, page 684, May 12, 1934.

71. Storage and Pumped Storage Hydro Plants. 1. *Storage.*—This may be of the type in which the entire accumulation of water and regulation of flow are handled from one very large reservoir alone, or where additional and auxiliary storage is provided at several other points. Boulder Dam, with its enormous reservoir capacity of 30,500,000 acre-ft., is probably the outstanding example of the first type. This capacity would be sufficient to store the entire average flow of the Colorado River for 2 years. It will be utilized for three functions: (1) 9,500,000 acre-ft. for flood control, (2) 5,000,000 to 8,000,000 acre-ft. for a silt pocket, and (3) 12,000,000 to 15,000,000 acre-ft. for active or regulation storage for power and irrigation.[1]

The Duke Power Company's extensive system in the development of the Catawba River in the Carolinas is a very notable example of the serial-reservoir type aimed at full utilization of the power possibilities of the stream.[2] The project utilizes the fall of 1,050 ft. in 210 miles of river. The 10 dams on the system create a storage reservoir of 55,000 acres, which, when drawn down within the limit set by the maintenance of water-wheel efficiency, renders available from storage nearly 814,000 acre-ft. of water. Nearly 35 per cent of the total storage passes through all the 12 power plants of the system. The watershed enjoys a large rainfall, 45 to 70 mean inches per year; still it is not uniform in distribution month by month or year by year. The input flow to the large headwater reservoir at Bridgewater has varied from 70 c.f.s. in 1925 to 83,400 c.f.s. in 1916. In spite of these extreme variations, the reservoir system shown in Fig. 93 has supported the larger thermal system with nearly 600,000 hydro horsepower with an average annual output of over one billion kilowatt-hours. The availability of storage and low increment cost of added capacity in hydro stations make it possible to utilize economically an installed rating far above the average load. With short-time use, such stations may furnish very large peak capacities. The plants mentioned above and recent developments on the Susquehanna River, Conowingo, and Safe Harbor typify this phase of hydro practice.

2. *Pumped Storage.*—Where the streams do not bring down sufficient inflow to the reservoir, the total storage can be increased

[1] See *Bull. Bur. Reclamation,* June 1, 1933.

[2] See *Elec. World,* July 12, 1930.

by pumping back to the reservoir water that has come down through the turbines or that is otherwise available in the tail water pool. The water so pumped back and stored when used with the relatively high heads, for which alone this type of development is economical, provides a peak capacity for the

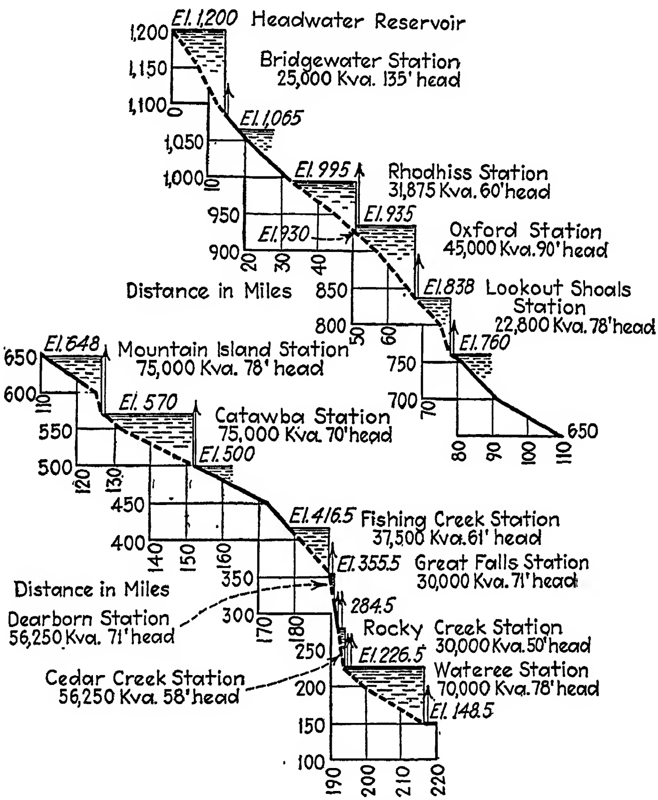


FIG. 93.—Serial development of Catawba River by The Duke Power Co. (*How Catawba Plants Utilize Entire Flow, S. T. Henry, Elec. World, July* 12, 1930.)

system. Naturally, the water is pumped into storage during the valley of the load using off-peak energy from the system. Since the over-all efficiency of the plan is about 60 per cent, 100 kw.-hr. must be used to recover 60 kw.-hr. on the peak. However, if other plants are downstream, then the over-all conversion economy derived from all the plants is raised. For a numerical

determination of the net gain in kilowatt-hours obtainable by pondage useful for regeneration under drawdown conditions in relation to minimum natural inflow and for viarous ranges of pond ratios, the reader should consult "An Analysis of Hydro Regeneration," by F. A. Allner, *Electrical Engineering*, August, 1933. As there developed, the approximate amount of additional hydro capacity that may be installed at low increment cost, in excess of that capacity which is rendered firm on the system load by minimum natural inflow, is determined thus:

Let Fig. 94 represent the load duration curve for a typical heavy load day, the total demand being supplied by a combined hydro-steam

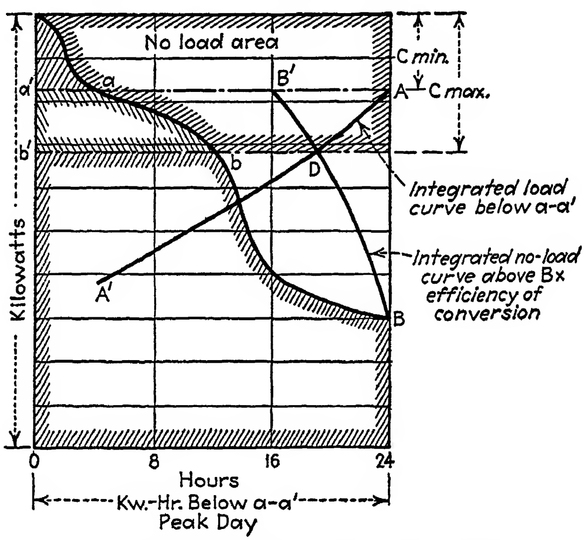


FIG. 94.—Load duration curve to determine firm hydro capacity for minimum inflow and regeneration. (*Economic Aspects of Water Power, F. A. Allner, Trans. A.I.E.E., March,* 1933.)

system with the hydro supplying the cross-hatched upper portion of the load area at times of minimum regulated river flow. C min. is the hydro capacity corresponding to this minimum flow rendering firm peak service on this load system. The integrated load area (kw.-hr.) below the horizontal line aa' is plotted to a suitable scale as a curve starting at point A in such a manner that the abscissa of any point on the curve AA' represents the load area lying above a horizontal line through that point and below line aa'. From point B, the minimum load of the load-duration curve, a curve BB' is drawn such that any abscissa of it

represents the integrated no-load area (kw.-hr.) below a horizontal line through the point of the curve multiplied by the over-all efficiency of conversion, steam-electric to hydroelectric energy. A horizontal line drawn through the point of intersection D of these two curves will divide the load curve into an upper area which can be supplied by water power produced by natural inflow and regeneration, and into a lower area in which the steam plants will operate at 100 per cent load factor.

In this determination, it has been assumed that the regenerative cycle is not limited by the capacity of the upper or lower pond or by excessive variations in operating head. It is to be noted that the installed hydroelectric capacity C max. is approximately equal to the amount of off-peak steam capacity available, B. A larger hydroelectric capacity may be installed and render firm peak service if other steam plants in the system have off-peak steam energy available during pumping hours.

An American example of 1928 is the Rocky River development of the Housatonic River where the natural flow to the 8-sq. mile storage lake is augmented with water pumped back through the single penstock against a head of 200 to 230 ft. Secondary steam-generated energy is used for the two 8,100-hp. motor-driven centrifugal pumps which have the task of supplying the difference between the 5.9 billion cu. ft. useful capacity of the reservoir and the 1.5 billion cu. ft. which the drainage area will supply naturally in an average year. The water outflow from the reservoir is used by a 30,000-kva. generating unit.[1] This unit together with the pumped storage actually added 40,000 kw. of yearly firm capacity to the system. On account of the presence of the lower plants, the over-all conversion economy derived from all plants is raised to nearly 80 per cent.

For a description of the 1932 Safe Harbor plant, see Sec. 29.

A large number of pumped storage plants have been built or planned in Germany, France, Sweden, Italy, Austria, and Switzerland because of the inherent advantages of the high heads obtainable there. The pumps in these installations have very large capacities, some reaching maximum ratings of 36,000 hp. The following plants are notable:[2]

[1] See *Elec. World*, May 12, 1928; and ALLNER, F. A., Economic Aspects of Water Power, *A.I.E.E. Trans.*, March, 1933.

[2] See ANGUS, R. W., Hydraulic Practice in Europe, *Trans. A.S.M.E.*, June, 1932.

Plant	1932, kva.	Reservoir capacity		Mean head, ft.	Turbine, hp.	Pump, hp.
		Upper million cu. ft.	Lower million cu. ft.			
Schwarzenback, Germany	46,000	507.0	12.8	1,148	62,500	25,100
Niederwartha, Germany	107,500	71.4	71.4	469	120,000	100,000
Herdecke, Germany	140,000	57.0	56.0	508	194,000	102,600
Bringhausen, Germany	144,000	27.0	27.1	954	164,000	116,000
Schluchsee-Hausern, Germany[1]	197,000	.....	....	625	197,000	106,800
Bleiloch, Germany[2]	....	.....	....	140	60,000	48,000
LacBlanc, France[2]	....	.....	....	357	108,000	95,000
Zapelle, Italy[2]	....	.....	....	1,180	14,700	9,440

[1] See *Elec. World*, July 2, 1932.
[2] Courtesy of René P. Chauvet, Morges, Switzerland.
NOTE.—For a very complete list, the reader is referred to *Power*, January, 1934, p. 22.

If present developmental work is successful, the same hydraulic unit may be used both for generating and for pumping, which will permit a decided simplification and saving of space in the new station designs.

72. Problems.

1. One manufacturer offers a line of machines costing \$ = \$10,000 + \$25 kw. and guaranteed to give the following hourly operating costs \$ = \$2 + \$0.006 kw.-hr.

A second manufacturer offers a line costing \$ = \$7,000 + \$25.5 kw. and \$ = \$1.90 + \$0.0061 kw.-hr.

kw. indicates machine rating.

kw.-hr. indicates output.

Taxes are 1.5 per cent, money use 5.5 per cent, expected life 20 years in either case.

a. Which machine is the *cheaper* to *own* in 5,000-kw. size; in 10,000-kw. size?

b. In what size are they equal as to ownership cost?

c. Which is the more economical to operate in 5,000-kw. size at 30,000,000 kw.-hr. annual output; at 6,000,000 kw.-hr. annual output? Machines run 8,760 hr. a year.

d. At what annual output will they be equally *economical?*

e. Write an expression involving size and output (kilowatt and kilowatt-hour) as a criterion by which one may select the better line for any given use.

2. Plot the following steam consumption rates for turbogenerator units in terms of pounds of steam used per hour at the various loads. Fit an approximate straight line to the points for each unit, and determine its underlying and marginal steam consumption per hour.

Unit	Rate		Conditions
	Kilowatts	Lb. per kw.-hr.	
5,000 kw.	2,000	15.45	225 lb. steam
	3,000	14.00	150° superheat
	4,000	13.30	28″ vacuum
	5,000	13.45	
8,000 kw.	3,200	15.25	225 lb. steam
	4,800	13.90	150° superheat
	6,400	13.60	28″ vacuum
	8,000	13.60	
12,500 kw.	5,000	14.40	225 lb. steam
	7,500	13.10	150° superheat
	10,000	12.50	28″ vacuum
	12,500	12.60	
	15,000	13.00	
20,000 kw.	8,000	14.50	200 lb. steam
	12,000	13.25	100° superheat
	16,000	12.80	28.5″ vacuum
	20,000	12.85	
30,000 kw.	15,000	12.07	200 lb. steam
	18,000	11.77	120° superheat
	22,000	11.40	29″ vacuum
	25,000	11.27	
	28,000	11.47	
	30,000	11.63	
50,000 kw.	20,000	11.05	265 lb. steam
	30,000	10.40	200° superheat
	40,000	10.05	29″ vacuum
	50,000	10.45	

3. The hourly steam rates of two 10,000-kw. turboalternators are 12,000 lb. + 12 lb. per kw. − 0.0001 lb. kw.^{2} How would you divide a 16,000-kw. load between them? How, if the third term were plus?

4. Two steam turbines each of 20,000-kw. capacity drive a total load of 30,000 kw. The steam rates in pounds per hour are: $R_1 = 2{,}000 + 10\text{ kw.}_1 - 0.0001\text{ kw.}_1^2$ and $R_2 = 5{,}000 + 7\text{ kw.}_2 - 0.00005\text{ kw.}_2^2$. What is the best division of load? What, if the third terms are plus? What, if the second terms are 13 kw.$_1$ and 10 kw.$_2$, respectively?

5. A stream has a steady flow capable of developing 250,000 kw.-hr. per day. How many kilowatts will it provide on the peak of Fig. 71? How many on the base? What kilowatt-hours of storage would be necessary in each case? Plot the mass and flow curves similarly to Fig. 89*D*.

6. A stream of 400,000 kw.-hr. steady daily flow can be used to supply part of the load indicated by Fig. 71. It is estimated that the hydro development will cost as follows per year:

5,000-kw. units at \$ 60,000 + \$0.50 hr. + \$0.002 kw.-hr.
7,500-kw. units at \$ 80,000 + \$0.63 hr. + \$0.002 kw.-hr.
10,000-kw. units at \$100,000 + \$0.75 hr. + \$0.002 kw.-hr.

If the stream is used to full capacity, select the size of standard machine which should be used.

Steam power can be generated at the following annual costs:
10,000-kw. units at \$50,000 + \$3.0 hr. + \$0.006 kw.-hr.
15,000-kw. units at \$67,500 + \$3.5 hr. + \$0.0055 kw.-hr.
20,000-kw. units at \$80,000 + \$4.0 hr. + \$0.005 kw.-hr.

Select the unit best suited to the steam-plant part of the load.

7. Let Fig. 71 represent the electric service in the city of X, which has a local steam plant and can purchase service from two near-by transmission systems.

Local steam service costs per year \$60,000 + \$5 kw. + \$0.003 kw.-hr.

A Falls Co. service costs \$0.005 kw.-hr.

B Power Co. service costs \$15 per horsepower per year.

a. To what extent would you use the outside services?

b. Is it worth while to retain any local generation?

c. If the local plant were abolished, how would you divide your purchase?

8. On the basis of the typical daily duration curve of Fig. 71, determine the annual value of the *A* and *B* per kilowatt costs of old apparatus kept to use on the peak of the load, when the base 20,000 kw. is replaced by more modern apparatus whose annual service costs are

$$\$ = \$10{,}000 + \$8 \text{ kw.} + \$0.006 \text{ kw.-hr.}$$

The old apparatus produces a kilowatt-hour at \$0.007.

9. Plot the chronological load curve, the load-duration curve, the integrated-duration curve, and the total mass curve for the following data:

Time	Load, Kw.	Time	Load, Kw.
12 midnight	8,000	1 P.M.	25,000
1 A.M.	7,000	1:30	27,000
2	6,000	2	28,000
3	6,000	3	28,000
4	7,000	4	30,000
5	6,000	4:30	36,000
6	12,000	5	40,000
7	25,000	5:30	34,000
8	29,000	6	25,000
9	27,000	7	20,000
10	26,000	8	15,000
11	27,000	9	10,000
11:30	24,000	10	10,000
12 noon	22,000	11	8,000
12:30	10,000	12	8,000

10. For the load-duration curve of Fig. 95:

A. Service costs:
$\$5{,}000 + \$10\ \text{kw}_A + \$0.006\ \text{kw.-hr.}_A$

B. Service costs:
$\$X + \$24.6\ \text{kw}_B + \$0.002\ \text{kw.-hr.}_B$

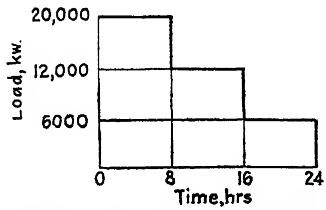


FIG. 95.—Load-duration curve for Problem 10.

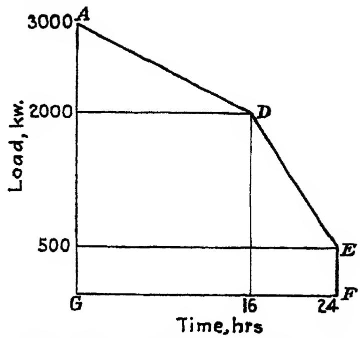


FIG. 96.—Load-duration curve for Problem 11.

How would you use these services on this load?
Compute the value of X which justifies the use of both services.

11. A public-service company considers two sources of electric power: A steam plant with annual costs of \$14,000 + \$24 kw. + \$0.006 kw.-hr. An hydroelectric development costing \$20,000 + \$45 kw. + \$0.002 kw.-hr.

The steam plant will be located within the city which is to be served. There are two possibilities for hydro plant:

1. At a point 6 miles distant from the load center, where the maximum development is limited to 900 kw.
2. At a point 30 miles from the city, where the power supply is unlimited.

The daily curve for the city demand may be substantially approximated by Fig. 96.

a. Choose between the power sources, and determine the extent to which the selected ones will be developed.

b. Show the total annual costs for the various plans you consider. A transmission line will cost \$6,000 per mile, with fixed charges at 12 per cent.

12. For \$5 coal in bunkers and new modern plants of 30,000-kw. units or larger (fixed charges at 15 per cent on \$112 per kilowatt), assume the following costs:

Plant factor, per cent	100	90	80	70	60	50	40	30	20
Coal	0.360	0.360	0.362	0.365	0.370	0.375	0.390	0.415	0.460
Operation	0.042	0.047	0.053	0.062	0.072	0.084	0.100	0.120	0.145
Fixed charges	0.192	0.213	0.240	0.274	0.320	0.384	0.480	0.640	0.960
Total cts. per kw.-hr.	0.594	0.620	0.655	0.701	0.762	0.843	0.970	1.175	1.565

For a utility system which has a load curve as shown in Fig. 97, the base load of 121,000 kw. and 1,960,000 kw.-hr. per day is carried by the steam plants. The peak load of 82,000 kw. and 1,210,000 kw.-hr. is taken by the hydro plants. There is an hydro site available from which 30,000 kw. and 200,000 kw.-hr. per day could be obtained at a total cost of 0.780 ct. per kilowatt-hour, if used on the steam peak as shown in Fig. 97. Assume 280 full working days per year. This new pondage would increase the flow at off-peak times for the other hydro plants by 14,300,000 kw.-hr. for the year.

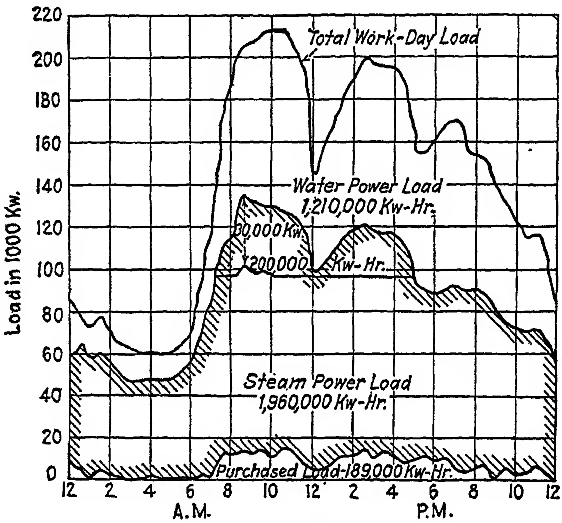


FIG. 97.—A public utility typical load chart, combined water and steam power, for Problem 12.

a. What would the proposed hydroplant save per year?

b. Are the 82,000 kw. of hydro economically used on the peak, or would they be better used on the base?

(From E. M. Burd, Water Power in the Middle West, *Michigan Eng.*, September, 1928.)

13. At a hydro site with an average annual output of 400 million kw.-hr., 85,000 kw. of firm capacity may be installed. Total cost is estimated at $185 per kilowatt with fixed charges at 10 per cent. For cost of hydro operation, see Fig. 52 text. There would be 50 miles of transmission with transformer stations costing $2,500,000 with total annual charges of 13 per cent. The transmission efficiency would be 95 per cent for demand and energy. An alternative steam plant at the load center would cost $16 per kilowatt year + 2.5 mills per kilowatt-hour.

a. What is the annual saving in favor of the cheaper power supply?

b. If the steam plant costs $120 per kilowatt, what total rate of return could be earned on the extra investment required for the hydro (including transmission) project?

14. It is estimated that a city will need additional capacity of 250,000 kw. and total energy of 2,230 million kw.-hr. for the coming year. Steam plant will cost $135 per kilowatt with fixed charges at 11.5 per cent and operating costs of 5.68 mills per kilowatt-hour. An hydro plant can deliver in the city 150,000 kw. firm capacity and 1,150 million kw.-hr. at 6.2 mills per kilowatt-hour. If used, this will save $82,000 a year in coal and supplies for the steam plants which will operate at 6.63 mills per kilowatt-hour. What will the hydro plant save the utility a year?

(From Combined Energy Generation by Ezra B. Whitman, *Trans. A.S.C.E.*, 1939, p. 1123.)

CHAPTER V

ECONOMIC CONDUCTOR SECTION—POWER DISTRIBUTION SYSTEMS

73A. Distribution Costs.—In the period of rapidly growing loads up to 1929, the emphasis in original design seemed to concentrate on the large power stations that had to be added continually to the utility systems to care for the predicted loads of the future. With the great drop in the industrial demands after that date and the cessation of new station construction, engineers found time to restudy their distribution systems and to analyze their economic and engineering characteristics in terms of modern requirements and the latest types of equipment and building methods. The recent widespread installation of electric ranges, refrigerators, oil-burner motors and water heaters, together with an almost universal demand for radio and electric-clock time service have made it necessary for the utility companies to guard their distribution areas most carefully against service interruptions and to furnish extremely good voltage and frequency regulation. Developments in faster relays, reclosing and repeating fuses, step regulators, capacitors, and pole-mounted regulators, together with improvements in tap-changing and induction regulators, have assisted in the required revisions. Very thorough and wise planning is justified, therefore, in order that the residential systems shall be rehabilitated and extended so as to be entirely adequate and economical for their new duties. That such attention is warranted from an investment point of view is readily seen by a glance at Table 7, Annual Capital Expenditures for New Construction, in Sec. 18. Here the expenditures for extension of the distribution system have been the major items in the budget for many years.

It should be emphasized that the cost of generating electric energy, as detailed in Chap. II, is only a small part of the total cost of electric service, which must also include transmission (Chap. XI) and the large item of distribution. Figure 98 shows

the cost of distribution (excluding the cost of energy) in relation to customer use for various customer densities.

Howard P. Seelye of the Detroit Edison Company makes the following estimates of investment and cost:[1]

1. Distribution of system investment on a large system having a diversified load: power plants, 30 to 35 per cent; local distribution, 30 to 35, per cent; bulk power transmission, 10 per cent; subtransmission, 10 per cent; distribution substations, 15 per cent.

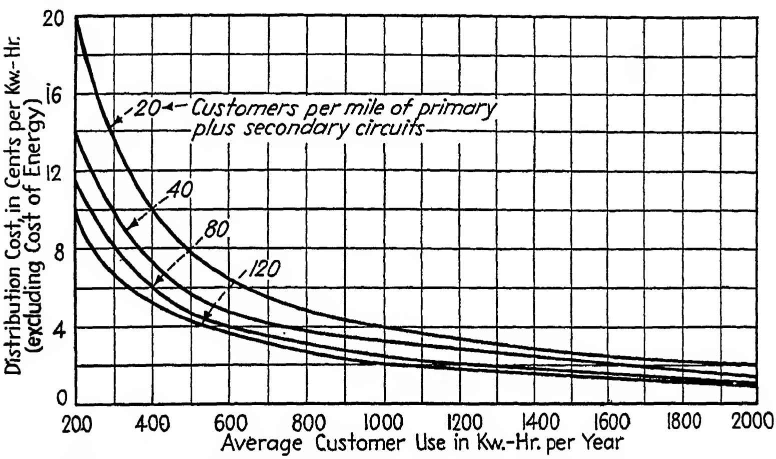


FIG. 98.—Cost of distribution (excluding cost of energy) in relation to customer use for various customer densities. (*Economic Aspects of Energy Generation, by Philip Sporn, A.S.C.E. Trans.* 1939.)

2. Component parts of distribution investment to serve a typical residential load: feeders, 5 per cent; poles and fixtures, 29 per cent; primaries and secondaries, 25 per cent; transformers, 15 per cent; services, 8 per cent; meters, 18 per cent.

3. Typical distribution cost to serve residential customers in a rapidly growing community: power plant, transmission, and substation, 33 per cent; distribution, 33 per cent; commercial, 34 per cent.

There has been much discussion of distribution costs in the last few years, notably a symposium held in New York City on "What Electricity Costs in the Home and on the Farm," by the Institute of Public Engineering on Jan. 20, 1933. The

[1] See Facts on Distribution Costs, by Howard P. Seelye, *Elec. World*, June 20, 1936.

Federal Power Commission also has made a national survey of the cost of distribution of electricity (Power Series 3 1936).[1] This interest has developed because of the newer analyses mentioned above and also because of the importance of this factor in the Federal plans to supply power to municipalities and districts to be resold over their own distributing systems. Table 23, Distribution Costs for Residential Service, shows 10 determinations of the dollars investment in distribution per customer, together with the fixed charges on this investment and the operating expenses for various usages of kilowatt-hours per year. Here distribution costs are defined as those between the outgoing wires of the distribution substation to and including the customer's meter.

The student should understand that, as the table plainly shows, there are many variables in such costs as collected from different localities. Even if an absolutely uniform basis of accounting were used, there would still be the difference in wage scales, in service standards, in geography and storm exposure necessitating more or less "heavy loading" construction, in customer density and usage, in amount of underground construction, etc. Moreover, in published figures such as these, there is the uncertainty of just what items have been included under each heading. For example, column 4, Commercial Expense, includes fixed charges on office space and uncollectible bills which are not usually included in this accounting classification. Utilization, commercial, and new business expense will, of course, vary to a marked degree and depend upon company policies and selling plans.

Some of the new ideas presented for distribution design are the interconnection of lightning-arrester grounds and secondary neutral grounds, combining the primary and secondary neutrals, rapid reclosure of oil circuit breakers or serial replacement of fuses, transformers of lower impedance, longer spans, lighter poles, bare wire, increased loading of transformers, etc.[2]

73B. General Considerations.—The design of a power plant is controlled by external, quite as much as by internal, requirements; therefore it is necessary that we make a study of certain

[1] For review of the report by Barclay J. Sickles, Wisconsin Public Service Commission, see *Jour. Land Pub. Util. Econ.*, August, 1937.

[2] See *Elec. World*, May 26, 1934, p. 769, and Nov. 10, 1934, p. 27.

TABLE 23.—DISTRIBUTION COSTS FOR RESIDENTIAL SERVICE

	1	2	3	4	5	6	7	8	9	10
Reported by	Kelly	Pike	Bary	Marshall and Snow	Panter	Lacombe	Lacombe	Lacombe	Lacombe	Reed
Date	1930	1930	1933	1930	1932	1934	1934	1934	1934	1934
District covered	Niagara Hudson	N.Y. State Utilities	N.Y. State	1 sq. mile Detroit	Los Angeles	Co. 1, city	Co. 2, city	Co. 3, city	Co. 4, spread	City
Overhead or underground construction	Overhead	Overhead	Overhead	Both	Overhead	Both	Both	Overhead	Overhead	Overhead
No. residence customers	200 per mile	...	...	2,240	...	56,202	474,399	93,761	138,207	4,608
Kw.-hr. per customer, per year	...	550	550	550	537	757	643.5	700	658	560
Includes street lights	Yes	No	...	No	...	No	No	No	No	No
Investment per customer:										
Distribution lines	$13.00	$30.00	$ 61.60	$31.10	$25.50	$24.71	$21.32	$33.16	$39.73	$32.27
Distribution transformers	8.37	6.50	8.56	7.72	6.00	7.39	8.33	7.61	8.98	6.29
Services	8.50	15.00	24.75	4.18	19.00	6.48	6.89	6.95	11.54	13.00
Meters	9.87	Included	Included	9.77	Included	8.89	10.24	11.25	14.98	18.00
General equipment	2.69	3.50	6.47	4.23	3.00	17.15	12.58	14.25	11.90	14.09
Working capital	...	...	2.52							
Total	$42.43	$55.00	$103.90	$57.00	$53.50	$64.62	$59.36	$73.22	$87.13	$83.65
Fixed charges, per cent	13.5	11.5	11.5	14.0	7.6					
Fixed charges, amount	$ 5.72	$ 6.35	$ 12.23	$ 7.98	$ 4.07					
Operating expenses:										
Distribution expense	$ 1.92	$ 2.00	$ 2.96	$ 1.60	$ 2.72	$ 1.88	$ 2.18	$ 1.69	$ 3.76	$ 1.91
Utilization expense	1.57	0.50	0.70	1.65	0.66	1.07	1.30	0.70	0.31	0.75
Commercial expense	1.87	1.80	3.00	6.93	1.48	2.91	3.79	2.31	4.11	3.12
New business expense	0.96	1.25	1.80	1.32	0.92	0.77	0.24	2.50	2.54	1.73
Uncollectible bills	0.03	...	...	Included						
General expense	3.28	1.05	1.61	Included	1.17	6.06	2.07	3.78	4.84	2.11
Total expense	$ 9.63	$ 6.60	$ 10.07	$11.50	$ 6.95	$12.69	$ 9.58	$10.98	$15.56	$ 9.62
Grand total per customer per year	$15.35	$12.95	$ 22.30	$19.48	$11.02					

References:

1. KELLY, COL. WILLIAM, Economics of the Power Industry, *N.E.L.A. Bull.*, April, 1931.
2. Distribution Costs Subjected to New Analysis, *Elec. World*, Mar. 18, 1933, p. 348.
3. BARY, C., Distribution Costs of Electric Service, *Elec. World*, Aug. 19, 1933, p. 248.
4. MARSHALL AND SNOW, Distribution Costs—Residence Service, *N.E.L.A. Bull.*, June 9, 1931.
5. Distribution Costs Subjected to New Analysis, *Elec. World*, Mar. 18, 1933, p. 348.
6 to 10. LACOMBE, C. F., Beware of Averages in Distribution Cost Studies, *Elec. World*, July 21, 1934, p. 76.

	1926	1927	1928	1929	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939
Average residence consumption United States	428	444	460	499	543	578	597	595	624	672	727	793	845	890

conditions affecting the system of distribution from the plant to the point of utilization. (1) The distribution system is the all-important factor in determining the frequency as well as the voltage at which the plant must deliver service, and whether the plant shall supply alternating or direct current. (2) The character of the distribution itself has a profound bearing upon the plant location. (3) The design of a very important part of the distribution system internal to the generating plant, *viz.*, the bus bars, must be determined by the same principles as are involved in the selection of conducting copper for that portion of the distribution system lying outside the plant walls.

In the analytic determination of the distribution system, we must bear in mind just what is included in this distribution system. For our present purposes, we shall include all those elements involved in getting the service from the station bus bars to the point of utilization, including motors and auxiliaries as part of the distribution system, in so far as the investment cost and the operating costs of the motors are influenced by the character of the distribution system.

Whether our distribution from the power house is overhead or underground, whether it is alternating or direct current, the distribution costs will be made up of two components—investment costs and operating costs—each of these, in its turn, being dependent on an initial underlying quantity, independent of the load, and on another quantity dependent on the usage.

The way in which these costs are made up may be somewhat clearer if we confine our discussion, initially, to a direct-current system of transmission and distribution, and consider first the matter of investment in such a system as we have described, made up of an *underlying* independent cost and of another *marginal* cost dependent on the load or usage. It is evident that if we have to transmit electric service from one point to another we shall have to have some sort of supporting structure whether our conductors carry much or little energy, much or little power. In the case of overhead transmission, we shall have to have our pole lines set in place with the labor cost attendant thereon, and have them equipped with crossarms and insulators. In the case of underground transmission, we shall have to dig and subsequently refill our trench, laying some concrete and at least one duct therein. Moreover whether the cable used in underground

transmission is large or small, the investment will in part consist of insulation independent of the cable size, part of the cost of drawing in the cable will be that involved in getting men onto the job of drawing it, irrespective of the force involved, which is a function of the size of the cable, and a certain amount of lead sheathing will be involved whether the cable is small or large.

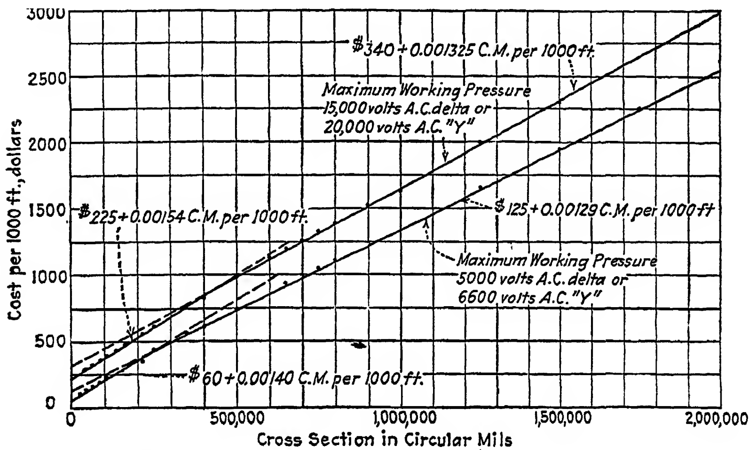


FIG. 99.—Cost of varnished-cambric insulated cable (list less 55 per cent), single-stranded conductor, single-braided, for 5- and 15-kv. delta, in form $a + b$ cir. mils. (*Catalog GEA.*—600.)

In addition to the foregoing, the extent of utilization of the cable or overhead wires will determine the size and hence involve investment in marginal copper. These two factors of transmission investment for a given type and voltage can be determined by plotting commercial conductor costs as ordinates and circular mils as abscissas. A straight line will very closely approximate the points so determined. Its slope will give the b factor and its intercept on the dollars-per-mile axis will give the a factor in the expression

$$\text{Conductor cost in \$ per mile} = a + b \text{ cir. mils.} \tag{63}$$

To this must be added the cost of installation of the conductor. Figure 99 gives the cost in the form of Eq. (63) of single-conductor varnished-cambric insulated cable for 5,000 and 15,000 volts delta, in sizes from No. 6 to 2,000,000 cir. mils, per length of 1,000 ft.

The operating costs involved in supplying a given load curve over an existing distribution already designed and installed will similarly be made up of an underlying and a marginal cost. The underlying cost will represent the carrying charges on the power-station feeder panel and switches, the labor attendant thereon, and the proper portion of the other station costs that cannot properly be allocated to either power or energy. The operating cost of our distribution system includes, in addition to the foregoing, certain marginal costs dependent on the extent and manner of usage, these being the costs involved in the fact that power and energy are lost in the very act of transmission and such losses for a given cable depend on both the demand and the energy utilized at the point of delivery.

Now the layout of a proper distribution will nicely balance all the variable elements of cost by selecting that voltage and conducting section which will give the best transmission cost, including thereunder any reactive effect on the power house itself and the utilization apparatus at the point of delivery.

For a given voltage, the problem is merely to select that number of circular mils per ampere that will give the minimum aggregate of annual investment and operating costs. Now it is clear that increase or decrease in the number of circular mils used for a given job will not affect the underlying investment or operating costs, since it can have no effect on the size of trench dug, the investment in the station feeder panel, or the number of poles erected. On the other hand, although this is true for a given system of distribution, so far as concerns the determination of the size of conductor to be installed, still the underlying costs will not be the same for a three-phase alternating-current high-voltage system as they would be for a two-wire direct-current low-voltage system, and this difference must be taken into account in choosing between systems. Our present task is to determine what copper section is called for by each given system, and later, having chosen the proper copper section for each system under consideration, then to choose between the systems.

The amount of copper to be installed will depend purely and simply on the number of amperes transmitted and not on the voltage or on the length of transmission, excepting as the voltage may influence the marginal copper cost given in Eq. (63) above. This must be apparent, for if we can afford to install a certain

number of circular mils to transmit 1 amp. for 1 mile, the investment and the energy loss will be the same for any other mile, therefore demanding a cross section independent of the length of conductor. Again, if it is good economy to use 1,500 cir. mils cross section in the transmission of 1 amp., it will pay us exactly as well to use 3,000 cir. mils for the transmission of another ampere in addition to the first, or 4,500 cir. mils for the transmission of 3 amp., etc. With certain significant qualifications, Kelvin's rule applies here, *viz.*, that the most economical number of circular mils per ampere will be obtained when the annual carrying charges on the investment in copper per ampere are just equaled by the annual energy cost due to losses involved in the transmission of that ampere.

However, our actual annual power service cost consists of three items as given in Chap. IV, Eq. (40).

$$\text{Annual service cost in } \$ = A + B \text{ kw.} + C \text{ kw.-hr.}$$

It is apparent therefore that, in addition to the annual energy loss, we shall have a power loss representing roughly a certain portion of station investment and other similar elements rendered unavailable for other demand. We should then compute two separate conducting sections, one based on the application in Kelvin's rule of the annual power cost involved in the maximum I^2R, the other based on the annual energy loss determined from the average I^2R. The proper copper section to install will be the square root of the sum of the squares of the two sections so determined.

74. The Ideal Conductor Section.—There are four general factors affecting conductor design: (1) economic, (2) thermal, (3) regulation, (4) structural. If our economic design is adequate, it will usually be found that the line can carry its load without overheating or excessive voltage drop, except for secondary distribution in small towns, and that it is structurally strong. In certain special cases, however, other criteria than the economic may govern. For example, the economic current for a No. 00 23,000-volt underground cable may be 200 amp., but owing to the type of duct construction used and the radiating facilities to dissipate heat, the current may have to be limited to 100 amp. to prevent damage to the cable insulation by overheating. Again, in the case of the conductor for a series incandescent

lighting circuit carrying, say, 4.4 amp., the economic section may be 8,000 cir. mil, or a No. 11 wire. Obviously, no conductor less than a No. 8 or No. 6 would be structurally strong enough to string on a pole line. In case it is desired to improve the regulation of a line, induction regulators, capacitors, synchronous condensers, etc., are available and should be considered before departing from the ideal section.

The use of smaller or larger conductors will not affect the power actually delivered at the load—kilowatts—or the energy taken by the load—kilowatt-hours. The power plant does have to supply, in addition to the demands of the load, the power wasted in transmission and similarly the energy wasted in transmission, these being, respectively,

$$\text{kw.} = \frac{nRI_m^2}{1{,}000} \quad \text{and} \quad \text{kw.-hr.} = \frac{nR}{1{,}000}\int I^2 dt \tag{64}$$

where n is the number of conductors, R the resistance of each, I_m the maximum current during the year, and I the current at any time t during the year, changing from time to time witn the variations in the load.

To build the line, we have to survey, acquire right-of-way, design, supervise, and erect structures. Much of this cost will represent an expense independent of either the number or size of conductors. Let us call this cost G per mile, and the annual charge on it will be fG, where f is the percentage of fixed charges.

The cost of insulators and pins, tie wires, stringing and tying-in conductors and a small part of the conductor cost will be dependent on the total conductor mileage, *i.e.*, on the number of conductors, and the length of the circuit. The carrying charges on this investment we shall indicate by $f'nla$, a being the underlying cost per conductor mile, and l the length in miles.

In addition to these annual costs, it is obvious that we shall have to pay for mere size in our conductors. The carrying costs on the conductor section may be represented by $f''nlb$ cir. mils, where b is the increment cost per circular mil per unit of length. The total fixed charges in each case, f, f', and f'', will have a common per cent for cost of money but will probably differ as to taxes, insurance, and depreciation. Thus galvanized-steel towers and right-of-way may be good for 40 years, but stringing and conductor costs may serve for only 15 or 20 years.

Noting that the size of our conductors inversely affects the resistance, we may substitute for R in Eq. (64) its equivalent; thus,

$$R = \frac{\rho l}{\text{cir. mils}} \tag{65}$$

ρ (the specific resistance) per circular-mil-mile for hard-drawn stranded copper = 57,500, for aluminum = 91,600 at 20°C. The total annual cost of generation and delivery thus becomes

$$\$ = [A + B \text{ kw.} + C \text{ kw.-hr.} + fGl + f'nla] + f''nlb \text{ cir. mils} + \left(\frac{BI_m{}^2}{1{,}000} + \frac{C\int I^2dt}{1{,}000}\right)\frac{\rho nl}{\text{cir. mils}}. \tag{66}$$

None of the square-bracketed terms involves conductor section, the first unbracketed term involves a cost proportional to the section, the last terms cost inversely proportional to the section. Cost will then be indicated as a function of circular mils by superimposing a horizontal line, which represents the value of the bracketed terms, an inclined line through the origin to represent the cost of the conductor section, and a rectangular hyperbola asymptotic to the axes to represent the power and energy losses in the line. The resultant curve is an oblique hyperbola, as in Fig. 100, whose lowest point indicates minimum cost and, hence, ideal conductor section. Analytically, we may determine this section by the usual process.

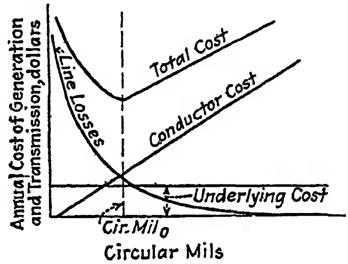


FIG. 100.—Variation of annual cost of generation and transmission with the size of the conductor.

$$\frac{d\$}{d \text{ cir. mils}} = f''nlb - \left(\frac{BI_m{}^2}{1{,}000} + \frac{C\int I^2dt}{1{,}000}\right)\frac{\rho nl}{\text{cir. mils}^2} = 0 \tag{67}$$

giving as our criterion for ideal section

$$\text{cir. mils}_0{}^2 = \left(\frac{BI_m{}^2 + C\int I^2dt}{1{,}000}\right)\frac{\rho nl}{f''bnl} \tag{68}$$

where cir. mils_0 denotes the ideal section in circular mils. So long as we have to deal with similar load curves,

$$\int I^2dt = KI_m{}^2,$$

and we have

$$\frac{\text{cir. mils}_0}{I_m} = \sqrt{\left(\frac{B + CK}{1{,}000}\right)\frac{\rho}{f''b}}. \tag{69}$$

That is, the ideal number of circular mils per maximum ampere is directly proportional to the square root of the power cost, the energy cost, the square of the *form factor* of the load curve, and the specific resistance of the conductor material. It is inversely proportional to the square root of the annual percentage of charges on the conductor and the marginal cost of the conductor.

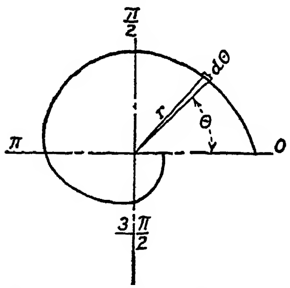


FIG. 101.—Polar load duration curve, to determine $I_{\sqrt{\text{m.s.}}}$

Note that the ideal section for any given set of conditions is independent of number of phases, or circuits; voltage, power, or power factor except as these affect the current; or length of the line. It is simply a section per ampere dependent on service costs, shape of load curve, nature of conductor, cost of conducting material, and fixed charge rate f''. These latter are likely to remain constant for any given property, and so one may establish a section per ampere to be modified only as they may change.

It is convenient to substitute for $\int_0^{8760} \frac{I^2dt}{1{,}000}$ its equivalent 8.76 $I_{\sqrt{\text{ms}}}{}^2$ where $I_{\sqrt{\text{ms}}}{}^2$ is the annual mean square of effective current values. This may be determined by plotting a yearly load—or load-duration curve—in polar form with 360 deg. = 1 year and with amperes as radii. The area of this curve is found by using a planimeter, this equals π times the mean square current. For in any closed polar figure, as Fig. 101, the total area being

$$\text{Area} = \int_0^{2\pi} \tfrac{1}{2}r^2d\theta, \tag{70}$$

the summation of a series of infinitesimal triangles of height r and base $rd\theta$. To get the mean square radius, divide by π, then

$$R_{\sqrt{\text{ms}}}{}^2 = \frac{1}{2\pi}\int_0^{2\pi} r^2d\theta. \tag{71}$$

But the area of an equivalent circle would be area $= \pi R^2$ (equivalent). Therefore,

$$R^2\ (\text{equiv.}) = \frac{\text{area}}{\pi} = \frac{1}{2\pi}\int_0^{2\pi} r^2 d\theta = R_{\sqrt{\text{ms}}}{}^2. \tag{72}$$

Figure 102 shows the load data of the typical daily curve of Fig. 71 plotted as a polar curve, curve I being plotted from the duration curve, whereas curve II is plotted from the chronological curve. The measured area averages 10.72 sq. in.; hence the radius of the equivalent circle is 1.846 in. which represents 26,250 kw. Thus the ratio of the kilowatts corresponding to the r.m.s. radius to the maximum kilowatts is as 26,250 is to 36,600, or 0.717, and the square of this ratio is 0.515. The voltage and power factor being assumed to remain constant, 0.717 would also be the ratio between the r.m.s. amperes and the maximum amperes.

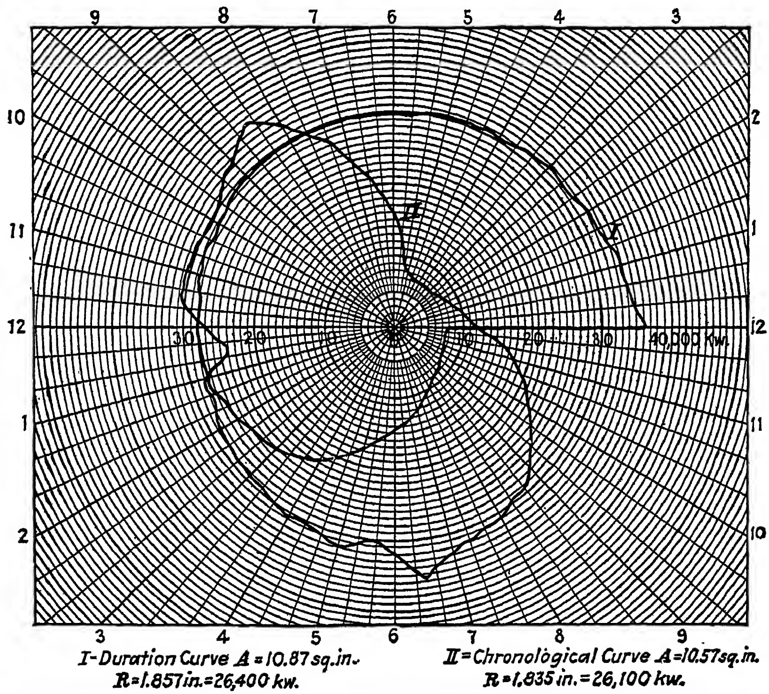


FIG. 102.—Polar load curves: I, from the duration curve, II, from the chronological curve, of Fig. 71.

Thus if the load of Fig. 71 is to be transmitted at 0.8 power factor, three phase, at 24,000 volts,

$$I_m = \frac{36{,}600}{\sqrt{3} \times 24 \times 0.8} = 1{,}100 \text{ amp.} \tag{73}$$

Let the costs be assumed as follows:

B = \$15 per year per kw.
C = \$0.005 per kw.-hr.
ρ = 57,500 for copper per cir. mil mile.
f'' = 0.15 for the total fixed charges.
b = \$0.0084 for the marginal cost of copper per cir. mil mile in a suitable cable for this work.

Then according to Eq. (68), the ideal circular mils would be

$$\text{cir. mils}_0 = \sqrt{\left(\frac{BI_m^2}{1{,}000} + 8.76CI_{\sqrt{\text{ms}}}^2\right)\frac{\rho}{f''b}} \tag{74}$$

$$= I_m\sqrt{\left(\frac{B}{1{,}000} + 8.76C0.515\right)\frac{\rho}{f''b}} \tag{75}$$

$$= 1{,}100\sqrt{\left(\frac{15}{1{,}000} + 8.76 \times 0.005 \times 0.515\right)\frac{57{,}500}{0.15 \times 0.0084}} \tag{76}$$

$$= 1{,}100 \times 1{,}315 = 1{,}445{,}000 \text{ cir. mils} \tag{77}$$

and the ideal circular mils per ampere as in Eq. (69) would be

$$\frac{\text{cir. mils}_0}{I_m} = 1{,}315 \text{ cir. mils.} \tag{78}$$

We have now

$$\text{cir. mils}_0^2 = \left(\frac{BI_m^2}{1{,}000} + 8.76CI_{\sqrt{\text{ms}}}^2\right)\frac{\rho}{f''b}. \tag{79}$$

This would give

$$\text{cir. mils}_B^2 = \left(\frac{BI_m^2}{1{,}000}\right)\frac{\rho}{f''b} \tag{80}$$

when

$$C = 0 \quad \text{or} \quad I_{\sqrt{\text{ms}}} = 0,$$

and

$$\text{cir. mils}_C^2 = (8.76CI_{\sqrt{\text{ms}}}^2)\frac{\rho}{f''b} \tag{81}$$

when

$$B = 0 \quad \text{or} \quad I_m = 0.$$

Noting that cir. mils_B is the section occasioned by the transmission of power, cir. mils_C the section occasioned by the transmission of energy, and cir. mils_0 that due to both, we observe further that

$$\text{cir. mils}_0{}^2 = \text{cir. mils}_B{}^2 + \text{cir. mils}_C{}^2, \text{ as in Fig. 103.}$$

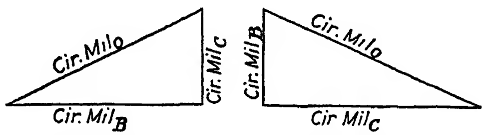


FIG. 103.—Relation between conductor section for power (cir. mils_B), section for energy (cir. mils_C), and total section (cir. mils_O).

Therefore, cir. mils_0 = cir. mils_B approximately, or cir. mils_0 = cir. mils_C approximately. These approximations are of very considerable value in practice.

We may modify slightly the criterion for ideal section, as given in Eq. (79), thus:

$$f''b \text{ cir. mils}_0 = \left[\frac{BI_m{}^2}{1{,}000} + 8.76CI_{\sqrt{\text{ms}}}{}^2\right]\frac{\rho}{\text{cir. mils}_0}, \tag{82}$$

i.e., the annual charges on the ideal copper are just equal to the annual cost of the line losses per conductor (Kelvin's law). Substitute this in the expression for cost of generation and delivery, Eq. (66), thus giving an ideal *total* cost

$$\$_0 = \begin{bmatrix} A + B\text{ kw.} \\ +C\text{ kw.-hr.} \\ +fGl \\ +f'nla \end{bmatrix} + nl\left[f''b \text{ cir. mils}_0 + \left(\frac{BI_m{}^2}{1{,}000} + 8.76CI_{\sqrt{\text{ms}}}{}^2\right)\frac{\rho}{\text{cir. mils}_0}\right] \tag{83}$$

= underlying costs + conductor charges + line losses.

Now since for ideal design the conductor charges and the line losses are equal, *i.e.*,

$$f''bnl \text{ cir. mils}_0 = \left(\frac{BI_m{}^2}{1{,}000} + 8.76CI_{\sqrt{\text{ms}}}{}^2\right)\frac{\rho nl}{\text{cir. mils}_0}, \tag{84}$$

then

$$\$_0 = \begin{bmatrix} A + B\text{ kw.} + C\text{ kw.-hr.} \\ +fGl + f'nla \end{bmatrix} + 2f''bnl \text{ cir. mils}_0, \tag{85}$$

or total cost_0 = underlying cost + marginal cost_0, where ideal marginal cost = $\Delta\$_0$.

75. Effect of Design Error.—Note that under ideal conditions of design the marginal, *i.e.*, those costs which vary with the size of the conductor, operating and investment costs are equal, and the total marginal cost is $2f''bnl$ cir. mils_0, as is shown in Fig. 100. This will be found of much importance in the sequel.

It will be of interest to note the effect of design error *on the marginal cost* which may be designated by

$$\Delta\$_1 = nl\left[f''b \text{ cir. mils}_e + \left(\frac{BI_m{}^2}{1{,}000} + 8.76CI_{\sqrt{\text{ms}}}{}^2\right)\frac{\rho}{\text{cir. mils}_e}\right] \quad (86)$$

$$= nl\left[f''b \text{ cir. mils}_0 \frac{\text{cir. mils}_e}{\text{cir. mils}_0} + \left(\frac{BI_m{}^2}{1{,}000} + 8.76CI_{\sqrt{\text{ms}}}{}^2\right)\frac{\rho}{\text{cir. mils}_0}\,\frac{\text{cir. mils}_0}{\text{cir. mils}_e}\right] \quad (87)$$

$$= (nlf''b \text{ cir. mils}_0)\left[\frac{\text{cir. mils}_e}{\text{cir. mils}_0} + \frac{\text{cir. mils}_0}{\text{cir. mils}_e}\right] = \frac{\Delta\$_0}{2}\left[E + \frac{1}{E}\right] \quad (88)$$

where cir. mils_0 represents ideal section, cir. mils_e represents an actual but unideal section, and E is an *error ratio* $= \dfrac{\text{cir. mils}_e}{\text{cir. mils}_0}$. We have then,

for excess error, $\dfrac{\text{cir. mils}_e}{\text{cir. mils}_0} \propto \dfrac{\Delta\$_1}{\Delta\$_0}$,	for deficit error, $\dfrac{\text{cir. mils}_e}{\text{cir. mils}_0} \propto \dfrac{\Delta\$_1}{\Delta\$_0}$
1.20:1.0166	0.80:1.025
1.25:1.025	0.75:1.04
1.50:1.083	0.50:1.25
2.00:1.25	0:∞
3.00:1.667	
4.00:2.125	

These values are shown graphically in Fig. 104.

It will be seen that so long as the error is not great, but little departure from ideal costs will appear as a result of incorrect design. In this case—and in most other design—the engineer who doubts his fundamental assumptions and wishes to be *conservative* will be truly *conservative* if, contrary to the usual implication of the word with reference to capital expenditures, he deliberately modifies his computed ideal section in the way of generosity rather than by *skimped* design.

We have seen above, Eq. (85), that with ideal design the marginal costs are $\Delta\$_0 = 2f''bnl$ cir. mils$_0$. But also by Eq. (69)

$$\text{cir. mils}_0 = I_m \sqrt{\frac{B + CK}{1{,}000} \frac{\rho}{f''b}} = I_m \, cir. \, mils_0 \tag{89}$$

where *cir. mils*$_0$ is the ideal section per ampere for our "job." Hence we have

$$\Delta\$_0 = 2f''bn \, cir. \, mils_0 \, [lI_m], \tag{90}$$

or the marginal cost of transmission is dependent on the ampere miles which for a given system is the same thing as being dependent on the kilovolt-ampere miles and inversely on the voltage.

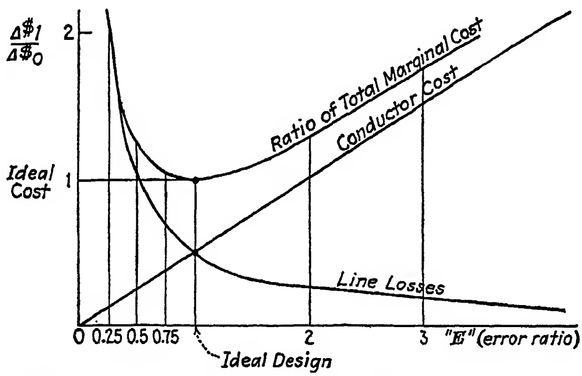


FIG. 104.—Ratio of actual to ideal marginal cost for error in conductor section.

Since a given shape of load curve and other local conditions indicate a fixed number of circular mils per ampere, it is evident that the resistance drop in each conductor will be the same for large or small amounts of power, for high or low voltages, for direct- or alternating-current systems, for single-phase, two-phase, or three-phase lines, whether one or plural circuits in parallel are used. The resistance drop per conductor will be proportionate to the line length. The percentage voltage loss in resistance using the ideal circular mils per ampere will be inversely proportional to the line voltage—the effect of voltage on the (RI/E) drop per conductor is unique, in no other respect can the drop be varied without doing violence to sound economics.

76. Limits for Size of Underground Cables.—In the previous discussion of conductor size, it has been assumed that the section may vary continuously, in fact, discontinuities will appear, since

Table 24.—Resistance and Weight of Conductors at 20°C.—Stranded Conductors[1]

Size cir. mils and B. & S. gauge numbers	Diameter, in.	Area of cross-section	Copper		Aluminum	
		Cir. mils	Ohms per mile	Weight per 1,000 ft., lb.	Ohms per mile	Weight per 1,000 ft., lb.
1,000,000	1.150	1,000,000	0.0575	3,060	0.093	940
950,000	1.125	950,000	0.0605	2,910	0.098	893
900,000	1.100	900,000	0.0638	2,750	0.103	846
850,000	1.060	850,000	0.0676	2,600	0.109	800
800,000	1.035	800,000	0.0719	2,450	0.116	752
750,000	1.000	750,000	0.0766	2,300	0.124	705
700,000	0.965	700,000	0.0822	2,140	0.133	658
650,000	0.930	650,000	0.0884	1,990	0.143	611
600,000	0.895	600,000	0.0957	1,840	0.155	564
550,000	0.855	550,000	0.1045	1,680	0.169	517
500,000	0.813	500,000	0.1150	1,530	0.186	470
450,000	0.771	450,000	0.1280	1,380	0.207	423
400,000	0.725	400,000	0.1440	1,225	0.233	376
350,000	0.680	350,000	0.1640	1,070	0.266	329
300,000	0.630	300,000	0.1920	920	0.310	282
250,000	0.575	250,000	0.2300	765	0.373	235
0000	0.522	211,600	0.2720	647	0.430	199
000	0.465	167,800	0.3430	514	0.556	158
00	0.414	133,100	0.4320	408	0.700	125
0	0.369	105,560	0.5450	323	0.882	99
1	0.328	83,690	0.6870	256	1.110	78.7
2	0.290	66,370	0.8680	203	1.400	62.4
3	0.260	52,630	1.0900	161	1.770	49.5
4	0.232	41,740	1.3800	128	2.230	39.2
5	0.207	33,090	1.7400	101	2.820	31.1
6	0.185	26,250	2.1900	81	3.540	24.7

[1] From Still, "Electric Power Transmission," McGraw-Hill Book Company, Inc.

the number of standard cable sizes which are to be kept in stock for the various voltages must be reduced to a minimum, even if some excess copper is installed now and then.

For the transmission of very large currents, the evident advantages from the viewpoint of continuity of service and ease in handling and supporting conductors would subdivide the total circular mils into several conductors and circuits. This would be true even for overhead transmission where it would be physically possible, but not advisable, to carry an enormous section in one conductor. Instead of insisting upon the exact theoretical circular mils determined by his solution, with its special manufacture, long delivery time, and high cost, the engineer will select, in general, the nearest standard conductor, or multiples of a standard conductor, as shown in Table 24, for his installation. For the overhead transmissions, the conductors rarely exceed 1 in. in diameter.

In the installation of underground conductors, however, a very positive restriction is met in the fact that there is a maximum size of cable that can be pulled into a standard duct. If it is desired to use a section larger than this, it is necessary to purchase a second cable to be installed in a second run of duct. Early duct was 3½-in. inside diameter but later 4-in. duct was substituted, and Chicago uses some of 5 in. for its heavy transmissions. For the 3½-in. diameter duct then, the maximum size of modern shielded three-conductor, paper and lead cable for 22-kv. grounded neutral is about 500,000 cir. mils each. For the 4-kv. feeders, one utility uses three-conductor 350,000-cir. mil cable. On the direct-current systems, the sizes used depend upon the services, 1,000,000, 1,500,000, and 2,000,000 cir. mils being used, with large customers fed from mains on both sides of the street. On account of the thickness of insulation required and for special reasons, the oil-filled cables have been developed in the single-conductor type up to 154 kv. These have either the hollow core or channels directly under the sheath. The three-conductor oil-filled cables, with channels in the three filler spaces, are available up to 66 kv. For information as to the maximum diameter of cable to be installed in various sizes and kinds of ducts the reader is referred to Table I, page 110, of the N.E.L.A. "Underground Systems Reference Book."

A comparison of costs for the distribution system of a residence block in a city of the Great Lakes district is as follows:

Overhead, run on rear alleys	$ 500
Underground, buried cable	2,000
Underground, duct and cable	4,000

In spite of this increase in cost and of the severe space limitations, more and more of the urban utility's transmission and distribution system is continuously being forced underground. This is due to the congested condition of the city streets, the question of sightliness and clearance for fire fighting, as well as for better protection and greater safety for the important circuits themselves. Although finding a distinct improvement underground in the latter features, unfortunately the distribution engineer, in general, finds no corresponding betterment in the matter of room but must worm his duct lines and manholes in among a maze of subways, sewers, water and gas mains together with their services, and other duct runs, at any and all places and levels.

The improvement in performance of the high-voltage underground systems operating at more than 7.5 kv. has been steadily maintained since 1926 and for the year 1929 was reported by N.E.L.A. as 11.7 total cable and joint failures per 100 miles of cable. Thirty-four per cent of the failures were in manholes which contain only about 3 per cent of the cable.

For 1938,[1] the trouble rates per 100 miles of cable were 6.3 for all high-voltage cable, 1.7 for all high-voltage joints, and 0.6 for potheads integral with apparatus. Approximately half of the cable troubles were assignable to four causes: initially defective sheath, 13 per cent; deterioration in cable over 10 years old, 7.2 per cent; sheath corrosion, 19 per cent; and external mechanical damage, 8.9 per cent.

For the first run-of-duct as discussed under General Considerations, the carrying charges will be higher than for successive ones. Thus we shall have investment costs as follows:

$$\$ \text{ per year} = (fG + f'Na' + f''Nna'' + f''Nnb \text{ cir. mils})l \quad (91)$$

where n is the number of conductors per cable, N the number of cables, and the cir. mils are those in each conductor.

It is evident that, up to the continuously ideal section, we shall have a series of minima at the discontinuities which occur wherever it is necessary to lay another duct in order to get in more conductor section, as in Fig. 105. These minima may represent costs greater or less than that at P. Each discontinuity, then, must be studied independently.

[1] See *E.E.I. Pub.* G-5, January, 1940.

It may be noted that it is a matter of indifference whether all cables are of equal size, whether all but one are of maximum size, or whether any other distribution of section is adopted. We are concerned only with the number of cables and the aggregate conductor section.

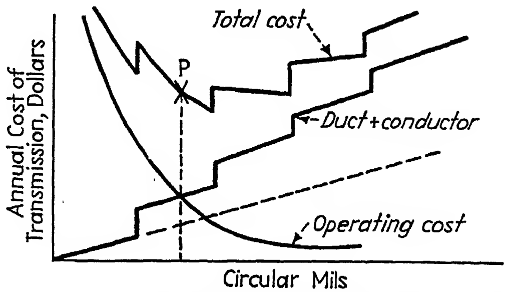


FIG. 105.—Annual cost of transmission with varying size of conductor, showing discontinuities for underground cables.

77. Heating of Underground Cables.—A conductor on an overhead transmission system can generally carry a current far in excess of its economic current without undue heating because of its advantageous position for radiation. In an underground cable, however, the permissible current for safe heating will generally be much less than the economic values. The A.I.E.E. Standards, No. 30, of April, 1937, prescribe the following maximum safe temperatures for the insulation of a wire or cable at the surface of the conductor:

For impregnated paper..........	$(90 - E)$°C.
For varnished cambric...........	$(75 - E)$°C.[1]
For rubber insulation...........	$(60 - 0.25E)$°C.

where E is the r.m.s. operating e.m.f. in kilovolts between conductors in the case of belted multiple-conductor cable or between conductors and ground in the case of single-conductor cable and shielded multiple-conductor cable. Further, the temperature is to be taken as 85° for voltages below 5 kv., and as 60° for voltages above 30 kv. The heating will depend on the size of the conductors, the thickness and material of the insulation, the watts loss in the cable, the location of the cable in the duct bank, the

[1] See also Specifications for Varnished Cambric Insulated Cables, Insulated Power Cable Engineers Association, July, 1939.

number of ducts and cables, the nature of the soil surrounding the duct bank, and on the variation of the electrical load on the cable itself and on the adjacent cables. Figure 106 shows the characteristic temperature curves of cable and duct in terms of the daily variation. It is reported that investment savings of 9 per cent

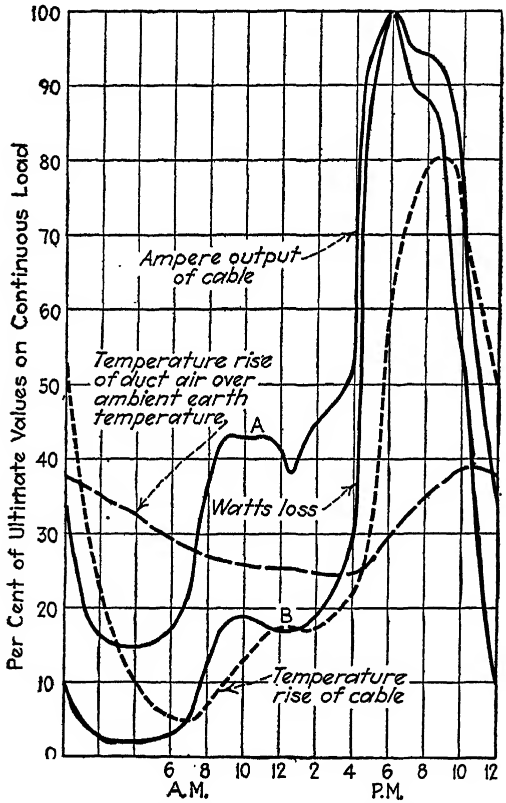


FIG. 106.—Daily variations in ampere load and temperatures of cable and duct. (*Calculations of Cable Temperature, by W. B. Kirke, A.I.E.E., May* 7, 1930.)

may be made by increasing cable ratings 15 per cent above standards.[1]

Since all the temperature rises, *viz.*, the duct above the earth, the sheath above the duct, and the copper above the sheath, are referred to the earth temperature as a base, it is important to know the characteristic variations of the reference temperature.

[1] See Load Ratings of Cable, by Herman Halperin, *A.I.E.E.*, October, 1939.

When the ground temperature, for any depth below the surface, is plotted as a function of time a cyclic curve results, apparently sinusoidal in shape and repeating itself each year. This is shown in Fig. 107 which gives the ambient earth temperature for metropolitan New York. It follows somewhat the daily mean air temperature when averaged over a reasonable time but there is

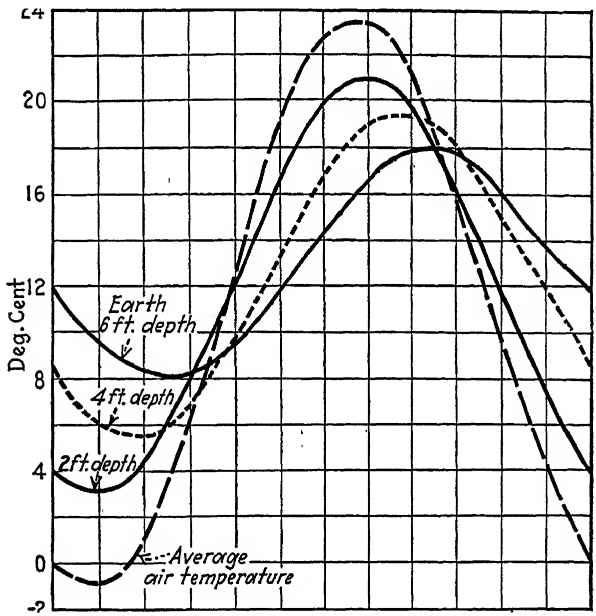


FIG. 107.—Ambient earth temperatures for metropolitan New York, typical for almost any year. (*Calculation of Cable Temperatures, by W. B. Kirke, A.I.E.E., May* 7, 1930.)

a time lag of two weeks to a month. The cooler ground during the winter months is thus of considerable assistance in carrying the peak loads of the year.

Using the recommended thicknesses of insulation adopted by the Insulated Power Cable Engineers Association, Donald M. Simmons gives an extensive treatment of the resistance, reactance, and capacity of cables, their current-carrying capacity, temperature rise, etc., in a series of papers on "The Calculation of the Electrical Problems of Underground Cables" published in the *Electric Journal*, May to November, 1932. Also complete tabulations of the allowable amperes per conductor for various

voltages are given in *E.E.I. Publication* A-14, of July, 1933, entitled "Current Carrying Capacity of Impregnated Paper Insulated Cable." This report lists cables for load factors of 50, 75, and 100 per cent in underground duct banks containing 3, 6, 9, or 12 cables in the outside ducts and for any load factor from 50 to 100 per cent in air. Figure 108 shows the allowable amperes per conductor plotted against the number of loaded cables in the duct bank for certain cables selected from the foregoing report.

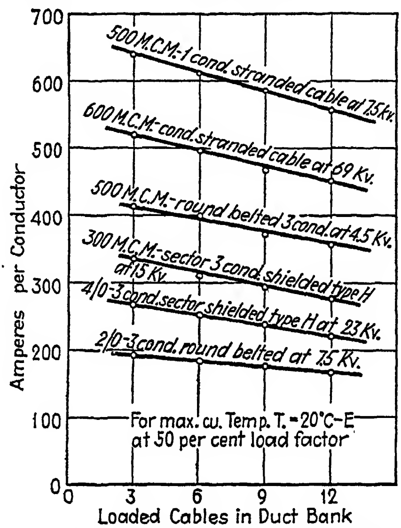


FIG. 108.—Current-carrying capacity of impregnated-paper-insulated cable. (*E.E.I. Pub. A-14, July,* 1933.)

78. Number of Ducts in a Duct Line.—To determine the most economical number of ducts that could be placed together in one duct line, we must consider the excavating, back filling, paving, etc., proportional to the width of the duct line or practically proportional to the square root of the number of ducts; the materials and labor as proportional to the number of ducts; the transportation, tools, etc., as the same for all sizes. If the number of cables in the duct bank is large, then the allowable current per cable may be small on account of the increased heating. Figures 109 and 110 show the annual costs per 1,000 ft. versus the number of ducts and cables for two different types of cables, as determined by Reyneau and Seelye,[1] according to

[1] "Economics of Electrical Distribution," McGraw-Hill Book Company, Inc.

the numerical values that they applied. The curve for total annual cost is a summation of the separate charges. The cost per ampere per 1,000 ft. is the total annual cost divided by the total allowable current carried. For the time and conditions under which these particular costs were derived, the curves

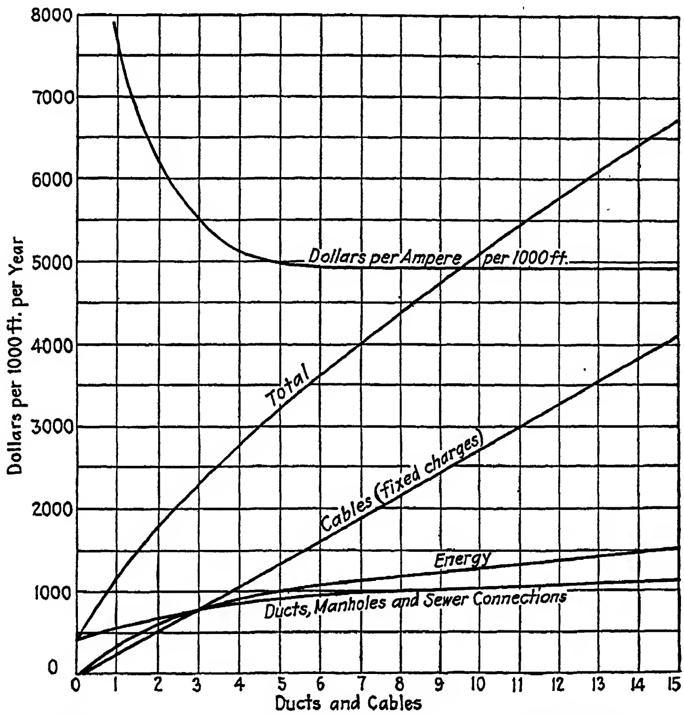


FIG. 109.—Annual costs per 1,000 ft. versus number of ducts and cables for No. 00, three-conductor, 23,000-volt cable.

indicate clearly the high cost of building less than six ducts in a run and that the total cost per ampere at 6- or 8-duct size is no more than that of a 16-duct run. Consequently the smaller size might be built now and later on, when necessary, a second bank of the same size, thus saving the investment charges on empty ducts over the growing period.

In the *Electrical World* for June 2, 1934, John Bankus has developed an analysis, "To Evaluate the Cost of Underground Transmission," which considers the determination of the eco-

nomic voltage and conductor size in paper-insulated cable for underground transmission systems of capacities up to 50,000 kva. and voltages between 11 kv. delta and 57.1 kv. Y. The annual costs have been taken to include yearly charges for station

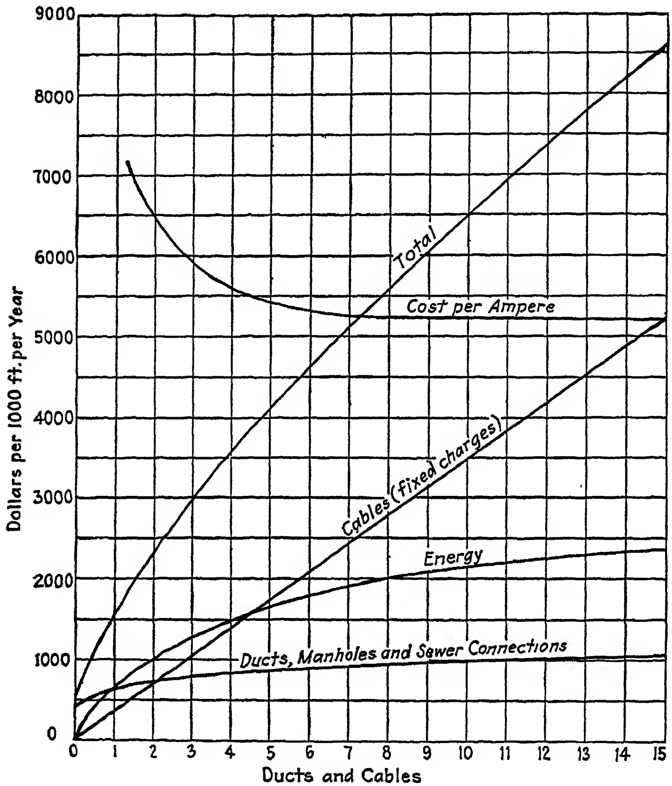


FIG. 110.—Annual costs per 1,000 ft. versus number of ducts and cables for 450,000 cir. mil, three-conductor, 4,600-volt cable.

equipment, subway and cable, and the energy losses in cables and transformation. Figure 111, from the foregoing data, shows the total annual costs per mile for various cables plotted against the kilovolt-amperes transmitted, assuming a single bus station at each end of the transmission line. The author emphasizes two points: (1) the economy resulting from the increase in the transmission voltage, (2) the saving resulting from increasing the conductor size for a given voltage.

79. Cable Data.—For the minimum average thickness of insulation and the thickness of lead sheath recommended by the Insulated Power Cable Engineers Association Standards for cables at various rated voltages, the reader is referred to "Calculation of the Electrical Problems of Underground Cables," by

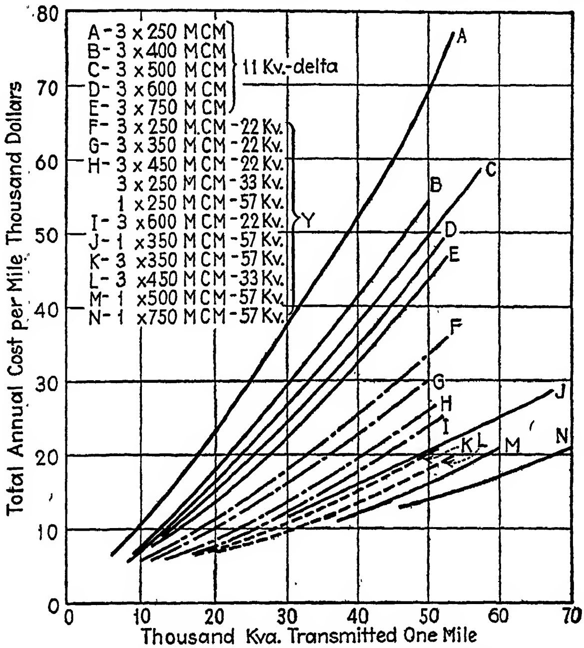


FIG. 111.—Total annual cost per mile for paper cables with single bus stations at each end. (*To Evaluate the Cost of Underground Transmission, by John Bankus, Elec. World, June 2, 1934.*)

D. M. Simmons.[1] If these values are known, the diameters are as follows:

(*a*) $D_o = D_i + 2L$,

(*b*) where $D_i = d + 2T$, for single-conductor cables.

(*c*) $D_i = 2(d + 2T) + 2t$, for two-conductor cables, round duplex.

(*d*) $D_i = \left(1 + \dfrac{2}{\sqrt{3}}\right)(d + 2T) + 2t$, for three-conductor cables.

(*e*) $D_i = (1 + \sqrt{2})(d + 2T) + 2t$, for four-conductor cables, in which D_o is the outside diameter, D_i the inside diameter of the lead sheath, d the conductor diameter, T the con-

[1] *Elec. Jour.*, May to November, 1932.

TABLE 25.—DATA ON PAPER-INSULATED UNDERGROUND TRANSMISSION CABLE

Item	Type	Size, cir. mils	Operating voltage	Insulation size 1/64 in.			Thickness of lead sheath, in.	Overall diameter of cable, in.	Estimated capacity, amp.	Max. temp.,[1] °C.	1930 cost per 1,000 ft. f.o.b. Portland	Weight per 1,000 ft.	Remarks
				Kind	Conductor	Belt							
1	3 C-S	250,000	11,000 delta	Paper	14/64	Type H	8/64	2.31	305	83.7	$1,409	7,856	
	3 C-S	400,000	11,000 delta	Paper	14/64		9/64	2.62	410	83.7	1,986	10,730	
	3 C-S	500,000	11,000 delta	Paper	14/64		9/64	2.79	455	83.7	2,284	12,200	
	3 C-S	600,000	11,000 delta	Paper	14/64		9/64	2.95	510	83.7	2,631	13,640	
	3 C-S	750,000	11,000 delta	Paper	14/64		9/64	3.15	575	83.7	3,123	15,700	
2	3 C-S	250,000	22,000 wye	Paper	18/64	Type H	8/64	2.55	290	77.3	$1,647	8,830	Neutral grounded
	3 C-S	350,000	22,000 wye	Paper	18/64		9/64	2.77	345	77.3	2,062	11,054	
	3 C-S	450,000	22,000 wye	Paper	18/64		9/64	2.95	410	77.3	2,422	12,590	
	3 C-S	600,000	22,000 wye	Paper	18/64		9/64	3.19	470	77.3	2,954	14,800	
3	3 C-S	250,000	22,000 delta	Paper	22/64 to 23/64	Type H	9/64	2.90	287	77.3	$1,830	10,500	
	3 C-S	350,000	22,000 delta	Paper	22/64 to 23/64		9/64	3.08	340	77.3	2,219	12,500	
	3 C-S	450,000	22,000 delta	Paper	22/64 to 23/64		9/64	3.24	405	77.3	2,642	13,800	
	3 C-S	600,000	22,000 delta	Paper	22/64 to 23/64		9/64 to 10/64	3.50	465	77.3	3,248	16,984	
4	3 C-S	250,000	33,000 wye	Paper	25/64	Type H	9/64	3.11	270	70	$2,064	11,340	Neutral grounded
	3 C-S	350,000	33,000 wye	Paper	25/64		9/64	3.30	345	70	2,428	13,100	
	3 C-S	450,000	33,000 wye	Paper	25/64		10/64	3.50	390	70	2,886	15,500	
5	3 C-S	250,000	33,000 delta	Paper	32/64	Type H	9/64	3.42	265	70	$2,721	9,765	One manufacturer recommends oil-filled cable for this voltage. This costs approximately 10 per cent more than cable which is specified
	3 C-S	350,000	33,000 delta	Paper	32/64		10/64	3.66	340	70	3,262	12,200	
	3 C-S	450,000	33,000 delta	Paper	32/64		10/64	3.84	380	70	3,574	14,100	
6	1 C	250,000	57,100 wye	Paper	44/64	Type H	8/64	2.26	314	60	$1,281	6,850	Neutral grounded
	1 C	350,000	57,100 wye	Paper	44/64		8/64	2.36	384	60	1,431	7,650	
	1 C	500,000	57,100 wye	Paper	44/64		8/64	2.50	475	60	1,649	8,560	
	1 C	750,000	57,100 wye	Paper	44/64		9/64	2.71	596	60	2,177	9,977	

NOTE.—The foregoing diameters are the maximum specified by any manufacturer. Capacities assume one cable in duct system, ambient 20°C., and sheath currents not allowed to flow. Cost figures are average for three manufacturers in 1930.

[1] Corresponding to capacity specified.

ductor insulation thickness, t the belt thickness, and L the thickness of the lead sheath—all in inches.

Table 25, gives the over-all diameters, ampere capacities, weights, and costs per 1,000 ft. for paper-insulated cables for transmission[1] at voltages 11 kv. delta to 57.1 kv. Y.

80. Aluminum Conductors.—Aluminum has come to be widely used for main-line overhead transmissions. The following facts should be noted in comparing this material with copper:

	Aluminum	Copper
International annealed copper standard, conductivity, per cent	....	100
Conductivity (annealed copper standard 20°C.) for hard-drawn conductors, per cent	61	97
Resistance, stranded, hard-drawn, per cir. mil-mile, ohms at 20°C	91,600	57,500
Temperature coefficient per °C. at 20°C	0.0039	0.00382
Density	2.705	8.890
Tensile strength, lb. per sq. in	20,000–30,000	50,000–67,000
Coefficient of expansion, °C	0.000023	0.0000167
Modulus of elasticity, lb. per sq. in	9,000,000	15,000,000

Aluminum is, in general, not used as a solid conductor, but is made up in stranded form. For long spans an aluminum cable, steel reinforced, called *A.C.S.R.*, is made up of central strands of "extra high strength" or "plow" steel, double-galvanized, and then the aluminum strands are placed over the steel, around which they fit closely by reason of the stress resulting from the load on the cable. The steel is relied upon for mechanical strength only, and its addition to the conductivity of the aluminum is not counted. Table 24, Resistance and Weight of Conductors, gives the properties of stranded aluminum conductors.

81. Alternating or Direct Current.—Decision as to the character of the electric service, whether alternating or direct, is essentially a problem in utilization and transmission. If the preponderating character of the machines used is such as to require adjustable speed in the motors, direct current must be utilized, provided the extent of the plant is not such as to make

[1] See BANKUS, J., To Evaluate the Cost of Underground Transmission, *Elec. World*, June 2, 1934, p. 790.

the transmission losses prohibitive. The voltage will in such case be approximately 230, giving a standard voltage for motors, and, when used on the three-wire system, permitting the use of standard 115-volt lamps. There is no reasonable excuse for the use of higher voltages, and the only other standard direct-current voltage, *viz.*, 115, is inadmissible in a plant supplying general power service.

If, however, the distances involved are at all considerable, as in a manufacturing plant with a multiplicity of buildings, and if the need for adjustable-speed motors is confined to one or two departments, then alternating current will be utilized, permitting the application of 440 volts directly to the motors or the use of higher distribution voltages conveniently stepped down at the various centers of utilization to 220 or 440.

In the case of city service of a general character, the generation will undoubtedly be as alternating current, and at a pressure suited directly to the distribution voltage of the community, since any voltage that it is feasible to distribute through the city streets will not be in excess of the 13,200 which may be developed directly on the generators. The one exception to this statement would be that of the very large community using high-voltage transmission to distributing transformer stations, in which case generation would be at a moderate pressure, the service being stepped up to the high transmission potential at the power-generating station.

A great many cities have direct-current service in the central part of the town, with alternating-current service in the surrounding portions. Even in such a case, the selection between direct- and alternating-current generation will be thought over advantageously in view of the expense of transmission of direct currents from any reasonable power-house location into the center of the city, and in consideration of the difficulty of securing reasonable sizes of direct-current generators for modern steam-turbine drive. The use of two characters of generation in a plant is to be deprecated because of the loss of flexibility in the use of units. In a case of this kind, it is much better to have all the generation as alternating current, and to use local converters in the direct-current portion of the town.

If for a typical 230-volt direct-current system, we substitute an alternating-current system involving a reasonably high

voltage primary, and with only negligibly long secondary leads, the general character of our transmission system, economically speaking, will be approximately the same as before. However, we shall have fewer amperes to transmit and hence a smaller power and energy loss and a smaller marginal copper investment. Against this advantage of going to the higher voltage, we shall have to set certain compensating disadvantages distributed between investment and operating conditions. The investment disadvantages are the cost of step-down transformers in place, including primary cutouts and labor of hanging, the cost of the transformer iron losses based on the cost of service at the transformer, the higher cost of better insulation in the primary leads, if underground, and possible higher investment in power-station switchboard apparatus. Another compensating disadvantage lies in the copper losses in the transformers which, however, will be a very slight portion of the total cost of transmission.

As between a 230-volt direct-current system and, let us say, a 2,300-volt alternating-current system stepping down to 220 or 440 volts for motor utilization, our choice of system will depend on the aggregate cost of transmitting the service. If the load is very small, it is evident that the disadvantages probably will outweigh considerably any operating saving resultant from the reduction in the number of amperes to be transmitted, particularly so if the transmission is comparatively short. Again the load factor plays a considerable part in the determination as between systems of transmission. Manifestly, if the usage of electric service is of very short duration, the energy saving by the use of a high voltage will be negligible in comparison with the extra investment necessary to effect that saving, while if the electric service is used continuously 8,760 hr. a year the energy saving by the use of high voltage will be relatively great and may perhaps justify increase in investment.

The influence of the size of maximum demand becomes apparent from the fact that for a given load factor the savings due to higher voltage will be roughly proportional to the demand, while the cost of additional insulation for the primary leads and a large part of the cost of transformers with their iron losses will remain fixed. Therefore, it will take a load of some appreciable size to justify the selection of higher voltage. Now, it is perfectly apparent in the foregoing that the character of the power plant

materially affects the selection of the transmission voltage, since a very expensive power plant utilizing high-priced fuel will naturally give a higher cost for the service delivered from the station bus and therefore much more readily justify high-voltage transmission than would an inexpensive plant using cheap fuel.

"Load concentration" also affects the matter of distribution voltage. Obviously, excess investment involved in going to higher voltages would be greatly increased were there a very large number of small taps each requiring its own step-down transformer. Also, the steady iron losses of these step-down transformers would be much increased, so that, in general, the use of higher voltages will best justify itself when the load concentration and the load factor are high, the transmission distance long, and power service relatively expensive.

A generating voltage of as high as 6,600 volts is almost always practicable on power-plant generators of 100 kw. or over, but with modern high-speed machines the use of any transmission voltage higher than 13,200 may involve the use of step-up transformers at the generating end of the distribution system. The investment and operating costs of such transformers, with any extra switching devices that they may occasion, must be added to the cost of the transmission line. Further, a slight increment of step-down transformer cost may be expected, although, unless step-down transformers are very small, the cost is roughly independent of the primary voltage over a very large range of voltage.

It may be, as in a mill using "power" in large individual blocks, that a voltage of as high as 2,300 may be taken directly onto the motors. In this case, the compensating disadvantages of going to a higher voltage as against our datum voltage of 220 consist merely in the slight additional cost of motors and auxiliaries, in the additional insulation cost if underground transmission is used, and, unless the motors are of rather large size, in an increase in the service consumption, due to decreased motor efficiency. If the transmission is long, and if the amount of power service involved is large, such a transmission may easily justify itself, while the net advantage would be slight with a short small-service transmission. In the latter case, the intangible features of dependability and human safety would be the determinant ones. Evidently, in this case, no transformer com-

plications arise excepting those due to the incidental small lighting load.

82. Direct-current Voltages.—The voltages most commonly in use are:

115 or 120 for lighting, small power, and field excitation.

230 or 240 for power, lighting, and excitation.

600 for power and urban electric railways.

1,200 to 1,500 for interurban electric railways.

2,400 to 3,000 for electric trunk lines.

83. Alternating-current Voltages.—For generation and distribution, the following values are available: 120–208, 230, 440, 660, 2,300, 4,000, 4,600, 6,600, 11,000, 13,200, 16,500, and 22,000. The last two voltages have been used on some modern American generators, and English practice has built some 200,000 kw. of generators for as high as 33,000 volts.

A joint committee report approved by N.E.L.A. and N.E.M.A. makes the new rated voltages for transmission systems (the mean receiving voltages) as follows: 6,600, 11,000, 13,200, 22,000, 33,000, 44,000, 66,000, 110,000, 132,000, 154,000, 220,000, 330,000.[1] The Boulder Dam–Los Angeles line, however, is 287,000.

There is no difference in the cost of generators wound for 220 or 440 volts as compared with 2,300 volts. There is an increase of cost, however, when the voltage is above 2,300 since the standard test voltage for all machines is twice the normal voltage of the circuit plus 1,000 volts (A.S.A. Standard C50-1936).

For large city loads, a low-voltage transmission would require excessively large cables, so that systems of 13,000, 23,000, 27,000, and 33,000 volts are found to be more economical. The last voltage named is being used in Chicago in 350,000-cir. mil conductors. Even higher cable voltages are used for special transmissions. For example, the Cleveland Electric Illumination Company has installed a 500,000-cir. mil single-conductor lead-covered cable to operate at 66,000 volts through 8 miles of urban territory on their line between Cleveland and Akron. Also, the Commonwealth Edison Company of Chicago and the Consolidated Edison Company of New York have installed single-

[1] See Switchgear Standards Simplify Selection and Application, *Elec. World*, Aug. 8, 1931, p. 243.

conductor hollow oil-filled cable for 132 kv. These conductors are of 600,000 cir. mils of copper in the Chicago installation and will give a carrying capacity for one underground circuit of 91,000 kva.

In order to select the most economical combination, calculate and compare the fixed and operating costs of generation and distribution at the voltages under consideration. Note that for voltages in excess of 13,000 volts, step-up transformers may need to be included at the generator end of the line. All costs that are affected by the change of voltage must be considered, *e.g.*, generating plant, insulators, inductors, switches, arresters, and transformer plant (see Fig. 111).

There is a "thumb-rule" method of arriving at the proper voltage by taking 1,000 per mile. On a basis of 2,000 cir. mils per ampere, the resistance for 1 amp. would be 27 ohms per wire per mile, and the circuit drop would be 54 volts, or 54/1,000 = 5.4 per cent. This would be about the loss for a 225-mile line operating at 220,000 volts. Obviously, then, it would be absurd to incur such a loss in a transmission of only 1 mile. If any considerable block of power were to be transmitted, a higher voltage than this would be used, say 2,300, 4,600, or even 13,200 volts, and the loss would be correspondingly decreased.

Professor Alfred Still[1] recommends the following empirical formula for preliminary estimates, stating that the results given generally agree with modern practice:

$$\text{Line pressure in kilovolts} = 5.5\sqrt{L + \frac{\text{kw}}{100}}$$

where L is the distance of transmission in miles, and kw is the total kilowatts to be transmitted.

84. Voltage Selection.

Let E = kilovolts of line.

kw. = power to be transmitted at power factor of $\cos\theta$.

$$\text{cir. mils}_0 = cir.\ mils_0\, I = k_1 \frac{\text{kw.}}{E \cos\theta}.$$

$f'a_1$ = annual line costs independent of voltage.

$f''a_2E$ = annual insulator costs, etc., dependent upon E.

$fbk_1 \dfrac{\text{kw.}}{E\cos\theta}$ = annual marginal conductor cost.

[1] See "Electric Power Transmission," McGraw-Hill Book Company, Inc.

$k_2 \dfrac{\text{kw.}^2 R}{E^2 \cos^2 \theta}$ = annual operating cost.

For economic design, $fbk_1 \dfrac{\text{kw.}}{E \cos \theta} = k_2 \dfrac{\text{kw.}^2 R}{E^2 \cos^2 \theta}$.

Neglecting the influence of generating equipment and any preference of the user of the power, and considering only the transmission line, the annual cost becomes

$$\$ = \left(f'(a_1) + f''a_2E + 2fbk_1 \frac{\text{kw.}}{E \cos \theta}\right) nl. \tag{92}$$

Then for a minimum cost

$$\frac{d\$}{dE} = 0, \text{ and } f''a_2 = 2fbk_1 \frac{\text{kw.}}{E^2 \cos \theta}. \tag{93}$$

The ideal voltage

$$E^2 = \frac{2fbk_1 \text{ kw.}}{f''a_2 \cos \theta}, \text{ or } E = \sqrt{\frac{2fbk_1 \text{ kw.}}{f''a_2 \cos \theta}}, \tag{94}$$

i.e., the ideal voltage depends chiefly on the kilowatts of power that are to be transmitted.

Should the voltage be above such a value that it could no longer be generated directly, transformers would have to be added to the line, introducing additional costs of fk_3 kw. $+ f'''a_3E$.

In the paper, "Economy in the Choice of Line Voltages and Conductor Sizes for Transmission Lines," by E. A. Loew,[1] calculated curves are given from which the most economical line voltage can be read off for any length of line and transmitted amount of power. The assumed average costs, types of construction, etc., are aimed to agree with good engineering practice.

85. Frequency.—We have in the United States two standard frequencies, 25 and 60 cycles per second. In the absence of any peculiar controlling conditions, the latter would be selected for reasons of comfort and economy—comfort because with small high-efficiency lamps 25-cycle light causes a noticeable and objectionable flicker; economy because 60-cycle transformers and motors are cheaper, smaller, and more efficient than 25-cycle equipment. Single-phase transformers in sizes from 10 to 500 kva. are about 40 to 50 per cent more costly for 25-cycle than for 60-cycle designs. Motors from 5 to 200 hp. show price

[1] See *A.I.E.E. Jour.*, August, 1928.

differentials of 40 to 60 per cent in favor of 60 cycles for the usual speeds. Generating equipment costs 5 per cent less for 60 cycles than for 25 cycles. Formerly, the 25-cycle power was much to be preferred because of the difficulty of securing satisfactory 60-cycle rotary converters for railway use, together with the increased reactance of transmission lines operated at the higher frequency. At the time that New York, Chicago, and Niagara Falls decided on 25 cycles, the first two power companies had 80 per cent of their load in direct current, while the plants at the Falls were influenced by the large electrolytic load. These reasons have now disappeared, and recent additions to some of the 25-cycle plants have been made with 60-cycle generators, the two systems being interconnected by frequency-changer units so that power may be transferred from side to side of the system. Such units are now installed in a size of 35,000 kva. for the Brooklyn Edison Company and of 49,000 kva. for the Commonwealth Edison Company of Chicago. The use of interpoles along with other developments in modern rotary converters and the use of synchronous condensers have done away with earlier objections to the use of the higher frequency except as regards single-phase railway work. The single-phase railway is better on low frequency, consequently it would run at 25 cycles, or preferably 15 cycles. The history of frequency selection and standardization is written in a most interesting manner by B. G. Lamme, late chief engineer of the Westinghouse Company, in the *Electrical World*, Sept. 20, 1924.

In addition to the two standard frequencies in America, there is also a frequency of 50 cycles in several large systems in southern California. A few other systems of 40, 35, 33⅓, and 30 cycles are gradually being changed to standard frequency.

In Europe, the standard for single-phase railways has been made 16.66 cycles, and for general power and lighting purposes 50 cycles have been adopted.

The small isolated plant may depend on general city service for protection in case of its own disability. Under these circumstances, the determination of the character of service to be locally generated will depend entirely on the public-service supply available, with such provision for adaptation as may be justifiable through the use of voltage transformers or conversion equipment.

86. Number of Phases.—In the case of alternating-current generation and in the absence of any special requirements, the number of phases would be three, because three-phase distribution is considerably cheaper than that given by any other number of phases and because of the simplicity in motor-control apparatus and the more nearly standard character of the three-phase motor. Even with three-phase generation, in the distribution system the phases may be separated and single-phase service be supplied to territory requiring only lights and small motors of less than 10-hp. capacity, care being taken to secure balance of the load on the different phases.

87. General Types of Distribution Systems.—The present vast system of electric distribution has grown from the engineer's idea of how best to supply power to all the connected customers with a satisfactory quality of service and to do so as economically as possible, consistent with that quality. Naturally, such systems have to be capable of growth, and the ideal system will bring its high quality of service and good economy along, hand in hand with its growth. From 1925 to 1929, however, the increase of load was so great that many of the old systems were inadequate to carry it, and such lines had to be changed over to newer systems of greater capacity. For the future, the distribution engineer, while retaining the old reliability and economy, must design a system capable of very great extension and so arranged that maintenance and construction can be carried on with minimum interruptions to service. With the increasing amount and diversity of load in the average residence, there is no time, day or night, when a service interruption will not cause serious inconvenience to the customer. The engineering problem of the distribution of energy in any load area from the supply substations to the consumers is treated extensively in *E.E.I. Publication* A-11, July, 1933, entitled "Load Area Development." The report covers the detailed outline of the problem, system capacity and service continuity and the selection of circuit voltages. The appendices include three practical applications from practice: (1) sparsely loaded area, a section of northwestern Mississippi, by H. L. Melvin of Electric Bond and Share Company; (2) metropolitan load area, Buffalo, N. Y., by R. T. Henry of the Buffalo General Electric Company; and (3) heavy industrial

load area, Pittsburgh, Pa., by W. J. Lyman of the Duquesne Light Company.

In a survey of 95 power companies, the Overhead Systems Committee of the N.E.L.A.[1] found that the trend showed the following conditions:

Fifty-one per cent of the companies were using the 4,000-volt star distribution system for a large portion of their distribution load.

Twenty-four per cent tended to go directly from 2,200/2,400-volt delta systems to a higher delta voltage for their concentrated and rural load.

Four per cent operated a 2,200/2,400-volt delta system with 11,000/13,200-volt delta for concentrated load and rural service.

Twenty-one per cent had taken no steps to increase their distribution voltages, but with increase in load density many of these may change from their lower voltage delta systems to a 4,000-volt star system.

The committee divided all types of distribution circuits into three classes, including all the major differences of the various types of circuits without covering minor peculiar features which differentiate some circuits from the general types. The classification is as follows:

1. Primary System:
   a. Radial.
      (1) Main.
      (2) Feeder and main.
      (3) Segregated phase.
      (4) Duplicate service.
      (5) Parent feeder.
   b. Loop.
      (1) General distribution.
      (2) Bulk load distribution.
   c. Network.
2. Secondary System:
   a. Radial.
   b. Network.
      (1) Isolated.
      (2) Interconnected.

For the description of these various systems, their relative advantages and disadvantages, and the development of the

[1] See *Serial Rept.* 289–46, May, 1929.

modern systems, the reader is referred to Chaps. VI and X of the "Electric System Handbook,"[1] "Electrical Distribution Engineering,"[1] by H. P. Seelye, and the technical press.

The following analytical and economic studies will be of special value to the student in this field: "An Economic Study of Suburban Distribution," costs of several plans of increased distribution, 10-year period, for Somerville, Mass., by Sweetman and Corney; *Electrical Engineering*, January, 1934; "Planning a Distribution System," by J. F. Fairman, Consolidated Edison Company, in *Electric Journal*, June, 1938; "Economic Study Determines Best Network Arrangement," by Forrest and Wickersham, in *Electrical World*, May 4, 1940; "Keeping down Costs for Service Continuity," by W. J. Lyman, Duquesne Light Company, in *Electrical World*, July 13, 1940; and "A Capacitor Program for Distribution Betterment," by Seely and Jeynes, Public Service Electric & Gas Company, in *Electric World*, Aug. 10, 1940, *E.E.I. Publication* G-1, August, 1939; "A-C Networks Operation 1936–1937."

88. Conductor Economies of Standard Systems.—In order to establish these fundamental relations all compared with the single-phase circuit as a base, we shall consider the transmission of a given block of power, a certain kilowattage, to be delivered at E volts, according to the same *economic standards; i.e.*, each ampere will take its ideal number of circular mils as developed in Eq. (69).

1. *Single Phase.*—This will involve two conductors per circuit each carrying current of kw.$/E \cos\theta = I$ amp. The annual cost, as discussed in our consideration of the economic conductor section, will be made up of an underlying charge ($f'anl$) and a marginal charge ($2f''bnl$ cir. mils). The annual cost, then, will be

$$\$ = 2f'al + 4f''bl \text{ cir. mils}_0\, I \tag{95}$$
$$= 100\% \text{ underlying} + 100\% \text{ marginal.} \tag{96}$$

2. *Edison Direct-current or Single-phase, Three-wire.*—In case direct current is desired for lighting, with some power load at the standard 115/230-volts, or that 220/440-volts alternating current is considered, there is great economy of copper in making the distribution on the basis of the Edison three-wire system. This gives E volts between the two outer wires and neutral with

[1] McGraw-Hill Book Company, Inc.

$2E$ volts between the outer conductors, so that advantage is taken of transmitting the load at the double voltage. If care is taken to balance the load evenly between the two outer wires and neutral, the latter will carry only the unbalanced current. There is a possibility, however, of one whole side of the load being turned off, so that the Fire Underwriters require that the neutral shall be of the same size as the outer wires for interior wiring. On this basis for the *same loss*, the copper losses, compared with a two-wire circuit feeding the entire load at E volts, are

$$2I^2R_2 = 2\left(\frac{I}{2}\right)^2 R_3 \tag{97}$$

where R_2 and R_3 are the resistances of the conductors of the two- and three-wire circuits, respectively.

Then

$$R_3 = 4R_2, \quad \text{or} \quad \text{cir. mils}_2 = 4 \text{ cir. mils}_3 \tag{98}$$

and

$$\text{Total copper} = \frac{3 \times \text{cir. mils}_3}{2 \times 4 \text{ cir. mils}_3} = \frac{3}{8} = 37.5\%. \tag{99}$$

If the installation is outdoors where the switching is done in a block, as in white-way street lighting with each street side between the corresponding outer wire and neutral, the neutral may be made half-size with respect to the outer wires, when the system copper, in terms of the copper of the equivalent two-wire circuit, will be

$$\text{Total copper} = \frac{2.5 \times \text{cir. mils}_3}{2 \times 4 \text{ cir. mils}_3} = 31.25\%. \tag{100}$$

As between the use of a low-voltage alternating-current system and a 230-volt direct-current system, comparing a 220-volt three-phase system with the direct-current layout, so far as concerns lighting, there is no advantage in the utilization on either side. Alternating-current motors are somewhat more efficient, less expensive, and considerably more dependable, but if the major part of the utilization requires close and extensive speed adjustment an alternating-current system will prove wasteful of power and energy. If, however, the service utilization is of small magnitude, the polyphase alternating-current system will suffer a disability through the fact that at least three conductors have to be used as against two in the direct-current system. It may

easily be that the cost of purchasing and housing three smaller conductors may be greater than the cost of purchasing and housing two conductors of larger aggregate cross-sectional area.

3. *Quarter-phase, Four-wire.*—This system necessitates four conductors each carrying a phase current of $I/2$ amp., since kw. $= 2EI \cos \theta$.

The total annual cost will be

$$\$ = 4f'al + 8f''bl \textit{ cir. mils}_0\, I/2 \tag{101}$$
$$= 200\% \text{ underlying} + 100\% \text{ marginal.} \tag{102}$$

This transmission is in effect single phase with the advantage that it permits polyphase motor load as well as single-phase lighting.

4. *Quarter-phase, Three-wire.*—This system combines the two return wires of the A and B phases above into one conductor carrying the vector sum of the currents, *i.e.*, $I/\sqrt{2}$, and due to the interconnection gives a voltage, between outside conductors of $\sqrt{2}\,E$, as well as E volts per phase. Thus there are three conductors required, two for a current of $I/2$ amp., and one for $I/\sqrt{2}$ amp. The total annual cost will be

$$\$ = 3f'al + 2f''bl\left(2 \textit{ cir. mils}_0 \frac{I}{2} + \textit{cir. mils}_0 \frac{I}{\sqrt{2}}\right) \tag{103}$$
$$= 150\% \text{ underlying} + 85.25\% \text{ marginal.} \tag{104}$$

This system is less flexible than the four-wire and the regulation is poorer because load conditions on one phase affect the other phase, producing unbalanced voltages.

5. *Three-phase, Three-wire.*—This system takes three wires and has the great advantage of theoretical balanced current $I/\sqrt{3}$ and equal voltages in all phases. The kw. $= \sqrt{3}\,EI \cos \theta$. The total annual cost will be

$$\$ = 3f'al + 6f''bl \textit{ cir. mils}_0 \frac{I}{\sqrt{3}} \tag{105}$$
$$= 150\% \text{ underlying} + 86.5\% \text{ marginal.} \tag{106}$$

This system may be either delta or Y connected and is satisfactory where good load balances are obtained. If the load is unbalanced, the neutral will shift, making voltage regulation difficult.

It will be noted that the three-phase system exceeds the single-phase by 50 per cent in the underlying cost, but saves 13.5 per

cent in the marginal cost. There is, therefore, a certain kilowattage of power that will make the saving in the marginal cost just equal to the loss in the underlying cost, and this kilowattage will mark the economical dividing line between single- and three-phase transmission. To determine this balance point, the single- and three-phase costs will be equal, *i.e.*,

$$2f'al + 4f''bl\ cir.\ mils_0\ I = 3f'al + 6f''bl\ cir.\ mils_0 \frac{I}{\sqrt{3}}. \quad (107)$$

Dividing through by $2f''bl$, then

$$\frac{f'a}{f''b} + 2\ cir.\ mils_0\ I = 1.5 \frac{f'a}{f''b} + \sqrt{3}\ cir.\ mils_0\ I \quad (108)$$

and

$$\frac{0.5f'a}{f''b} = 0.27\ cir.\ mils_0\ I;\ \text{or}\ cir.\ mils_0\ I = 1.85 \frac{f'a}{f''b}. \quad (109)$$

But

$$cir.\ mils_0\ I = \frac{k \cdot \text{kw.}}{E \cos \theta}.$$

Hence,

$$\text{kw.} = 1.85 \frac{f'a}{f''b} \cdot \frac{E \cos \theta}{k}. \quad (110)$$

That is, for E kv. the dividing power is determined.

If compared as to *equal loss* in transmission, the single- and three-phase copper losses are equal, *i.e.*,

$$3\left(\frac{I}{\sqrt{3}}\right)^2 R_3 = 2I^2R_1 \quad (111)$$

where R_1 and R_3 are the resistances of each single-phase and three-phase conductor, respectively.

Therefore,

$$R_3 = 2R_1, \quad (112)$$

or the three-phase conductor has half the cross section of the single-phase conductor. The ratio of the total copper then, in the three-phase circuit is

$$\text{Copper} = \frac{(3 \times \text{cir. mils}_0/2)}{2 \times \text{cir. mils}_0} = 75\%\ \text{single-phase.} \quad (113)$$

In addition to the 220-volt three-phase system, there is one other low-standard power voltage ordinarily used in connection

with three-phase service, *viz.*, 440 volts. The 440-volt arrangement involves no additional insulation cost above that required with 220 volts and has the advantage of transmitting only one-half as much current for a given power usage. It enables the use of smaller motors than could be used directly on 2,300 volts and so is applicable to factories involving extremely small motor use. It should, however, require the use of totally enclosed switching apparatus as it is a little too high a voltage for general use with exposed conducting parts. The extra cost of such enclosure may offset the economic advantages if the number of motors in proportion to the aggregate service demand is very large. Moreover, 440 volts are not applicable to lighting service except through the use of transformers or autotransformers, so that if the lighting supply is the dominant feature 440 volts are inadmissible.

6. *Three-phase, Four-wire.*—This system uses a neutral wire, generally full-size, in addition to the three-phase wires, the voltages between phase wires being 4,000, which gives the standard voltage of 2,300 between phase and neutral wires. This scheme enables transmission to be made at 4,000 volts instead of 2,300, so that the current per wire is 0.577 of I three-phase, three-wire. For the *same loss* in transmission three-phase, three-wire, and four-wire,

$$3R_3 \times I_3^2 = 3R_4(0.577I_3)^2, \tag{114}$$

where R_3 and R_4 are the resistances per conductor of each system, respectively.

Then

$$R_4 = 3.0R_3, \text{ or cir. mils}_4 = \frac{\text{cir. mils}_3}{3}, \tag{115}$$

so that with neutral full size,

$$\text{Total 4-wire copper} = \frac{(4 \times \text{cir. mils}_3/3)}{(3 \times \text{cir. mils}_3)}$$

$$= 44.5\% \text{ of three-wire, three-phase.} \tag{116}$$

$$= 44.5\% \times 0.75 = 33\% \text{ of single-phase.} \tag{117}$$

This system improves the regulation of poor three-wire circuits and also increases the economical radius of distribution. In general, the neutral wire is grounded. The same feeders can be used for lighting and power service, and if the star points of the

star-delta step-down transformers are not connected to the neutral wire, the voltage of each phase can be regulated separately by induction regulators. The system is used in New York, Chicago, Baltimore, Boston, Toledo, Kansas City, Cincinnati, St. Louis, Louisville, Denver, Minneapolis, and Rochester.

89. Probems.

1. A three-phase factory feeder of T.B.W.P., 440 volts, installed in wiring tunnel, 400 ft. long, consists of three 500,000-cir. mil conductors, each costing \$0.46 per foot plus \$0.006 per circular mil mile installed. Fixed charges are at rate of 12 per cent. 2,500 cir. mils should have been used per ampere, but only 600 cir. mils was used.

a. How much is the original design error costing per year?

b. What is the net annual saving after stiffening the existing copper section up to the economic circular mils by using additional 500,000-cir. mil conductors?

c. At what number of circular mils per ampere actually installed would reinforcement to 2,500 cir. mils per ampere cease to be justified?

2. Poles for a three-phase 4,600-volt transmission line will cost \$15 to buy and set up. Estimated life, 15 years. Salvage value, \$3. Spacing, 160 ft. No. 000 bare copper wire (167,800-cir. mil section) can be bought for \$596 per mile; No. 0 (105,500 cir. mil) for \$398 per mile; No. 2 (66,370 cir. mil) for \$272 per mile. Wire will last for 20 years and can then be sold as metallic copper scrap for 15 cts. per pound. (1 lb. = 330,000 cir. mil ft.) The cost of stringing the wire including labor, tie wire, etc., is \$70 per wire per mile. Leasing, surveying, clearing, etc., cost \$2,000 per mile.

Electric service costs the central station company \$40,000 + \$6 kw. + \$0.0035 kw.-hr. annually. Life of line, 40 years.

Taxes, 3 per cent. No insurance. Money use, 6 per cent.

Design the conductors that should be used to supply power to a factory ½ mile from the main transmission line. The demand will follow that of our standard chronological load curve (Fig. 71), the peak load being 170 amp. This design should include

a. The ideal conductor section, in circular mils.

b. The conductors actually used, if standardizing on one size of foregoing wires.

c. The total annual cost for the line.

3. A load curve has maximum amperes = 2,230, r.m.s. amperes = 1,660, for three-phase 24,000-volt service. One conductor costs \$325 + \$0.0084 per circular mil mile in three-phase lead-sheathed cable. Generating costs are \$30,000 + \$15 kw. + \$0.005 kw.-hr. annually. Fixed charges (f' and f'') are 15 per cent. Take 57,500 as the resistance of 1 cir. mil mile of copper.

a. What is the economic number of circular mils per ampere?

b. If you standardize on three-phase 250,000-cir. mil cables, how many are required?

c. What is the marginal cost of the total transmission for 5 miles?

d. What is the underlying cost of the conductors for 5 miles? (*f'nla.*)

(NOTE.—Consider Nos. 4, 5, and 6 on a theoretical basis, neglecting any limitation of current flow due to heating.)

4. On a design basis. On a certain job, it is practicable to install nothing larger than "250,000-cir. mil three-phase" cable, *i.e.*, three-conductor cable, in which each conductor has a cross section of 250,000 cir. mils. The continuously ideal section is 800,000 cir. mils per phase. Would you install this? Why? What loss or saving per year would be involved in the use of three 250,000-cir. mil cables?

Assume the annual fixed costs of one conductor in place to be 0.15 ($560 per mile plus $0.0054 per circular mil mile).

5. See note, No. 4. The continuously ideal section calls for 2,000 cir. mils per ampere. Assume 250,000 cir. mils is the maximum size of three-phase cable that can be pulled. If the annual fixed costs on a mile of cable and duct are $260 plus $0.0005 per circular mil, what is the minimum and what is the maximum economical current to be transmitted over one such 250,000-cir. mil cable? Over two? Over three?

Generalize this in equation form.

6. See note, No. 4. For the same data as No. 5, what minimum aggregate section would be permissible in two cables? In three? Generalize.

7. A transmission line will require a copper section of 2,000 cir. mils per ampere. Copper costs 16 cts. per pound. On the assumption that the per pound cost is a measure of the marginal cost per circular mil mile, at what price per pound would aluminum be equally economical? How many circular mils of aluminum would you use per ampere?

8. If equal conductance costs 85 per cent as much in aluminum as in copper, what would be the relative amounts installed and the per cent economic advantage in using aluminum?

9. If copper sells at 24 cts. per pound, at what price should one be able to buy aluminum in order that the cost of transmission be equal?

10. If aluminum sells at 10 per cent less than the equivalent copper, what would be the saving in line losses by using aluminum; what would be the investment saving? Copper conductor installed per mile costs $30 + $0.004 cir. mil mile. $f = 10$ per cent. Electric service costs annually $1,200 + $18 kw. + $0.02 kw.-hr. Root mean square $I = 0.7\ I$ maximum.

11. On a basis of 2,000 cir. mils per ampere, determine the kilovolt-ampere capacity of an overhead three-phase 33,000-volt circuit of No. 0000; of a 44,000-volt circuit of the same size.

12. For $E = 2{,}300$, $f = 12$ per cent, $a = \$30$, $b = \$0.003$, $cm_0 = 2{,}000$, determine economic limit of single versus three-phase power transmission.

13. Verify the copper quantities given in the text of the chapter for single-phase, two-phase, and three-phase systems by actual solution of a transmission of some certain power at a selected voltage. Economic standards as at head of problems.

14. Prove the copper ratios given in the text for the Edison three-wire, direct-current, and single-phase systems for transmission of 10 kw. 500 ft. with 3 per cent loss. Load voltage 115.

15. A power house is to supply two three-phase loads in a line, the first of 15,000 kva. is 8 miles from the station, and the second of 5,000 kva. is 4 miles past the first load. The design calls for a 1,000,000-cir. mil cable in the 8-mile section and a 250,000-cir. mil cable in the next 4-mile section.

It is desired to use one size of cable throughout the run. What will be the best compromise section, and what will be the cost of the compromise in per cent of the ideal cost?

16. A transmission line has such a copper section as would deliver 2,200 kva. at 2,300-volts, three-phase using ideal current density. It has been overloaded until it now delivers 4,500 kva. at the same power factor. This has resulted in an extra line loss (above that which appeared at the 2,200-kva. load) of $700 annually.

It is proposed to change the line to a 4,000-volt four-wire system (2,300 volts line to neutral) by adding a fourth conductor half the size of those now installed. It will cost a total of $2,500 to buy and install the new conductor and arrange the line for 4,000-volt operation.

Assuming fixed charges at 12 per cent, will the change be economically worth while?

17. One overhead wire installed costs per mile $150 + $0.008 cir. mil mile. Fifteen per cent fixed charges exist. No. 0 (105,500 cir. mil) was installed to carry a three-wire three-phase load at 2,300 volts, 6 miles. By adding an equal-sized fourth wire on the same poles to act as neutral, and by rearranging the transformers, the voltage was changed to 2,300 volts line to neutral. If the ideal copper section is 2,000 cir. mils per ampere, compute the annual saving (if any) due to carrying the four-wire system's ideal load on four wires instead of on three wires.

18. If the marginal cost is $2,000 more per year to own and operate a transmission line as built than the $60,000 it would have been, had the line been correctly designed, how much more or less is the investment than the ideal? Fixed charges are 10 per cent annually. Explain all steps in the solution.

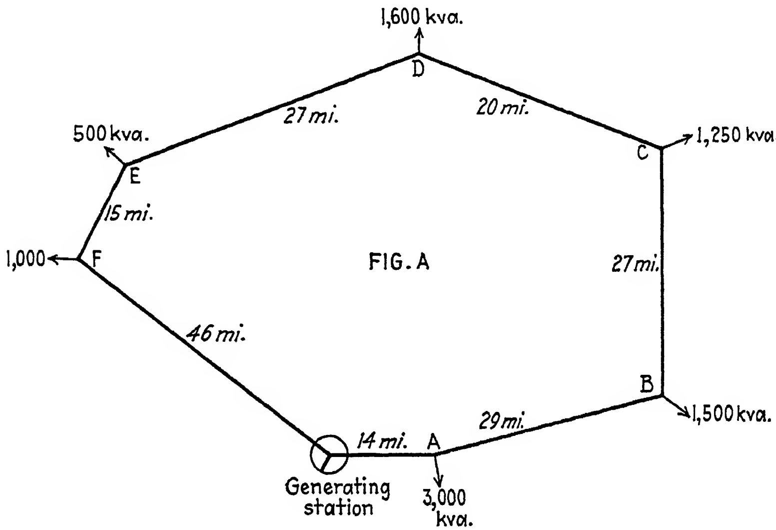
19. *a.* An automobile factory purchases its power at a power company substation, then transmits it to its own plant, 3,400 ft. distant. The power load is three-phase, 20,000 kw. at 0.65 power factor lagging. Under this condition, the voltage measured 4,750 volts at the factory. There are 12 three-phase cables in parallel; per 1,000 ft. of single conductor $R = 0.022$, $X = 0.0286$ ohm. What is the power loss in the cable, and how much of it could be eliminated by raising the load power factor up to 80 per cent? Plant operates 4,300 hr. per year.

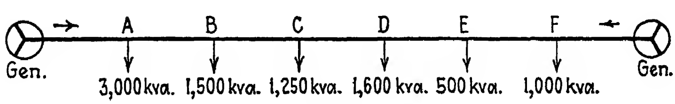
b. The customer considers spending $70,000 in order to increase the current-carrying capacity of his 3,400-ft. line, by adding 33⅓ per cent more cables of same size as the existing ones. This is necessary in order to care for future expansion.

As an alternative, two 5,000-kva. synchronous condensers have been recommended, for location in the factory. The total installed cost of these condensers is $50,000. Operating at full rating, the loss in each condenser is 125 kw.

The overhead charges on the cable investment may be figured at 12 per cent, and for the condensers 15 per cent. Power costs $25 per kw.-yr. demand + 0.96 ct. per kw.-hr. at 65 per cent power factor, with 1 per cent bonus for each 5 per cent improvement in power factor.

Which is the more economical proposition for the increased load, the condensers or more cable?





PROBLEM 21.—Loop feed system. (*Elec. Jour., October,* 1923, *p.* 386.)

20. Open versus closed delta for combined three-phase and single-phase loads. Assume transformers with proportional impedances as

Phase XZ, 37.5 kva. at 230 volts.

Phase ZY, 15 kva. at 230 volts.

Phase YX, 15 kva. at 230 volts.

The loads are

Three-phase, 32 kva. at 230 volts, 0.8 power factor.

Single-phase, 32 kva. at 230 volts, 1.0 power factor on phase XZ.

a. Draw the vector diagram, and determine total delta currents in each phase for a closed delta.

b. Do the same for open delta, phase ZY being open.

c. Do the same for open delta, phase YX being open.[1]

21. The loop-transformer system shown in Fig. *A* is to be fed from the generating station. Assume that the generator plant is divided and connected by a tie line which also supplies the loads, as in Fig. *B*. Then the loads are divided inversely as the impedances, *i.e.*, inversely as the distances. The line is 66 kv., three phase, 60 cycles. Copper costs 10 cts. per pound, has 25-year life, salvage 50 per cent. Money costs 6 per cent, taxes and insurance 3 per cent. Electric service costs \$6 kw. + \$0.0035 kw.-hr. All loads have r.m.s. amp. = 0.7 of maximum amperes.

a. Design the various conductor sections.

b. Since the loop may have to feed one way in an emergency, increase the size of any transmission section less than 2,000 kva. to that capacity.

[1] See Seelye, H. P., "Electrical Distribution Engineering," McGraw-Hill Book Company, Inc.

CHAPTER VI

POWER-PLANT LOCATION

90. How the Problem Differs, Large Plants and Small Plants. The determination of plant location, including under this general designation frequency-changer stations, substations, and transformer installations, along with primary power stations, is essentially a problem of transportation of the labor and raw materials into the plant and the finished product to the market. The cost of transporting labor in this field is usually quite negligible; therefore the problem must be studied from the point of view of the supply of coal, oil, and water, on the one hand, and of electric transmission, on the other. The methods to be used are radically influenced by the type of prime mover and the size of the plant.

The hydroelectric plant[1] is, of course, a special case wherein the location of the power house and its related structures of dam, headworks, tailrace, etc., is governed almost entirely by certain topographical and geological conditions that exist at various points on a stream. The choice among various possible locations, therefore, will be based upon the relative costs and efficiencies with regard to the production of energy in the plants and the relative costs of the electric transmissions.

In the fuel-burning plants, we may first consider the small plant using liquid fuel or noncondensing steam engines. In such a case, the cost of getting fuel into the plant and the ease of water supply are relatively insignificant factors, and the problem of plant location reduces almost entirely to an electric-transmission problem. Other things being equal, it is manifestly desirable that the plant be located at such a point that the investment in electric cables for transmitting the service from the plant to the points of utilization, together with the annual operating costs of such transmission, be a minimum. This end, roughly speaking, will be accomplished if the plant is located at or near the center of the load. If, however, such location takes the plant farther

[1] See MEAD, DANIEL W., and BARROWS, H. K., "Water Power Engineering," McGraw-Hill Book Company, Inc.

from the point of fuel supply, or farther from the point of supply of water for boiler feed or cooling purposes, the saving in the electric transmission may be more than offset by extra costs in the fuel and water supply. It is then necessary to determine the relative significance of these items before we can decide on the ideal location for the plant.

Other details enter into the determination of plant location after we have decided, through the study of electric transmission and fuel and water supply, just where the plant can best be situated. These details are realty cost, voltage regulation, and nuisance, although voltage regulation is pretty well covered under the determination of the economic balance between electric transmission and fuel and water supply.

It is the relatively high cost of transporting the enormous quantities of water used in the condensers of large steam prime movers that differentiates the problem of location of a condensing steam plant from the problem of location of a plant not dependent on a very liberal water supply. In general, the large condensing plant will have to be located at the water's edge, or very close thereto, and the location must be such that the water supply is ample and cool. A very slight difference in the temperature of the condensing water makes so marked a difference in the performance of a steam turbine that a change of location of many thousands of feet, or even a few miles, may easily justify the greater electric-transmission expense involved. To consider a concrete case, we may note that a 30,000-kw. plant might be expected to burn 30 tons of coal an hour; at a 65 per cent load factor, this would amount to nearly 171,000 tons a year. With coal in the neighborhood of $5 a ton, it is evident that a very slight change in the available temperature of condensing water might easily make a difference of $10,000 a year in the mere coal cost of service production, an amount that would very handsomely justify considerable electric-transmission expense. Further, the tremendous size of the water supply required must be appreciated. Considering a modern 60,000-kw. unit, its condenser would require approximately 45,000 sq. ft. of cooling surface which would need about 90,000 gal. per minute from the circulating water pumps.

The small plant, on the other hand, is practically independent of large water supply, and the expense of electric transmission

from a small plant is relatively very severe, owing to the impracticability of using high voltages, so that the electric-transmission features become dominant in plant location.

The same differentiation applies with reference to coal supply in cases where coal is the fuel used. It is imperative that a very large plant be located close to a railroad offering adequate service, or at least sufficiently close so that a privately owned spur can be run to such a railroad. The cost of railroad-track construction is so great in comparison with the cost of electric conduits or overhead lines as to overshadow any expense involved in electric transmission as against coal handling. On the other hand, a small plant will, in general, be supplied with coal hauled from railroad sidings or coal yards in trucks. The major part of the cost of such transportation is in the loading and unloading of the trucks, and, as these costs have to be carried in any case, the only additional cost involved in locating the small plant somewhat farther from the loading point is truck hire for the time required to cover such extra distance. This is a matter of fairly easy computation, based on the local costs of such truck hire, their carrying capacity, and their normal traveling speed. Again, it should be remembered that the small plant will be a low-voltage plant, so that extra trucking distance is not serious in comparison with the cost of extra electric transmission.

The location of the large electric station suffers one other limitation not experienced by the small station. The economic significance of the large station is much greater than the mere ratio of size would indicate in comparison with the small one. The large station will be likely to supply a community whose interests are so involved and so dependent on continuity of service that the coal supply must be absolutely continuous and independent of the railroads over a long period of time. It would be fatal to have a bridge wreck or a railroad or coal strike interrupt for so short a time as one day the fuel supply at a metropolitan railway supply station or central light and power station. Such a plant must, therefore, have adequate coal storage to carry over the worst possible contingency of this sort. This means that the plant location must be so chosen as to be closely adjacent to an ample space for the storage of fuel. The extent of such storage can be determined only by combination of the judgment of the engineers and those responsible for the com-

mercial policy of the business, and may represent anything from a one month's supply to a supply sufficient for a half year. Many of the central-station companies have yards capable of storing a supply sufficient for two or three months at average load.

The very large power plant will benefit by location adjacent to more than one source of coal supply. If a site can be found in close proximity to two railroads, the plant will be able to secure

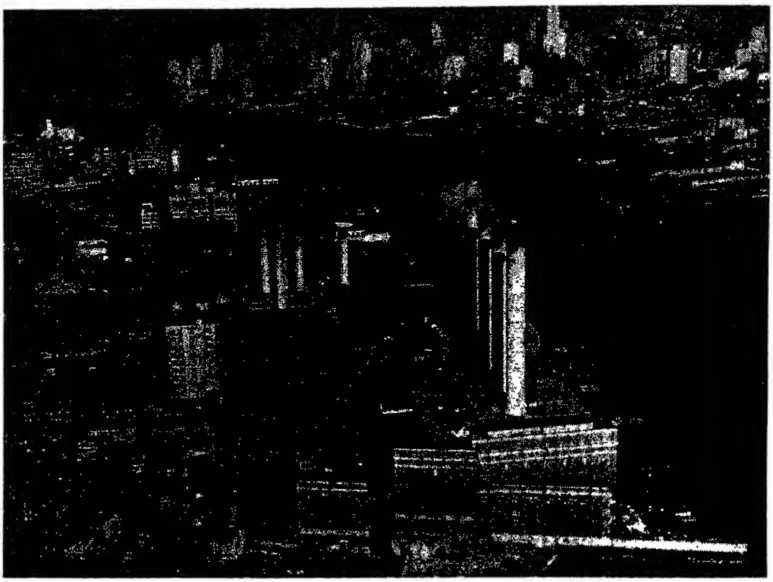


FIG. 112.—Hudson Avenue station with respect to its surroundings and part of the territory it serves. (*Courtesy of Brooklyn Edison Co.*)

its fuel without drawing on the coal storage in case of strike or physical disabilities on the part of one of the roads. If a navigable stream, canal, or tidewater location is available, such location should certainly be chosen, both because of the assurance of continuity of fuel supply and because of the economic advantages attendant on water-borne transportation. Fortunately, this consideration runs hand in hand with the need for a large supply of condensing water.

If it is at all practicable to do so, the location for the large plant may advantageously be so selected as to enable the running of loaded bottom-dump coal cars on elevated trestles through the storage yards and directly into the coal bunkers of the boiler

plant, thereby eliminating all handling of fuel except by gravity. Where water-borne transportation is depended on, the location of the plant on its site will have to be such as to enable direct handling of the fuel from scows or barges into the boiler-house bunkers so that the material in the storage yards will be more or less stagnant. Any other arrangement would necessitate handling of coal from the barges into storage and again from storage into the coal bunkers, obviously entailing considerable additional expense.

The very large plant is subject to still another limitation in comparison with the small plant, *viz.*, the cost of realty. A plant supplying public service in a large community could not readily be located anywhere near the middle of its load, as the middle of the load is very certain to be in just that part of the city where realty is the most expensive. Some considerable sacrifice of the economics involved in fuel supply and in the electric transmission must be made, in order to obviate the very serious cost of realty whose normal usage is for mercantile and office buildings.

A study of Fig. 112 will show how serious is the problem of crowded real estate for some large metropolitan plants, how limited may be the storage supply of coal, and how almost impossible it may be to have space for outdoor switching stations. On waterside locations, the foundations may also offer special problems, as at Hudson Avenue, where the subsurface soil was fill mixed with coarse sand to 10 ft. below mean high water, then very fine sand to 25 ft. below, with quicksand underneath down to the shale at a depth of 73 to 100 ft. Also at ebb tide, a current of 4.8 m.p.h. sweeps along the front of the site with a very considerable washing action. As is frequently done in soft locations, the plant was carried on a reinforced-concrete mat 10 ft. thick in places, imposed on piles driven to refusal. In this case, the heavy vibratory loads of the turbine room were placed inshore and the lighter static boiler-room load was placed next to the water front.[1]

The small plant naturally finds its use either in the public-service supply of small communities, where there is not any great range of realty costs, or within the confines of an industrial plant

[1] See Hudson Avenue, Generating Station, *Elec. Jour.*, May and July, 1925.

or building. The small plant, then, is almost free from the limitations of realty cost, with the exception that, if located inside a mercantile building, it must be kept away from the more valuable portions of the basement space which can better be used for merchandising purposes.

The general annoyance features of plants are experienced in different degrees but are the same in character for the large and for the small plant. The location of a large plant must never be such as to create inconvenience to its neighbors through the general noise of operation or tremors in the ground due to moving machinery or as a result of the discharge of smoke and cinders from the stacks. With adequate firing, however, the smoke nuisance has no right to existence and represents in any case improper plant management. The nuisance features of a small plant located within an industrial establishment or office building are likely to be of rather different character, affecting not so much the general public as the owners and the tenants of the building in which the plant is housed. The location within such a private property must be so chosen as to bring the fuel in and to take the ashes out away from any principal entrance to the building, and must be such as to avoid undue heat, noise, or dirt in any important portion of the building. In an industrial establishment, a power plant housed by itself within the yards constituting the works served is scarcely affected at all by this annoyance consideration.

The small plant housed within a mercantile building, hotel, or single-building factory will have its location very considerably affected by convenience of layout with reference to other features of the building. In a hotel building, for instance, the problem is primarily one for settlement by the architect, who can determine the relationship between the engine room and the hotel departments ordinarily housed in the basement.

With these limitations noted, and subject at all times to them, we are prepared to arrive at an analytic solution of the problems of plant location.

In the first instance, we shall neglect the influence of coal and water supply, assuming the plant location to be entirely a matter of electric distribution.

91. Marginal Transmission Costs.—The simplest study will be in the case of a series of buildings served by a plant through one

distribution cable or through a single circuit of overhead wires. Such a plant would be represented by an industrial establishment laid out on one or both sides of a main avenue along which the electric transmission is carried. It is clear that as the service would be through one run of duct or through one pole line the cost of conduit or supporting structure will not be influenced by the location of the plant, whether it be at one end or the other or at some intermediate point. Nor will the underlying costs such as the insulation and sheathing of the copper be affected, or any of those elements of copper cost which may be included in the a or basic component of the expression, see Eq. (63).

$$\text{Conductor cost per mile, \$} = a + b \text{ cir. mils.}$$

We are concerned, then, entirely with the marginal costs involved in the b term. For the sake of simplicity, we shall assume that the load factor of each building tapped off the main supply line is identical, and we note from the discussion in Chap. V that, with proper design of the transmission, we use a fixed number of circular mils per ampere, or, what is the same thing for all practical purposes, per kilovolt-ampere. We should also bear in mind that the annual value of power and energy loss is equal to the annual investment costs on the marginal copper installation, see Eq. (82).

It is evident that location of the plant at either end of the avenue would be undesirable, since all the service delivered by the plant would have to be transmitted to the first tap, a little less to the second tap, etc., the theoretical conductor section tapering off to a small quantity at the last point of delivery. Were our plant located at the middle of the avenue, the copper section would be of one-half size going in either direction from the plant and tapering off to a very small amount at each end.

Bearing in mind now that for a given system our transmission cost is a function of the length of transmission plus the kilovolt-ampere miles, we can start the study with a power plant delivering at but two points along the transmission and using, be it noted, a single set of conductors. The first item represents the underlying costs of conductor housing, which are directly proportional to the length, and the second item represents the marginal copper and operating costs, which are obviously dependent on the kilovolt-ampere miles. In the absence of influencing

factors other than electric transmission, the plant would evidently be located at the greater of the two loads, since conductor-housing costs would be the same for any location of power house at or between the two taps, and since if the power house is located at the point of greater load no marginal transmission cost is involved by this load. The only marginal transmission cost remaining is that due to the lesser load over the distance between the two taps. If both loads are equal, it is a matter of indifference as to which point is chosen for the power-house location.

If, however, there are three or more taps, the problem of best location becomes slightly more complex. The power house should, of course, be located at one of the points of utilization, in

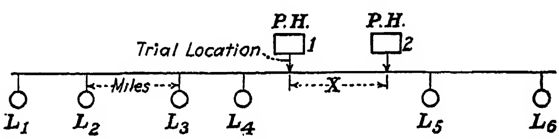


FIG. 113.—Power-house location for loads on a common feeder.

order that such service may go directly from the power house to the service tap, instead of through the transmission system, and the point chosen must be that one for which the total kilovolt-ampere miles of transmission are a minimum. To secure this condition, it is necessary only that the aggregate load on one side of the power house be as nearly as possible equal to that on the other side.

92. Concentrated Loads on a Common Feeder.—Assume a circuit with concentrated loads of L_1 L_2, L_3, L_4, L_5, and L_6 kva., situated as shown in Fig. 113.

If the power house is moved from the trial location shown as No. 1 to a new location No. 2 distant X miles to the right, a saving is effected with reference to the transmission of all loads to the right over distance X, *i.e.*, $(L_5 + L_6)X$ in kilovolt-ampere miles, but correspondingly a loss is suffered owing to the assumption of the transmission to the left of $(L_1 + L_2 + L_3 + L_4)\ X$ in kilovolt-ampere miles. Let $\$K$ be the cost of transmission per kilovolt-ampere mile. Owing to the change in location, there has been a cost increase of

$$D\$ = K[(L_1 + L_2 + L_3 + L_4) - (L_5 + L_6)]X. \qquad (118)$$

It is thus seen that the cost increase is proportional to the distance moved and that it is positive if the move is made toward the lesser loads, but it is negative—a saving—if the move is toward the greater loads. Since, in general, the sum of the loads to the left of the location minus the sum of the loads to the right of the location will not be zero, *i.e.*,

$$(L_1 + L_2 + L_3 + L_4) - (L_5 + L_6) \neq 0, \qquad (119)$$

there will be a saving by moving one way or the other within any transmission segment. Therefore, the ideal plant location must be at a load point.

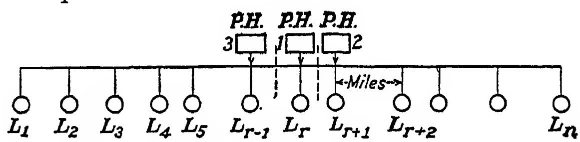


FIG. 114.—Power-house location for any number of loads on a common feeder.

Should the foregoing inequality reduce to zero, it makes no difference whether we locate at L_4 or L_5.

For any number of loads in line, as $L_1, L_2, L_3 \ldots L_r \ldots L_n$, if the trial location has been set down at any load as L_r in Fig. 114, there are three tendencies to establish the location; (1) to move to the right due to the weight of loads $(L_{r+1} + L_{r+2} + \ldots L_n)$; (2) to move to the left due to the weight of the loads

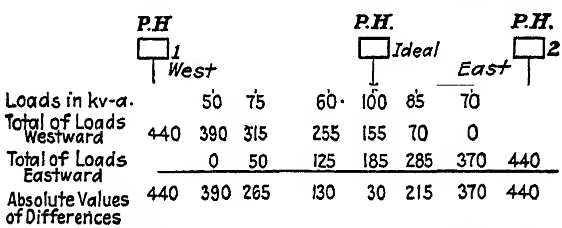


FIG. 115.—Power-house location for loads on a common feeder by method of "sectional differences."

$(L_1 + L_2 + L_3 + L_4 + L_5 + \cdots L_{r-1})$; and (3) to stay right at the trial location due to the weight of load L_r. If, therefore, load L_r is greater than the absolute value of the differences of the sum of the loads to the left and to the right of L_r, it is the ideal location.

93. Example, Loads on Common Feeder.—If there are loads as indicated in Fig. 115, assume the power house is moved to the

extreme west of all the loads. The transmission will be 440 kva. from the power house to the first load of 50 kva., where that load will be taken off. Then the transmission will be 390 kva. to the next load of 75 kva., where that load will be taken off. Similarly until the last load of 70 kva is reached and taken off.

Next the process is repeated with the station located at the extreme east of all the loads. Then the absolute value of the difference in the two summations is set down at each load point.

The absolute value of the "sectional differences" is less than the load at the 100-kva. load point *uniquely* and in accordance with the criterion above, this is therefore the ideal location. It is evident that, with the plant located at the 100-kva. load, the transmission to the right must be 155 kva., and to the left 185 kva. Were the plant to be moved 1 mile to the right, the transmission of 155 kva. miles would be obviated, but there would be incurred the transmission of 285 kva. miles to the left, making a net additional transmission burden of 130 kva. miles. Were the plant to be moved 1 mile to the left, the transmission of 185 kva. miles would similarly be obviated, but there would be incurred the transmission of 255 kva. miles to the right, making a net additional transmission burden of 70 kva. miles. Therefore, location of the plant at the 100-kva. load gives the *minimum kilovolt-ampere miles of electric transmission.*

It should be noted that this arrangement does not at all indicate equal transmission costs on either side of the ideal plant location, nor has anything been said about the distance between the loads, both of which would be called for by the so-called "center of gravity" method. On the contrary, movement of the left-hand load of 50 kva. above to a point 1,000 miles to the left would involve a tremendous transmission cost, but would not influence one iota the desirable location of the power plant, since any movement of it toward the 50-kva. load for the purpose of reducing the transmission cost would simply result in moving the plant farther away from the bulk of the load, and the same distance toward a very small portion of the load.

This numerical solution and the following graphical method are more complex in theory and description than they are in utilization. The justification for the graphical method of plant location lies in the fact that the foregoing tabular method does not enable one to visualize the influence of fuel and water-delivery costs

which can be readily included under the graphical method which follows.

94. Graphical Solution for Location.—In order to produce a graphical representation of delivery costs, plant location may be permitted to vary indefinitely and the effect observed.

Let $\$K$ be the marginal cost of transmission per kilovolt-ampere mile, and let any line of loads L_1 to L_n, as shown in Fig. 114, be considered. Assume a three-phase 6,600-volt circuit, using an economic section of say 2,500 cir. mils per ampere, with copper in place costing \$30 per mile plus \$0.004 per circular mil mile. Let the fixed charges be 10 per cent per annum. Then the current necessary to transmit 1 kva. would be $\frac{1{,}000 \text{ volt-amp.}}{\sqrt{3} \times 6{,}600} = 0.0875$ amp. At 2,500 cir. mils per ampere, the circular mils would be $0.0875 \times 2{,}500 = 219$. Then the annual marginal cost on the ideal copper would be fbn cir. mils $= 0.10 \times 0.004 \times 3 \times 219$. But for the ideal section, the energy losses are equal to the marginal conductor costs; therefore, the total marginal cost per kilovolt-ampere mile (K) is

$$2fbn \text{ cir. mils} = 2 \times 0.10 \times 0.004 \times 3 \times 219 = \$0.525 \text{ per kva. mile.} \tag{120}$$

Consider the plant located between loads L_{r-1} and L_r. The result of moving the plant a distance X, to the right toward load L_r within the line segment $_{r-1}$ to $_r$, is an increase of transmission cost

$$\$D_1 = K.X.[(L_1 + L_2 + L_3 + \cdots L_{r-1}) - (L_r + L_{r+1} + L_{r+2} + \cdots L_n)] \tag{121}$$

After passing load L_r, moving a distance X, to the right away from load L_r within the line segement $_r$ to $_{r+1}$, increases the transmission cost

$$\$D_2 = K.X.[(L_1 + L_2 + L_3 + \cdots L_r) - (L_{r+1} + L_{r+2} + \cdots L_n)] \tag{122}$$

and the difference in increase is

$$\$D_2 - \$D_1 = K.X.2L_r. \tag{123}$$

Therefore in moving the plant location past a load point, there is a change in the cost of transmission per mile proportional to twice the value of the load.

From this relation, the marginal cost of transmission can be represented by a funicular polygon whose sides have a change of slope proportional to twice the load that marks each vertex. It is apparent that the slope of the external chords of this polygon—since they represent the marginal transmission cost of the whole

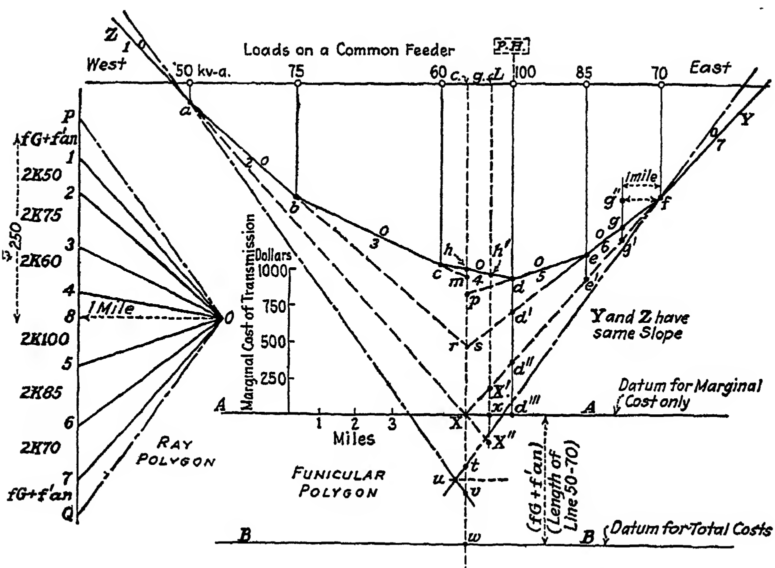


FIG. 116.—Graphical solution for location of power house, loads on a common feeder.

load from a point to the left of L_1 or to the right of L_n—must be equal. A ray polygon can, therefore, be constructed whose segments are successively $2KL_1$, $2KL_2$, etc., and whose focus is 1 mile from the center of the line of loads.

In Fig. 116, the horizontal line 50-70 indicates the distance in miles between the extreme loads, the location of the intermediate loads being shown to scale. On the vertical line 1-7, at the left, are laid off from 1 to 2, from 2 to 3, from 3 to 4, etc., twice the kilovolt-amperes taken at the respective load points, each multiplied by the marginal transmission cost per kilovolt-ampere mile and O, the focus of this ray polygon, is a point 1 mile distant from the mid-point of line 1-7. It is to be noted that, for the sake of

convenience in the drawing, the ray polygon is drawn to a different scale from that used in the funicular polygon. This is perfectly permissible because the ray polygon is used only to define the necessary slopes.

Beginning at any convenient point vertically under the right-hand load of 70 kva., lay off a line *fe* drawn parallel to 6-0. Through point *e*, vertically under 85 kva., *ed* is drawn parallel to 5-0. Similarly *dc* is parallel to 4-0, *cb* is parallel to 3-0, *ba* is parallel to 2-0, *aZ* is parallel to 1-0, and *fY* is parallel to 7-0. The lowest point of this polygon gives the ideal power-plant location, *i.e.*, at point *d*. This funicular polygon will truly represent the marginal transmission costs if its base datum is known. This latter will be found at the intersection of the external chords 1-0 and 7-0, *i.e.*, at the point *X*. Therefore, the distance dd''' gives, to scale, the total marginal cost of transmission from such a plant.

The point of the center of gravity *c.g.* of the system of loads is located vertically above *X* and is, in general, not at a load, and denotes a location for equal transmission costs eastward and westward. That *c.g.* does not, in general, lie at a load point will be seen if we note that our ray polygon, and hence the slopes in the funicular polygon, are independent of the distances between loads. If *c.g. happens* to lie at a load point, this will be seen to be accidental and not essential by considering that changing the length of *any* line segment would change the elevation but not the slope of one of the external chords of the funicular polygon and hence would shift *X* (and *c.g.*) laterally away from the load point. Thus again, the center of gravity method does not indicate the ideal location.

The proof that justifies the foregoing method of locating the datum line is not difficult. Triangle gfg' is by construction similar to triangle 6-0-7. Since the ordinate 6-7 represents twice the marginal cost of transmitting load 70 a mile, in the ray polygon where 8-0 equals 1 mile, then gg' will represent twice the marginal cost of transmission for this same load in the figure where $g''f$ denotes 1 mile; and, the distance *Xs* indicates twice the cost of sending load 70 a distance of *c.g.*-70 miles. Similarly, *sp* gives twice the cost of transmitting load 85 from a power house located at *c.g.*; *ph* twice that for transmitting load 100; and *Xr*, *rm*, and *mh*, two times the costs for loads 50, 75, and

60, respectively. That is, the line *Xsph* represents twice the cost of transmitting, from a power house located at *c.g.*, all the loads to the right of that point, and *Xrmh* twice the cost for the loads to the left of that point. Therefore, *c.g.* is the point of equal transmission costs to the right and to the left, and *Xh* the total marginal cost of transmission of all the loads from that point.

Again, as we consider location of the power house at some other point such as L, we perceive that twice our total cost of transmission for the loads to the right will be scaled by the line $X'h'$, twice that for the loads to the left by $X''h'$, and the total cost of transmission for all the loads by the average of these two. Since the slopes of the two lines XZ and XY are equal, $xX' = xX''$, and this average sum is the distance xh'. Also, since the change of slope as we pass the point d is, by construction, twice the cost per mile of transmitting load 100, we have the segment *de* representing cost variation as we move our plant along the line 100-85. A similar statement will hold as we pass any other load point, including f and a, and we have, finally, the complete funicular polygon representing accurately the cost variation as we move the power-house location anywhere along the line 50-70.

In considering only underlying costs, it is evident that, for the single-feeder circuit, there will be no advantage of any one location over any other location, so long as the power house is kept within the range of the loads; we must incur these costs for the length of line 50-70 in any case, regardless of location. As we go to the right, beyond load 70, or to the left beyond load 50, however, each additional mile involves an extra underlying cost, such as has been represented, heretofore, by $(fG + f'an)$. These costs may be included in our graphical construction, then, by depressing the datum line by an amount equal to the basic cost for the distance 50-70 and adding, beyond the points a and f, segments of the ray polygon of increased slopes $(fG + f'an)$.

This has been indicated by the dash-and-dot line *ftu* drawn through f parallel to OQ which adds the ordinate Xt below the marginal datum to represent the underlying cost of transmission from *c.g.* to load 70. Similarly line *auv* drawn through a parallel to OP adds the ordinate Xv below the marginal datum to represent the underlying cost of transmission from *c.g.* to load 50. Thus the total underlying cost for the length of line from load 50

to load 70 is the sum of ordinates Xt and Xv, or is equal to Xw. Therefore the bottom line of the diagram, B-B, is the datum for total costs of transmission.

95. Loads on Individual Feeders.—In contrast with loads all on a common feeder, the practice of supplying each load by an individual feeder may have considerable effect upon plant location. Considering underlying costs only, for loads as shown in Fig. 117, the plant should be located as nearly as possible at the middle load point. Thus, if the plant is moved 1 mile to the right, a saving is effected of the underlying cost on 2 feeder miles, but a loss of 4 feeder miles is suffered in the necessary extension of the feeders to the left. Therefore the move causes a net loss of the underlying cost of 2 feeder miles. If the plant is moved 1 mile to the left, a saving is effected of the underlying

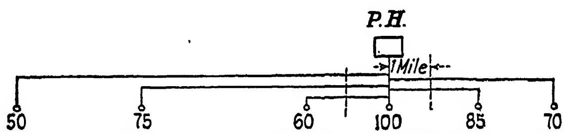


FIG. 117.—Power-house location, kilovolt-ampere loads on individual feeders.

cost on 3 feeder miles and a loss is incurred on 3 feeder miles, or there is no change in the underlying cost up to the arrival at load point 60. A move to the left of load 60 of 1 mile would result in a loss of 2 feeder miles. Therefore with an even number of individual load points, the plant should be located at either one of the middle loads.

From the foregoing, it is seen that the rate of underlying cost change, *i.e.*, the cost change per unit distance, itself changes by the amount $2(fG + f'an)$ whenever plant location moves through a load point.

When an odd number of loads is being considered, the plant should be located so that the number of circuits is the same in both directions.

The previous graphical solution of Fig. 116 will hold then, if instead of plotting $2KL_r$ in the ray polygon we plot $2KL_r + 2(fG + f'an)$ for each and every load. This modified construction takes account of both underlying and marginal costs. It should, however, be noted that the quantity $2(fG + f'an)$ is introduced only where a *change* in underlying cost occurs. Thus, for the single circuit supplying a plurality of loads, the quantity

$(fG + f'an)$ should be added to the first and the last load and the shape of the polygon will be correct for any location, even outside the range of the loads. The effect of such underlying costs will be to shift the desirable plant location toward the maximum number of individual services irrespective of their loads. It might easily be that a large number of comparatively small services each independently fed by high-voltage cable would

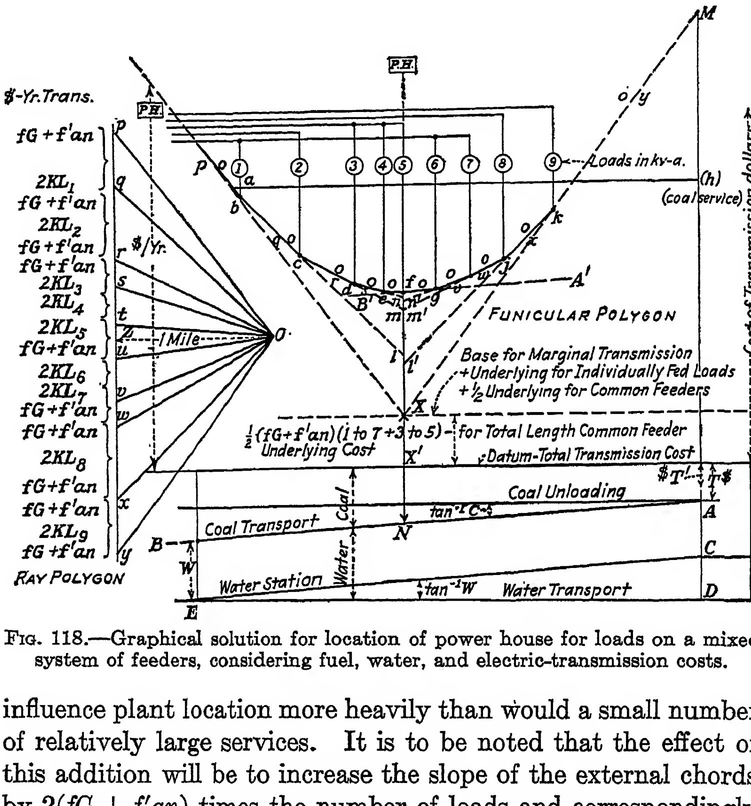


FIG. 118.—Graphical solution for location of power house for loads on a mixed system of feeders, considering fuel, water, and electric-transmission costs.

influence plant location more heavily than would a small number of relatively large services. It is to be noted that the effect of this addition will be to increase the slope of the external chords by $2(fG + f'an)$ times the number of loads and correspondingly to depress the point X in Fig. 116.

96. Loads on a Mixed System of Feeders.—When a transmission system carries a part of the load on a common feeder and other parts on individual feeders, the change in the rate of cost as the power house moves along the transmission line will be $(fG + f'an) + 2KL'$ when passing the first load on any common feeder, and again $(fG + f'an) + 2KL''$ when passing the last

load on the same feeder; where L' and L'' are first and last loads, respectively, on that particular feeder. Again, the change in the rate of cost for intermediate loads on a common feeder will be $2KL$, since no change in underlying cost occurs as the plant passes an intermediate load.

The polygon construction, then, is the same as before except that we plot in the chord of the ray polygon $(fG + f'an)$ and then $2KL'$ for the first load on each common feeder and $2KL''$ and then $(fG + f'an)$ for the last load on each such feeder, as in Fig. 118.

To show that the constructions in the ray polygon of Fig. 118 correctly represent the transmission costs, we may proceed to move the possible location of the station mile by mile along the main axis and study the various costs. Thus, for a location to the left of the loads, a move of 1 mile toward load 1 makes a saving on the underlying cost of 5 feeder miles, plus the marginal cost of transmitting all the load over the mile. This result and that of further moves are then tabulated as follows:

Move	Saving underlying + marginal	Change of saving underlying + marginal
Moving to the right—toward		
Load 1........	$5(fG + f'an) + K\Sigma_1^9 L,$	
Load 2........	$4(fG + f'an) + K(\Sigma_2^9 L - L_1)$	$(fG + f'an) + 2KL_1$
Load 3........	$2(fG + f'an) + K(\Sigma_3^9 L - L_{1+2})$	$2(fG + f'an) + 2KL_2$
Load 4........	$(fG + f'an) + K(\Sigma_4^9 L - L_{1+2+3})$	$(fG + f'an) + 2KL_3$
Load 5........	$(fG + f'an) + K(\Sigma_5^9 L - L_{1+2+3+4})$	$O + 2KL_4$
Load 6........	$O + K(\Sigma_6^9 L - L_{1+2+\dots5})$	$(fG + f'an) + 2KL_5$
Load 7........	$O + K(\Sigma_7^9 L - L_{1+2+\dots6})$	$O + 2KL_6$
Load 8........	$-(fG + f'an) + K(\Sigma_8^9 L - L_{(1+2+\dots7)})$	$(fG + f'an) + 2KL_7$
Load 9........	$-3(fG + f'an) + K(L_9 - L_{(1+2+\dots8)})$	$2(fG + f'an) + 2KL_8$
away from 9....	$-5(fG + f'an) - K\Sigma_1^9 L$	$2(fG + f'an) + 2KL_9$

The slope of the ray p-O is represented by the tangent $pZ/1$ mile. But by construction, Z is the mid-point of the cost ordinates of $2K\Sigma_1^9 L + 10(fG + f'an)$ or pZ is $5(fG + f'an) + K\Sigma_1^9 L$; therefore p-O has the proper slope. After passing load 1, the rate of saving should decrease, according to the foregoing tabular results, by $(fG + f'an) + 2KL_1$. The ray q-O has just such a decreased slope because the construction of the ordinates placed

$(fG + f'an) + 2KL_1$ between the points q and p. Similarly, each ray may be verified with the tabular data.

It should be noted that, in this case, the construction fails to include one-half of the underlying cost of the common feeders 1 to 7 and 3 to 5 between the extreme loads of each run. For, on a similar analysis to that made for Fig. 116, the intercepts noted represent the costs, as listed, of transmitting a particular load from a power house located at load 5:

Intercept	Transmission costs	Load
Xl	2 marginal + 1 underlying (5 − 1)	1
lm	2 marginal + 2 underlying (5 − 2)	2
mn	2 marginal + 1 underlying (5 − 3)	3
nf	2 marginal + 0 underlying	4
fn'	2 marginal + 0 underlying	6
$n'm'$	2 marginal + 1 underlying (5 − 7)	7
$m'l'$	2 marginal + 2 underlying (5 − 8)	8
$l'X$	2 marginal + 2 underlying (5 − 9)	9
$\Sigma = 2Xf$	= 2 marginal all transmitted loads + 2 underlying all special feeders + 1 underlying all common feeders.	

Then Xf = marginal cost for all loads + underlying cost for individually fed loads + ½ underlying cost for all common feeders.

Since the power house is located at load 5, there is no marginal or underlying cost for the transmission of that load, it being assumed that the load will be fed directly from the station bus bars.

In order that the transmission datum may be that for the total cost, it must be depressed below the datum through X by an amount equal to $\frac{1}{2}(fG + f'an)$ times the aggregate length of common feeder, 1 to 7 and 3 to 5. This brings the datum for total transmission cost to the line through X', as shown in Fig. 118.

97. Fuel-importation Cost.—We are now in a position to consider the effect of fuel-importation cost on plant location. Since the load to be carried is unaffected by plant location—except for the slight variation in ohmic losses, which we may neglect—the fuel tonnage is extremely nearly a constant.

The cost of fuel haulage and handling may be expressed as

$$\$ = T + C \text{ miles}^1 \qquad (124)$$

[1] Freight rate per ton = \$0.90 + \$0.0035 × miles haul. Frank F. Fowle, *Elec. World*, Mar. 12, 1938.

where T represents terminal costs such as breaking bulk from cars to trucks and unloading trucks, and C represents haulage cost per mile on the road going and coming. These costs may be added to the costs given by the polygon, or, more conveniently, we may change the base by adding them below the datum line.

Let it be assumed that the only source of coal supply is located at point h in Fig. 118 and that all the coal must be transported from h to the power-plant location somewhere between a and h. At point h must be added $\$T$, the underlying cost of coal handling, *i.e.*, loading and unloading of the trucks, switching of the cars to a power-house locomotive, etc. It is evident that the marginal cost of coal haulage will increase with the distance to be traveled, which cost may be represented by use of a line AB beginning immediately under h whose downward slope shall be the marginal cost per mile of the coal haulage.

The use of the straight line AB is not rigorously correct since it assumes a constant amount of coal to be handled irrespective of the plant location, whereas with the varying losses involved in transmission the amount of coal burned would decrease as plant location approached the ideal point, so that AB should be a line slightly concave upward. The aggregate of losses in any well-designed transmission is so small that this refinement would be entirely absurd.

The method of determining the ideal point of plant location is now to construct a line parallel to AB and tangent to the transmission polygon k,j,g,f,e,d,c,b, etc., and to select as the plant location the point of tangency. Such a parallel line is shown at $A'B'$ which comes tangent to the polygon at point f and therefore indicates the cost for transmission plus fuel importation for that location to be equal to $\$fN$.

If location of the power plant at some point as h, which might be a railroad siding, would involve a different terminal cost $\$T'$ due to a different method of handling, then fuel importation costs, plus transmission costs, might be shown by the addition of $\$T'$ to the datum of the polygon. It would be necessary to determine then whether the cost of location at h would be less than for the location at f, *i.e.*, if the length MT' (M being the intersection of Xk extended to intersection with a vertical line through h) were less than fN.

98. Water-supply Cost.—The matter of water supply in Fig. 118 can be taken care of in a similar manner by increasing the slope of AB by the marginal cost per mile of water transmission in case the source of condensing water is on the same side of the loads as the source of coal supply or, if the water supply is located at the other end of the line, by considering the marginal cost of water pumpage per mile as shown by the angle whose tangent is W.

In case the coal-receiving point or the point of water supply lies somewhere intermediate between b and k, the construction will then be similar to the construction used in arriving at line AB, except that in such case we shall have some other line sloping downward from the datum, one or the other of which must be used in testing for the ideal plant location.

A word of additional explanation concerning the cost of water supply may be desirable. The water supply for the plant will have to be pumped and possibly may have to be pumped to a certain elevation above its source in case the plant condensers cannot be located at water level. The cost of keeping pumps and their drive equipment and the cost of housing them together with their necessary attendance constitute an underlying cost. The marginal cost of water pumpage resides in the extra investment in pipe, the extra power service required to overcome the friction in an increasing length of pipe, and the extra cost of a pump adapted to enough higher pressure to overcome the pipe friction. All these marginal quantities will be found to be almost absolutely proportional to the length of piping. The water-supply cost may therefore be represented by $\$(W + w\text{-miles})$, and the increasing marginal cost in Fig. 118 is shown by the increase of ordinate between lines EC and ED. For location of the plant, then, in order to consider both fuel and water costs in addition to the underlying and marginal transmission cost, the ordinates from the datum to the total water and fuel cost line DE must be added to the ordinate from the datum to the polygon.[1]

The best location having been selected for the electrical transmission, fuel, and water supply, other conditions such as public nuisance or nonsystematic variation in realty costs may now be taken into account, either by noting the annual cost excess

[1] See LeClair, T. G., Plant Location in Relation to Coal, Water and Electric Load, *Elec. World*, Sept. 16, 1933, p. 375.

involved in taking a less economical location to avoid giving offence or by adding above the polygon the various carrying costs on different plots of ground.

99. Effect of Voltage on Location.—It may be seen that with high-voltage transmission the amount of current and copper involved in the marginal cost is very small, while, in general, the marginal cost of water pumpage and the marginal cost of coal haulage are relatively large. If both coal and water supply are at or near the neighborhood of point *h* in Fig. 118, the downward slope of *AB* will probably be greater than that of *kX* and so the plant will be located at point *h*. If, however, point *h* represents the point of coal supply, and the point of water supply is to the left of *a*, then undoubtedly *ED* will have an extremely large slope upward to the left and plant location will be very close to the point of water supply.

With a high-voltage transmission, the divisions of the ray polygon such as *p-q*, *q-r*, etc., are bound to be extremely small and therefore the whole transmission polygon will be quite flat and location of the plant to one side or the other of the ideal point will make but little difference from the point of view of electric transmission. With a low-voltage plant, on the other hand, distances *p-q*, *q-r*, etc., will be relatively large and the electric transmission will be the dominant factor in plant location. If the transmission development happens to lie along a navigable body of water, then the cost of water supply will be the same at any point between *a* and *k* and the marginal cost of the coal delivery will be probably negligible, representing only the trifling cost of towing barges or propelling coal freighters a few thousand feet more or less. Here then the transmission polygon even for a high-voltage plant assumes considerable significance.

With regard to the location of substations, the advantage of the higher voltages will lie with the alternating-current systems. For the direct-current network, the low voltage will call for many cables, large substations comparatively close together and in the central city load areas which include the most expensive realty. The alternating-current automatic network may have its substations at greater distances from the load and therefore possibly outside the downtown district. If the feeders are direct leads of 13 or 22 kv., the substations may be eliminated altogether.

100. Effect of Variable Load and Coal Facilities.—A similarly designed plant working on extremely variable loads will necessarily have a great deal of fuel consumption to keep boilers in readiness to carry the peak demand. All this fuel consumption fails to appear in the transmission polygon, but this coal must be transported, and so *in a plant carrying highly fluctuating loads the economical point of plant location will be found to be nearer the coal supply* than it would be were the plant to operate on steady loads, which would obviate the large "stand-by" coal consumption necessitated by carrying boilers ready for service, but not under full pressure.

Although the marginal cost of transmission of coal is now low, we should note that there is a wide range in such cost, depending on whether the coal is handled by the car load directly into the station or whether it must be carried in trucks. The marginal cost of hauling a loaded coal car an extra half mile or mile is very much smaller than would be the marginal cost if this same coal were dumped into 10- or 5-ton motor trucks and hauled that same distance over perhaps more or less unsatisfactory roads. Location near the coal terminal will be more highly important for the comparatively small plant than for the large plant equipped with facilities for rail haulage directly into the plant.

101. Rectangular Distribution of Loads.—Although the single, long, straight transmission is not typical, it is not by any manner of means so unusual as to be unique. A chain of properties supplied from one common steam plant might easily enough be located over a stretch of 50 miles, along a navigable stream or lake, in which case the analysis given in the immediately foregoing pages would be absolutely applicable. A similar analysis is pertinent and essential in the location of the power plant for a long interurban railroad or for a main-line electrification, with only the limiting condition that all possible points of plant location must be points of ample water supply. We assume that any traction property of the type referred to will be its own source of coal supply.

The more usual arrangement of distribution system for which a plant location must be determined is that of a city with streets approximately rectangular to one another, as shown in Fig. 119. Now it may be seen that the location of the power plant along the line *af* is not affected by what happens on the laterals, and so we

may proceed by considering each lateral circuit as a single point of load delivery and applying the method shown in Fig. 115. However, the location so determined does not tell us whether we would better move the main transmission to $a'f'$ or $a''f''$, nor indeed whether our main transmission would better run at right angles to the direction shown. To determine such a point, all that is necessary is to project individual loads on the laterals to such a line as jk and apply to this hypothetical distribution the

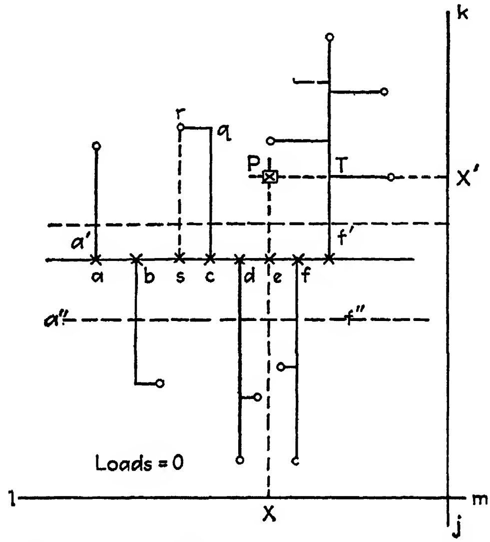


FIG. 119.—Location of power station for loads shown, and rectangular streets.

same test as was applied to the main line af. In actual fact, the layout of Fig. 119 would never be made by streets. Rather, individual load points unaffected by any question of transmission layout and plant location would be spotted on a map and two lines jk, lm would be drawn parallel to the main street axes, and the individual loads would be projected onto these axes and each such projection treated as an independent straight-line distribution system. The desirable plant location for each of these hypothetical distributions having been determined as at X and X', perpendiculars would be erected at these points and their intersection P would be the ideal plant location.

The justification for the foregoing construction is more or less evident, and few details remain to be indicated. If each point

served from the power house has its own individual feeder, it is perfectly evident that as between path *csr* and path *cqr* there is no choice. So it is perfectly permissible to use *r* projected on lines *lm* and *jk* without reference to the path traversed. If, however, *q* represents a street corner and *r* a service supply in the middle of a block, then transmission length *qr* will have to be considered in any case, and so for the sake of simplicity this length may be left out of our problem entirely and the load *r* assumed to be located at point *q*. Similarly, the whole distribution network beyond point *T* would be disregarded and its aggregate load assumed to be concentrated at point *T*. Similar reasoning will show that the problem of desirable plant location will be unaffected in such case by any consideration as to whether the main line of transmission is straight or broken, or whether it runs parallel to *lm* or *kj*. It is assumed, however, and with propriety, that no transmission is likely to be built with individual cables looping back on themselves, *i.e.*, running first north, then east, and then south, but that all transmission would be progressive along the direction of one or the other of the axes from the power house toward the point of delivery. Occasionally special considerations may necessitate such a looping back as shown at *qcdP*. In this case, there is evidently "no thoroughfare" by a short route from point *q* to *P*, and in recognition of this the engineer would assume the load *q* to be actually taken at point *c* for purposes of power-plant location since distribution length *cq* *must* be installed, and installed for a definite load irrespective of any question of power-plant location.

So far we have discussed the location of power plant for a territory with a rectangular arrangement of highways on the assumption of individual cables from the plant to the points of delivery. If, however, a single large cable is used along the main transmission *af* and branch cables are used along the laterals, the problem of plant location will involve the use of marginal costs based on the cost per kilovolt-ampere per mile on line *lm*, but using the marginal cost per mile of transmission plus the underlying cost per mile of transmission on axis *jk*. In such case, the specific solution of the problem will involve more or less obvious applications of the general principles laid out in the foregoing, but the engineer should satisfy himself that he has assumed the correct axis for the main transmission. If the trans-

mission system is not already laid out, this can be done very quickly by working through two solutions and then taking that one which gives the lower *aggregate* cost of transmission along both axes *lm* and *jk*.

In any practical plant-location problem, a great many modifications of the foregoing analysis will arise owing to departures of highways from an orderly mathematical arrangement and to other local conditions. These abnormal conditions can, and indeed must, be studied on their own individual merits, taking due cognizance of any modifying influences that they may have on the general problem.

So far the discussion of the best location of plant has been based on the assumption of a definite system of transmission from the plant to the utilization points. However, owing to the profound influence which the system used has on the best location with that given system, it will be found that a 230-volt direct-current plant might be located at a radically different point from that which would be chosen for a 2,300-volt three-phase alternating-current system and that the actual best location will have its effect on the total cost of transmission under any given system.

Had we carried out the studies as to the most efficient type of system to use, on the assumption that the plant would be located squarely in the middle of a transmission line of moderate length, and had this tentatively adopted system then required the location of the power plant nearly at the end of the distribution, it is evident that the cost of the electric transmission would be considerably higher than we had calculated. A system of distribution apparently less economical at first glance might, however, have indicated a power-plant location nearer the point assumed, in determining *as* between systems. Then, in actual fact, the initially calculated losses arrived at in rejecting the less desirable system would not have been greatly increased by the actual and subsequent plant location. If there has been a very close choice between the two systems of distribution in the first instance, the engineer should arrive at the ideal plant location for each of these systems, then recalculate the total transmission cost or determine it graphically by the process of plant location, and then compare the two systems with the plants as located. Ideally, this process of successive approximations should keep up through a never ending series, but in general one recheck will

indicate whether any change in choice of system has been necessitated by the actual problem of location.

102. Problems.

1. On the assumption that the secondary loads per foot of circuit are constant, locate a 25-kva. transformer most economically.

Fixed costs = 10 per cent.

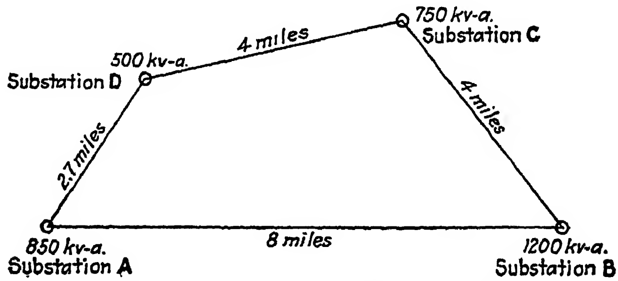


FIG. 120.—Location of power house to supply electric railway substations, Problem 2.

Conductor cost = \$90 + \$0.004 cir. mil mile (principal costs).

Cost of electric service = \$1,500 + \$16 kw. + \$0.006 kw.-hr.

Root-mean-square amperes = 0.6 max. amp.

Primary volts = 6,600, single-phase; secondary volts, 220.

Length of secondary feeder = 1,000 ft.

The primary line and the secondary loads are parallel.

a. When the primary line goes on past the location under consideration to feed other transformers.

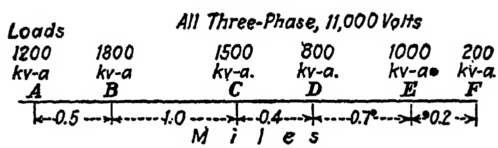


FIG. 121.—Location of power house for loads on a common feeder, Problem 3.

b. When the transformer considered is the last one on the primary.

2. For the electric railway substations shown in Fig. 120.

a. Find the center of gravity of the system and the total kilovolt-ampere miles of transmission necessary to supply the loads from a power station at that point. Use first rectangular feed runs vertical and horizontal, second direct feed lines.

b. Find the ideal location, and the total kilovolt-ampere miles of transmission from such a point, using only rectangular feed runs.

c. Find the ideal location and kilovolt-ampere miles of transmission if the feeders run only on the highways shown by the lines connecting the substations.

3. For the system of loads on a common feeder shown in Fig. 121: Copper costs \$30 + \$0.003 per circular mil mile. It is a 20-year project, with money use 6.5 per cent, taxes 1.7 per cent, salvage 75 per cent. Assume that all changes occur entirely in the marginal part of the line. The economic section is 2,500 cir. mils per ampere. How much more can be paid for real estate at the best location than for a site at the next load to the left. considering marginal cost only?

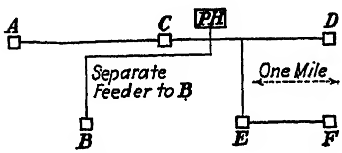


FIG. 122.—Location of power house for loads on a mixed-feeder system, Problem 4.

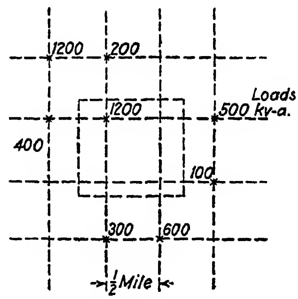


FIG. 123.—Problem 5, to locate the power house for loads with cable feed.

4. For the mixed feeder system of loads of Fig. 122: All maximum demands at 6,600 volts, three-phase.

A = 300 kva.
B = 200 kva.
C = 100 kva.
D = 400 kva.
E = 100 kva.
F = 200 kva.

The daily r.m.s. load is 70 per cent of the maximum. Fixed charges are at 15 per cent throughout.

With electric service costing \$20,000 + \$20 kw. + \$0.003 kw.-hr. per year and three-phase cable costing \$150 + \$0.004 cir. mil mile per conductor for purchase and installation, locate a steam power plant for the most economical transmission cost to serve the foregoing feeder system.

5. For the loads of Fig. 123: Locate a substation and draw the cable feeds to serve these loads. Real estate for a substation within the dotted square will cost \$6,000 per year; outside this area, the cost will be \$4,800 per year.

The annual marginal transmission cost (*2fbn* 1 cir. mils) is \$0.75 per kilovolt-ampere mile. The annual basic cost is \$200 per mile per cable. Heat losses limit the practical load to 500 kva. per cable.

6. For the loads of Fig. 124: In a city distribution system, it is desired to reduce the loads on substations A and B by transferring the loads designated to a new substation to be located most economically with respect to the

transmission for these loads. Loads taken from Station A are marked *x*, those from Station B are marked by a circle. Locate the new substation.

7. Locate the distribution transformer to serve the secondary loads on 9 poles, space 100 ft. apart, the total services swung from each pole being

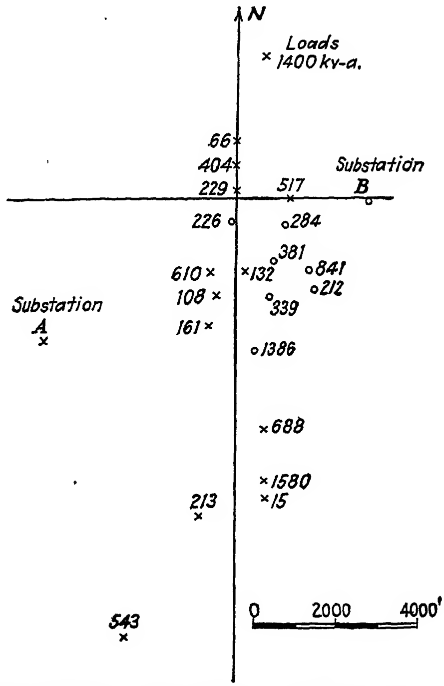


Fig. 124.—Locate the substation for loads to be transferred to a new substation Problem 6: *x*, from substation A; *o*, from substation B.

loaded as follows: No. 1 pole, 700 watts; No. 2, 600; No. 3, 600; No. 4, 100; No. 5, 1,100; No. 6, 100; No. 7, 500; No. 8, 400; No. 9, 600. (*a*) Give location and watt-feet transmitted for center of gravity, (*b*) same for ideal location.[1]

[1] From Shapiro, Leo, *Elec. World*, July 30, 1932.

CHAPTER VII

BUS SYSTEMS AND CURRENT-LIMITING REACTORS

103. General Conditions of Power Control.—Having determined the peak capacity and character of the load to be supplied, and selected the units that will most economically furnish the service, we must now consider how the assembly should be arranged for operation and control. This must permit economic loading of the units for the changing loads of the day, while gathering the power from the generators to the bus bars, on the one hand, and then distributing it from the bus bars to the various transformers, transmission and distribution lines, substations, and load devices on the other. The great requisite of any important bus system must be, of course, continuous and reliable service together with such flexibility that it is adapted to the particular operating conditions which prevail for the plant in question. Flexibility will aid also in the maintenance of power supply, since in case of failure of some unit of apparatus, or part of the bus system, loads may be fed by duplicate or special paths provided for just such emergencies. Quite naturally every extra feature complicates the layout, takes up room, adds to the already heavy cost of this vital portion of the plant, and is another operating hazard to be maintained. Therefore, the engineer must weigh carefully the advantages of each addition beyond the requisite minimum to be sure that every item justifies itself.

The student is referred to the "Electric System Handbook"[1] for diagrams and discussion of the standard bus systems.

After a plan has been evolved that will provide for all the expected switching operations, calculations should be made to determine the power concentrations on the various parts of the layout, for both normal and abnormal conditions. Modern station alternators have reached such huge sizes that a short circuit on the bus system may involve a tremendous capacity

[1] McGraw-Hill Book Company, Inc.

with danger to the safety of the structures and apparatus and, due to the prolonged heavy drop in voltage, the dropping of all synchronous load equipment from the system. Rather than trust the performance of the largest size of circuit breakers on such concentrations, it may be desirable to use current-limiting reactors to decrease the duty on the breakers and help to maintain service. Operating experience has amply demonstrated that the reactor is very valuable in thus permitting large capacities to be operated in parallel while providing protection for sections from each other, and in maintaining station voltage during line short circuits. It is generally necessary to hold the distribution voltage above 67 per cent of normal since synchronous motors drop out of step at this value under full load at rated power factor; fully loaded induction motors pull out at 63 per cent; alternating-current contactors drop out at 60 to 65 per cent; and undervoltage releases operate at 50 per cent.[1] By the use of high-speed relays, selective circuit-breaker action may be obtained and the trouble be confined to a minimum section of the system.

In the following paragraphs, examination will be made of the effectiveness of the reactor as used in various parts of the power system.

104. Definition of Current-limiting Reactor.—The A.I.E.E. Standards, Sec. 13-106, define a current-limiting reactor as "a form of reactor intended to limit the current that can flow in a circuit under short-circuit."

105. Rating of Current-limiting Reactor.—Article 13-156c specifies that "the rating of current-limiting reactors shall be expressed in kilovolt-amperes absorbed at the rated current and frequency on a circuit of specified voltage rating."

The reactance is usually rated in the "per cent" reactance that it introduces into a circuit. Such per cent is the ratio of the voltage drop across the reactor, for rated current flowing at rated frequency, to the Y voltage of the circuit. Thus, in a 6,600-volt (3,810 volts Y) circuit, a 6 per cent reactor would have a reactive voltage drop of $0.06 \times 3{,}810 = 228$ volts at the rated current and normal frequency of the circuit in which the reactor is inserted.

[1] See *Gen. Elec. Co. Bull.* GEA.-1116.

106. Heating of Current-limiting Reactor.—Article 13-200 specifies that for continuous rating, reactors shall have the following limiting temperature rise in degrees centigrade above the temperature of the cooling medium:

Class A insulation (cotton, silk, paper, and similar organic materials impregnated or immersed in oil).

Class B insulation (inorganic material such as mica and asbestos in built-up form combined with binding substances).

Type	Method of measurement	Class A insulation, °C.	Class B insulation, °C.
Air cooled	Resistance	55	75
	thermometer	60	80
Oil immersed	Resistance	55	
	thermometer	55	

107. Momentary Load Limitations of Current-limiting Reactor. Article 13-252 states:

Current-limiting reactors having an impedance of 3 per cent or more shall be capable of withstanding without injury for five seconds the maximum current that would result from any short circuit on the system with normal line voltage maintained at the supply terminals and with only the inherent impedance of the reactors in the circuit.

Reactors having an impedance of less than 3 per cent shall be capable of withstanding without injury for five seconds a current equal to 33⅓ times the rated current.

For the purpose of standardization the temperature of the copper under the short circuit conditions given above shall not exceed 250 deg. for Class A insulation or 350 deg. cent. for Class B insulation. . . .

108. Losses in Current-limiting Reactors.—Article 13-311 specifies, "The losses to be considered in current-limiting reactors shall be the load losses. These losses consist of I^2R losses in the winding due to load current, and stray losses due to stray fluxes in the windings and other metallic parts. The stray-load losses shall not include the loss produced by the magnetic field of the reactor in adjacent apparatus or materials."

109. Dielectric Test for Current-limiting Reactors.—Article 13-400(*d*) states, "Current-limiting reactors shall be tested between conductors and ground by applying 2¼ times the rated voltage of the circuit, plus 2,000 volts."

110. Reactor Construction.[1]—Current-limiting reactors are built in two types, the dry-type and the oil-immersed form. The former is an air-insulated type for use indoors at voltages up to 33,000, the windings being embedded in vertical supports of

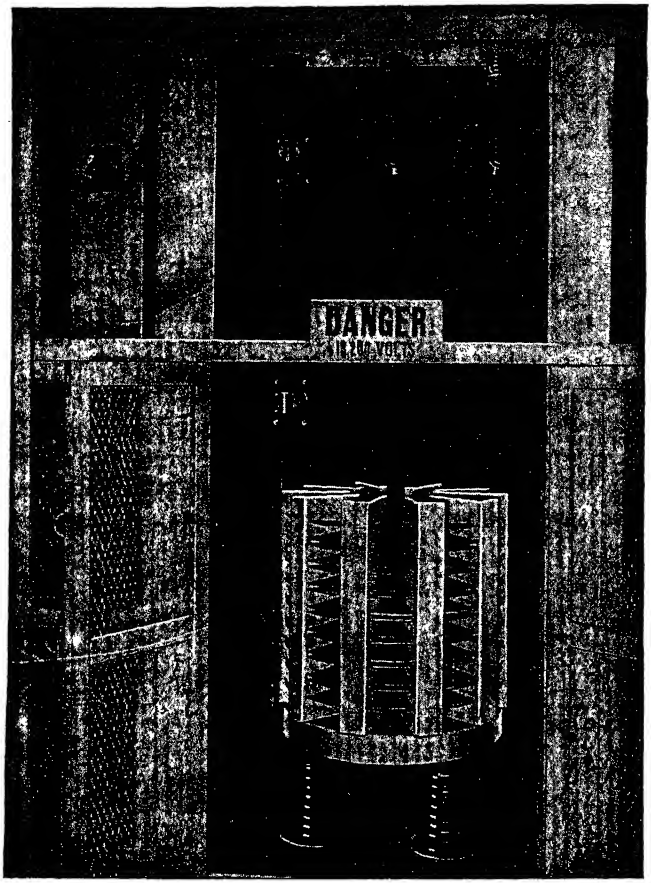
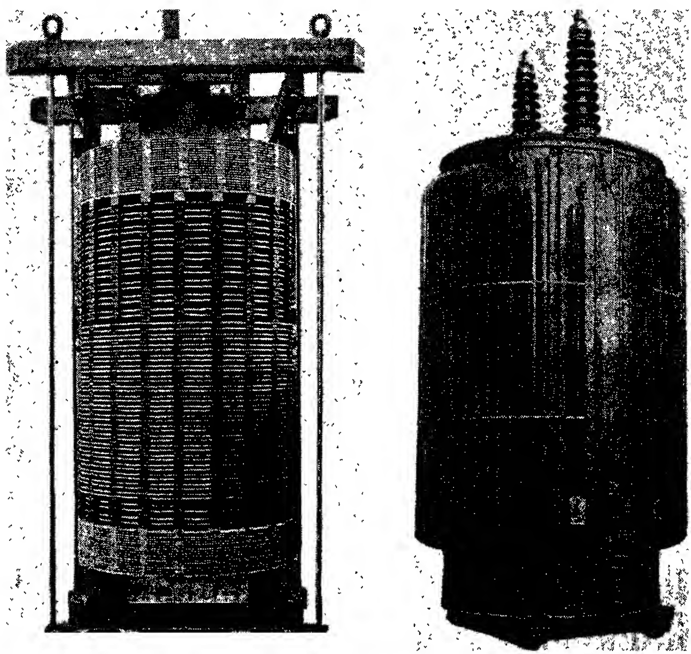


FIG. 125.—Typical installation of General Electric single-phase reactor in ventilated cell.

specially treated concrete. The completed winding with its supports is bolted to a concrete base, which is insulated from ground by porcelain footings. The conductors are insulated with fireproof asbestos braid. Figure 125 shows a typical installation

[1] See GEORGE, R. B., Development of Large Current-limiting Reactors, *Elec. Jour.*, August, 1931; and *Gen. Elec. Co.*, *Bull.* GEA-1116.

of a reactor of this type. For voltages above 33,000 indoors, or 25,000 outdoors, or where it is desired to isolate the phases without having individual cells for the reactors, the oil-immersed type is used. Here the windings are similar to those for transformers, and a nonmagnetic shield prevents the flux from entering the steel tank where it would cause heating and losses. Heavy



(*a*) (*b*)

FIG. 126.—(*a*) Interior view; (*b*) exterior view of Westinghouse 5,400-kva., single-phase, 60-cycle, OISC, 110,000-volt, oil-immersed current-limiting reactor.

copper rings or laminated iron strips on the tank wall assist in the shielding. Figures 126*a* and *b* show such a unit for 110,000-volt service.

111. Determination of Size.—The total reactance required in a circuit is determined by the maximum short-circuit current which can be allowed in that section of the system. Under a major operating fault, the current concentration at the weakest point in the system or the maximum allowable voltage drop on the bus for a feeder short will be determined. Then, for this limiting amperage the total required reactance, made up of all the react-

ances in the circuit, will be 100 times the ratio of the normal full-load current to the limiting short-circuit current. If the limiting amperes are five times normal, then the total reactance must be 20 per cent, and if inherent reactances of apparatus, lines, etc., provide some 15 per cent of this amount, then additional current-limiting reactors will be required for the remaining 5 per cent.

112. Generator Reactance Balanced Operation.—Considered in relation to short-circuit conditions, the following definitions from the A.I.E.E. Standards No. 2, August, 1932, Sec. 10.35. 100, 120, and 140, respectively, are important:

Short-circuit Ratio.—The short-circuit ratio is the ratio of the field current at rated open-circuit armature voltage and rated frequency to the field current at rated armature current on sustained symmetrical short circuit at rated frequency.

Direct-axis Synchronous Reactance.—This is the ratio of the fundamental component of reactive armature voltage, due to the fundamental direct-axis component of armature current, to this component of current under steady state conditions and at rated frequency. The per-unit direct-axis synchronous reactance equals the ratio of the field current at rated armature current on sustained symmetrical short circuit to the field current at normal open-circuit voltage on the air-gap line (*i.e.*, the extended straight line part of the magnetization curve).

Direct-axis Transient Reactance.—The direct-axis transient reactance is the ratio of the fundamental component of reactive armature voltage, due to the fundamental direct-axis component of symmetrical armature current, to this component of current under suddenly applied load conditions and at rated frequency, the value of current to be determined by the extrapolation of the envelope of the symmetrical current wave to the instant of the sudden application of the load, neglecting the high-decrement currents during the first few cycles.

The direct-axis transient reactance is determined from a symmetrical short circuit suddenly applied to the machine operating at no load and rated speed, and equals the ratio of the no-load voltage to the corresponding armature current given by the extrapolation of the envelope of the current wave to the instant of sudden application of the short circuit, neglecting the high-decrement currents during the first few cycles.

Since it is so important that modern alternators be capable of withstanding the stresses of full short circuit, Standard 3-130 specifies:

Short-circuit Requirements.—All synchronous machines shall be capable of withstanding short-circuit without injury, when operating under rated load. Test shall be made by abruptly short-circuiting for a period of 30 seconds, the machine when operating at no load, rated frequency and 10 per cent above rated voltage. This test shall be applied only when the armature coil bracing is in good condition and is advisable on new machines only.

113. Synchronous Machine Reactance Unbalanced Operation. Recently it has been necessary to expand the theory of the reactive voltages in synchronous machines and, in addition to the reactance coefficients covering balanced operation referred to above, to determine the constants for operation under the unbalanced conditions which prevail so commonly under short circuits in networks. Most of the development has been based on the method of symmetrical coordinates as originated by C. L. Fortescue and presented in his paper, "Method of Symmetrical Coordinates Applied to the Solution of Polyphase Networks," in the *A.I.E.E. Transactions* for 1918.[1]

This method provides for the resolution of any distribution of armature current into the superposed sum of three symmetrical components:

1. Balanced three-phase currents of normal-phase rotation, called *positive phase-sequence.*

2. Balanced three-phase currents of reverse-phase rotation, called *negative phase-sequence.*

3. Equal three-phase currents of equal-time phase, called *zero phase-sequence.*

It has the great advantage of isolating the parts of a circuit that pass only the particular phase-sequence currents concerned and permitting a special analysis to be made of that circuit. Thus only the positive phase-sequence currents are considered in three-phase shorts, only positive and negative phase-sequence currents for single-phase line-to-line shorts, but all three phase-sequence currents for single-phase line-to-ground faults.

Using this method, R. H. Park and B. L. Robertson contributed a critical survey of the detailed study of reactance characteristics in their paper, The Reactances of Synchronous Machines, in the *A.I.E.E. Transactions* for February, 1928. Table 26 is

[1] See Wagner, C. F., and Evans, R. D., "Symmetrical Components," McGraw-Hill Book Company, Inc.

a partial list of the reactance values listed in their paper as being representative for most machines of normal design.

TABLE 26.—TYPICAL VALUES OF REACTANCE—SYNCHRONOUS MACHINES[1]

Machine	Direct synchronous pos. ph.-seq., per cent	Direct transient pos. ph.-seq., per cent	Negative ph.-seq., per cent	Zero ph.-seq. per cent
Syn. motors				
High-speed..........	65–90	15–35	11–25	2–15
Low-speed...........	80–150	40–70	25–50	4–27
Syn. condensers........	Av. 160	40–50	25–32	4–10
Waterwheel generators..	60–125	20–45	25–60	2–21
Turbo-alternators				
Solid rotor..........	Av. 115	15–25	8–13	1–8
Laminated rotor.....	Av. 115	15–25	11–21	1–8

[1] From PARK and ROBERTSON, *A.I.E.E. Trans.*, February, 1928.

In Part 2 of their paper, covering "Theoretical Considerations," the authors point out that though, in accordance with conventional practice, the characteristic coefficients that express the factors of proportionality between reactive voltage and current are referred to as reactances, they have a special property. This is, that unlike a simple reactance these coefficient reactances are not independent of the character of the terminal circuit. Thus the effective negative phase-sequence reactance of a synchronous machine will vary with the nature of the circuit external to the machine, and to a slight extent the positive- and zero-phase-sequence reactances will also be affected.

Of the foregoing reactances, we are interested primarily in the transient, as it determines the value of the first rush of current which the circuit breakers and relays are installed to control. The relation between this initial current and the sustained short-circuit current will be discussed more fully in Sec. 141, Characteristics of an Alternator Short Circuit.

114. External Generator Reactors.—As shown in Table 26, modern alternators have, in general, transient reactances of the order of 15 to 25 per cent and hence are protected against short circuit even at their terminal leads. In remodeling a station, however, where it is planned to use some of the older machines of only 6 or 8 per cent reactance in parallel with modern units, it

may be necessary to add generator reactors for their protection. Except in such special cases, the use of generator reactors is not recommended because of the constant large load losses and reactance drop in the voltage regulation of the unit when it is in

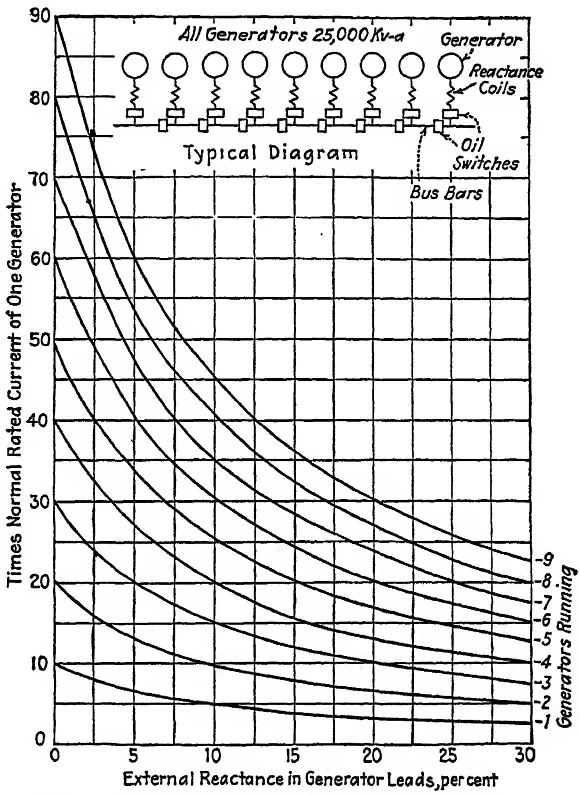


FIG. 127.—Relation between percentage external reactance in generator leads and maximum short-circuit on bus bars. Inherent reactance of each generator 10 per cent.

service. If used, they should be placed in the line leads as close to the generator as possible.

Figure 127[1] shows the relation between per cent external reactance added to the generator leads and short-circuit amperes in the single straight bus for different values of reactances, with one to nine generators connected to the bus bars. All generators

[1] From LYMAN, J., ROSSMAN, A. M., and PERRY, L. L., Protective Reactance, *A.I.E.E. Trans.* Vol. 33.

are assumed to be rated at 25,000 kva. and to have 10 per cent inherent reactance.

It is important to note that the first few per cent of reactance added are much more effective in reducing the short-circuit amperes than are subsequent additions. Beginning with the inherent reactance only of the generators, the addition of 5 per cent external reactance reduces the total short-circuit amperes by 33.3 per cent; the second 5 per cent reactance reduces it by

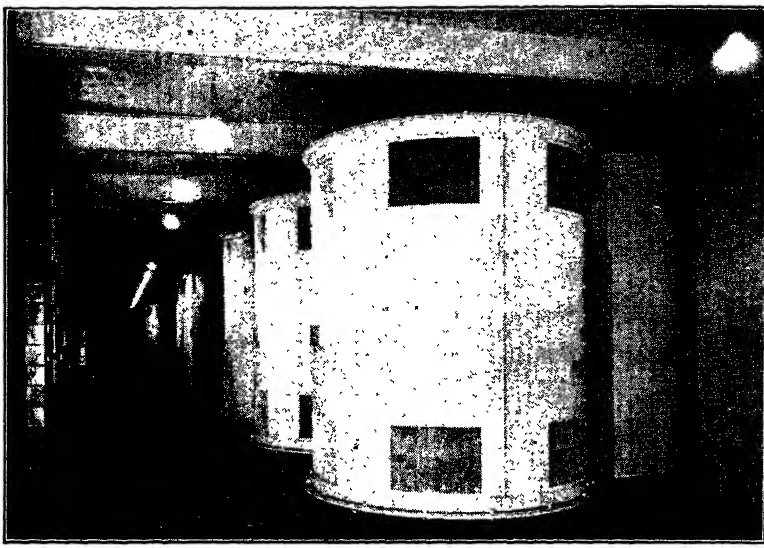


Fig. 128.—General Electric installation of cast-in-concrete reactors assembled inside metal casings. Rated type CLS-60, 2,500 kva., 1,385 volts, 1,800 amp. Reactors are placed between 75,000-kva. sections of a 300,000-kva., 24,000-volt bus. The reactors and bus casings are insulated by being supported on insulators and are connected to the ground bus.

16.7 per cent; the third 5 per cent reactance reduces it by 10 per cent; the fourth 5 per cent reactance by 6.7 per cent, etc., until the effect of further additions is negligible.

115. Bus-sectionalizing Reactors.—As was noted above, when a bus becomes so large that, for continuity of service, etc., it is necessary to divide it into several sections, protection may be had when these sections are separated by reactors. This will permit any heavily loaded section to draw part of its load from adjacent sections, but in the event of a short circuit on the bus, or on a feeder connected to the bus, the momentary short-circuit current will be limited to that amount which one

bus section can furnish plus only a small amount supplied by the adjacent sections. The voltage of the section upon which the short circuit takes place may fall to zero, and thus the reactors connecting the two adjacent sections will each consume the total voltage. For section reactors of 25 per cent each then, the total voltage (100 per cent) would be consumed when four times the normal full-load current flows over them; and hence both adjacent sections would supply eight times normal full-load current of one machine to the short-circuited bus section. If each section had three generators, and each generator had a short-circuit current of eight times normal full-load current, then the current

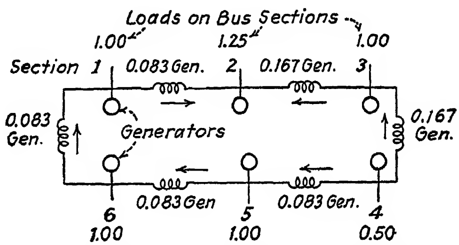


FIG. 129.—Distribution of energy flow on a sectionalized bus, six generators on bus. All generators fully loaded except No. 4, which is three-fourths loaded.

supplied from the two adjacent sections would be the equivalent of the short-circuit current of one machine, and the total short-circuit current would be that due to four alternators. Figure 128 shows a typical installation of bus-sectionalizing reactors.

In an article entitled "The Use of Power-limiting Reactances in Large Power Stations," in the *General Electric Review*, June, 1913, C. M. Davis shows how a sectionalized system might be operated under normal running conditions.

Consider a generating station having 18 machines of equal output connected to a six-section ring bus, three units per bus section. Assume that during a certain portion of the day, the total output of the station equals 5.75 times the full load of one generator, and call this value 5.75 gen. Figure 129 shows an assumed distribution of load. Bus Section 2 is overloaded by 0.25 gen. while Section 4 has 0.50 gen. to spare. The overload on Section 2 is drawn from both adjacent sections; but, as the generators of each of these sections are under full load, the energy is drawn from those next adjacent, and so on until the next underloaded section is reached. Between Sections 2 and 4 there are two reactances in series. Thus of the 0.25 gen. additional required by

Section 2, ($\frac{4}{6} \times 0.25$ gen.) = 0.167 gen. comes from the right and ($\frac{2}{6} \times 0.25$ gen.) = 0.083 gen. comes from the left. These values are indicated at the arrow heads in the figure. The normal operation under this condition is full load on each generator, except that on Section 4, which is running at three-quarters load.

In Fig. 130, the load is assumed to have increased to the full output of 10 generators, and to be distributed as shown. If the generators are connected as indicated, the amount of energy

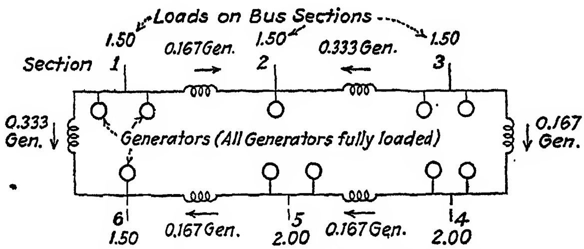


FIG. 130.—Distribution of energy flow on a sectionalized bus, 10 generators on bus. (*Gen. Elec. Rev., June*, 1913.)

transferred from one section to another may be obtained as follows:

On the basis of watts flow,

$$W_{1-2} + W_{3-2} = 0.5 \text{ gen., then } W_{3-2} = 0.5 \text{ gen.} - W_{1-2} \quad (125)$$
$$W_{3-2} + W_{3-4} = 0.5 \text{ gen., and } W_{3-4} = 0.5 \text{ gen.} - W_{3-2} \quad (126)$$
$$W_{1-2} + W_{1-6} = 0.5 \text{ gen., and } W_{1-6} = 0.5 \text{ gen.} - W_{1-2}. \quad (127)$$

When the loads and generators are all balanced, the voltage differences become zero and the summation of flow around the bus is zero.

Then going clockwise,

$$W_{1-2} - W_{3-2} + W_{3-4} + W_{4-5} + W_{5-6} - W_{1-6} = 0. \quad (128)$$

Substituting,

$$W_{1-2} - (0.5 \text{ gen.} - W_{1-2}) + [0.5 \text{ gen.} - (0.5 \text{ gen.} - W_{1-2})]3 - (0.5 \text{ gen.} - W_{1-2}) = 0 \quad (129)$$

and

$$2W_{1-2} - 0.5 \text{ gen.} + 3W_{1-2} - 0.5 \text{ gen.} + W_{1-2} = 0. \quad (130)$$

Therefore,

$$6W_{1-2} = 1.0 \text{ gen.}, W_{1-2} = 0.167 \text{ gen.} \quad (131)$$

Then,

$$W_{3-2} = 0.333 \text{ gen.} \quad (132)$$
$$W_{1-6} = 0.333 \text{ gen.} \quad (133)$$
$$W_{3-4} = 0.167 \text{ gen.} \quad (134)$$

The sectionalizing reactors for a straight bus should have about 5 to 15 per cent reactance based on the capacity of a bus section with a current-carrying capacity equal to that of one bus section. The number of sections into which a bus should be divided depends largely upon the individual system and the conditions under which it is expected to operate. Since the largest oil switches can be expected to open safely approximately 2,500,000 kva. under the usual conditions of short circuit, no concentration should be permitted that will allow a short circuit to form in excess of this circuit breaker interrupting capacity. In general, then, a generator bus should be divided into such a number of sections that the normal capacity of any one section should not exceed 50,000 kva.

116. Sectionalizing Reactors with Single or Double Straight Bus.—Let the power-limiting reactors be installed in the bus bars between groups of three generators per group, and assume that all nine generators are running. Then Fig. 131[1] shows the relation between per cent reactance in the bus bars and the short-circuit amperes on the bus. Curves *a* and *b* assume a fault in the center of the bus bars, and *c* and *d* assume a fault at the end of the bus bars. It is to be noted that with the double bus bars twice as many reactors are required as with the single bus, but since with the double bus each reactor carries but half the current carried by each reactor in the single bus, the total kilovolt-amperes required is the same in either case.

Figure 132[1] shows the same relations as Fig. 131, but with the reactance installed between the individual generators. In comparing similar curves in Figs. 131 and 132, it is to be noted that a given total per cent of reactance has a greater choking effect when distributed between adjacent generators than when concentrated by being placed between groups of three generators per group, and that this difference increases as the amount of reactance is increased. For example, the authors point out that

[1] From Lyman, Rossman and Perry, Protective Reactance, *A.I.E.E. Trans.*, Vol. 33.

in comparing the b curves, 16 per cent total bus reactance divided into two units of 8 per cent each limits the current to 48 times normal rated current of one generator, whereas the same reactance divided into eight units of 2 per cent each will limit the current to 44.5 times. Likewise 32 per cent reactance in two units of 16 per cent each limits the current to 40.5 times, while

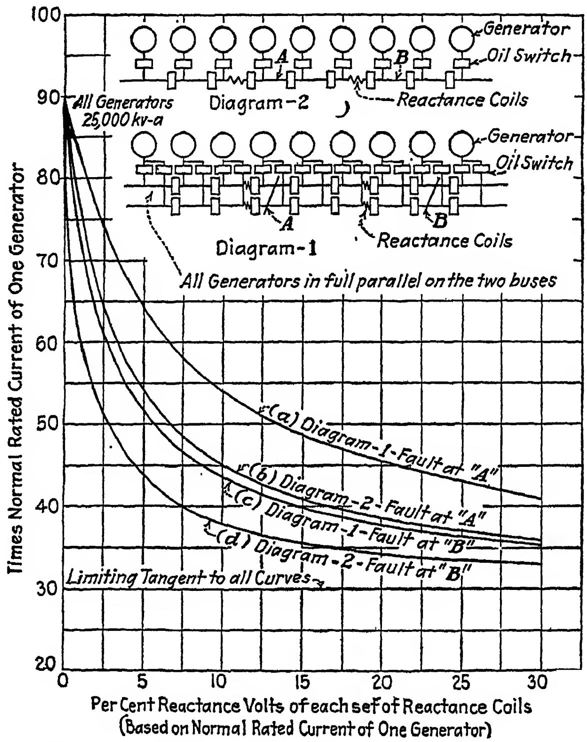


FIG. 131.—Relation between reactance in bus and short-circuit amperes for single and double straight bus. Reactors between generator groups. Inherent reactance of each generator 10 per cent.

in eight units of 4 per cent each it limits the current to 33 times. Similarly, 60 per cent reactance in two units of 30 per cent each limits the current to 36 times, and in eight units of 7.5 per cent each it will limit the current to 25.5 times.

If it is desired to limit the short-circuit current of the nine 25,000-kva. generators to 30 times the normal rated current of one generator, each reactor having half the current-carrying capacity of one generator, and remembering that the kilovolt-

ampere-reactance rating varies as the square of the current, it will be seen from the *a* curve, Fig. 131, that with reactors between groups of three generators, infinite reactance would be required,

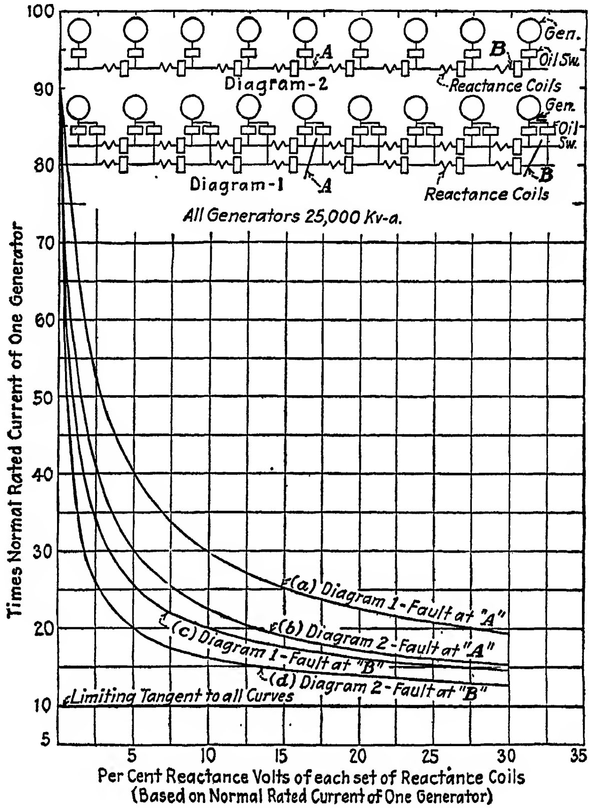


FIG. 132.—Relation between reactance in bus and short-circuit amperes for single and double straight bus. Reactors between each generator. Inherent reactance of each generator 10 per cent.

whereas with reactance between adjacent generators (see *a* curve, Fig. 132) the total reactance required is

$$0.10 \times 16 \times \left(\frac{25{,}000}{4}\right) = 10{,}000 \text{ kva.} \tag{135}$$

where (25,000/4) is the rating per reactor.

117. Reactors in Series and in Parallel.—Reactors in series should be considered as having the same current flow through them, which on a given system voltage means the same kilovolt-

ampere rating. The various reactances being now expressed by the same kilovolt-ampere capacity, their percentages are added to give the total series reactance. Thus if the following reactors are in series,

3% on 5,000 kva.
5% on 7,500 kva.
6% on 15,000 kva.

let all be based on a capacity of 15,000 kva., then they would rate, respectively,

	9% on 15,000 kva.
	10% on 15,000 kva.
	6% on 15,000 kva.
Total	25% on 15,000 kva.
	or 12.5% on 7,500 kva.
	or 8.3% on 5,000 kva.

or proportionately for any other capacity.

Reactors in parallel should be considered as having the same voltage drop across them; *i.e.*, their reactance drop in terms of the given system Y voltage will yield the same "per cent reactance." The various reactances being now expressed on the same per cent reactance, their kilovolt-ampere capacities are added to give the total capacity at that per cent. Thus if the following reactors are in parallel,

3% on 5,000 kva.
5% on 7,500 kva.
6% on 15,000 kva.

let all be based on a 6 per cent rating, then they would rate, respectively,

	6% on 10,000 kva.
	6% on 9,000 kva.
	6% on 15,000 kva.
Total	6% on 34,000 kva.
	or 2.64% on 15,000 kva.
	or 1.32% on 7,500 kva.
	or 0.88% on 5,000 kva.,

or proportionately for any other capacity.

By applying these elemental processes, the values of short-circuit kilovolt-amperes for the curves of Fig. 131 and 132 are obtained by the following typical method, based on Fig. 133:

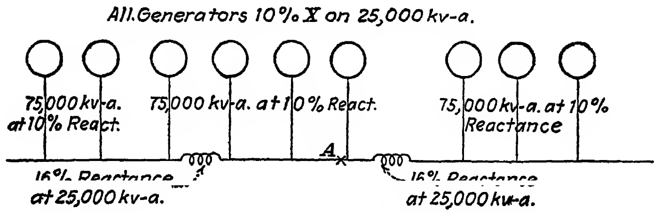


FIG. 133.—Diagram for single straight bus. Fault at A.

Consider a fault at A.

From the left bus section,

Three generators in parallel, each 25,000 kva. at 10%	=	75,000 kva. at 10%	(136)
Bus reactance in series, 25,000 kva. at 16% . .	=	75,000 kva. at 48%	(137)
Sum	=	75,000 kva. at 58%	(138)
From the right bus section, an equal amount due to symmetry	=	75,000 kva. at 58%	(139)
These two in parallel, sum	=	150,000 kva. at 58%	(140)
Which on a basis of 10%	=	25,900 kva. at 10%	(141)
From the center bus section, in parallel	=	75,000 kva. at 10%	(142)
Total at A, all in parallel,	=	100,900 kva. at 10%	(143)
For a fault, when 100% of the voltage is consumed	=	1,009,000 max. kva.	(144)

or

$$\frac{1,009,000}{25,000} = 40.4 \text{ times normal rated current of one generator.} \tag{145}$$

118. Sectionalizing Reactors with Ring Bus.—If the two buses of the double straight bus are extended and tied together with a bus-tie circuit breaker, then we have a so-called *ring bus.* Reactor short-circuiting switches and a transfer bus may also be

included, as shown in Fig. 134.[1] The former provide a means of shunting the reactors when they are not required, and the latter permits any bus section to be fed from an adjoining section without passing current through the bus reactors. It is to be noted that as long as the ring connection is maintained any power that is transferred from section to section passes through two sets of reactors in parallel. Figure 135[1] shows the total short-circuit current, in per cent of short-circuit current from a single

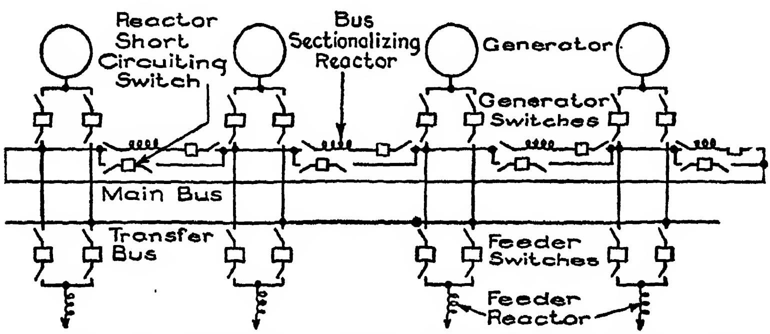


FIG. 134.—Ring-bus arrangement showing transfer bus, bus, and feeder reactors with reactor short-circuiting switches.

generator, for various ratios of bus reactance to generator reactance with different numbers of bus sections.

119. Sectionalizing Reactors with Star Bus.—As is shown in Fig. 136,[1] this bus system has great flexibility combined with simplicity and maintains its voltage better than the ring system when under a short circuit. Since the current path is always through two of the reactors in series when power is transferred from one generator to the main bus section of another machine, the maximum variation in bus-section voltage is constant for any given reactors, regardless of the number of bus sections. For a given maximum allowable short-circuit current, the reactors may have less drop than those for a ring system because the power sent to the short-circuited section by the other sections must all pass through the one sectionalizing reactor of the shorted section. The reactors, however, must have a greater current rating than those for a ring system, the maximum being equal to the current rating of one bus section. The transfer bus and additional

[1] From *Gen. Elec. Co. Bull.* A-1116, Current Limiting Reactors.

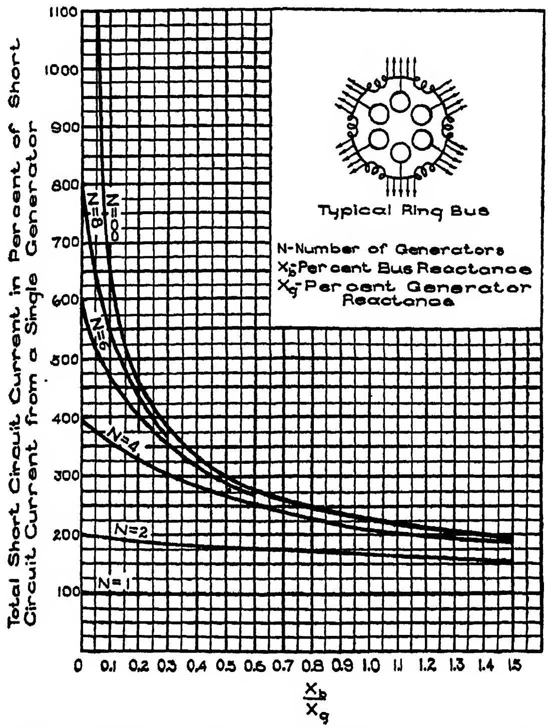


FIG. 135.—Effect of bus-bar reactors on reduction of short-circuit current, for ring bus.

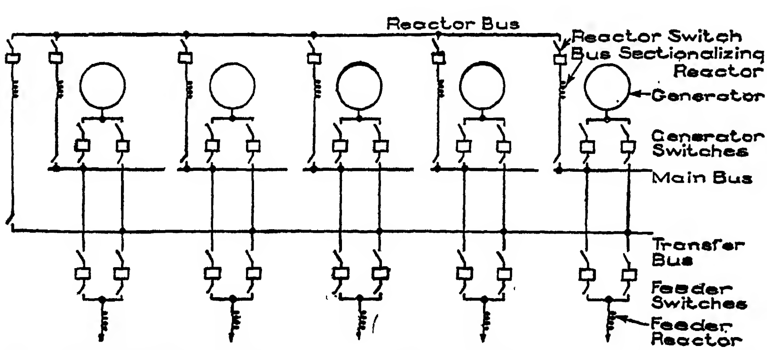


FIG. 136.—Star-bus arrangement, including transfer bus, bus sectionalizing reactors, and feeder reactors.

reactor shown permit any bus section to be taken out of service without a change in operating conditions.

Figure 137[1] shows the total short-circuit current, in per cent of the short-circuit current from one generator, for various ratios of bus reactance to generator reactance for any number of bus sections. All generators are assumed to have the same reactance

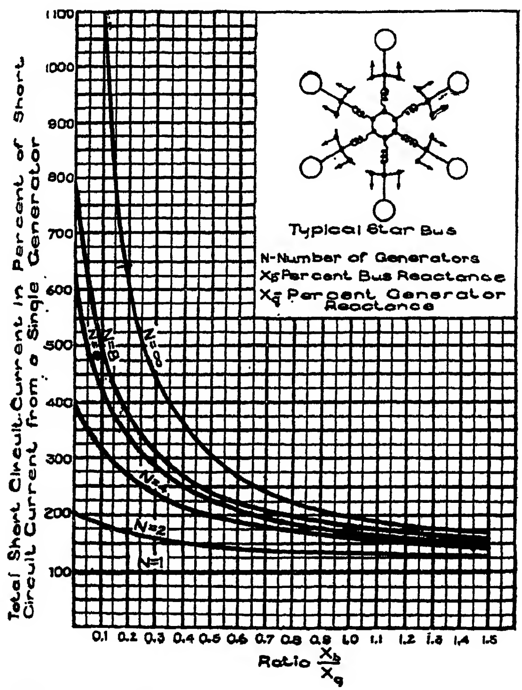


FIG. 137.—Effect of bus-bar reactors on reduction of short-circuit current. Star-bus construction.

and capacity, and only one generator is connected to each bus section.

120. Bus System with Double-winding Generators.—The phenomenal increase in the capacity of the individual alternator units recently installed in the larger power stations would have presented an urgent need for very large bus reactors and switch gear of tremendous interrupting capacity if arranged as in the old bus systems. In order to keep the generator currents of these

[1] From *Gen. Elec. Co. Bull.* A-1116, Current Limiting Reactors.

huge units within the capacity of the disconnecting switches and circuit breakers so far developed, the machines were designed with two windings. These are completely independent, being arranged in alternate slots and so connected under the poles that the terminal voltage of the corresponding phase of each winding

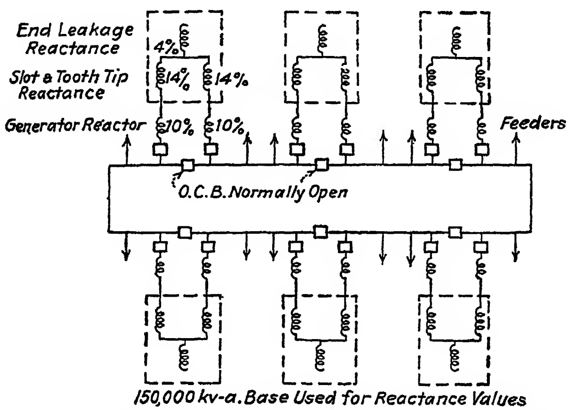


FIG. 138.—Diagram of bus system with double-winding generators.

is in phase with and numerically equal to that of the other winding. By using the double winding with the bus arrangement shown in Fig. 138, it is possible to eliminate the bus reactors because there is normally no electrical connection between the bus sections. Any transfer of power from one bus section to another must be made through the two generator circuits. Thus there will be the series resistance of these two circuits interposed between the bus sections with a corresponding reduction in short-circuit current on either bus. Since the two windings are in the same structure, energized from the same magnetic field, they cannot fall out of step, and the reactance introduced between bus sections does not reduce the system stability. Not only does the double-winding generator eliminate the bus-section reactor but it halves the ampere capacity of the circuit breaker and materially reduces its necessary interrupting capacity.

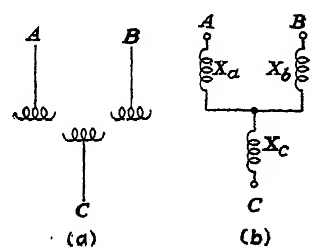


FIG. 139.—Equivalent circuit reactance of three-winding transformer.

The characteristics of the double-winding generator are calculated by the "equivalent-circuit theory" given by A. Boyajian in his paper, "Theory of Three-circuit Transformers."[1] Figure 139*b* shows the equivalent circuit for one phase of the three-circuit transformer of Fig. 139*a*. Then the reactances of the equivalent circuit are

$$X_{AC} = X_a + X_c \tag{146}$$
$$X_{BC} = X_b + X_c \tag{147}$$
$$X_{AB} = X_a + X_b \tag{148}$$

so that

$$X_a = \left(\frac{X_{AB} + X_{AC} - X_{BC}}{2}\right) \tag{149}$$
$$X_b = \left(\frac{X_{AB} + X_{BC} - X_{AC}}{2}\right) \tag{150}$$
$$X_c = \left(\frac{X_{AC} - X_{BC} - X_{AB}}{2}\right). \tag{151}$$

If sources A and B both feed into a short circuit at C and we neglect the resistance, the short-circuit reactance may be written, instead of the impedance, as

$$X_{\text{short}} = \frac{1}{\dfrac{1}{X_a} + \dfrac{1}{X_b}} + X_c \tag{152}$$

which may be written in the form

$$X_{\text{short}} = \left(\frac{X_{AC}X_{BC} - X_C^2}{X_{AB}}\right). \tag{153}$$

The short circuit then is

$$I_{\text{short}} = \left(\frac{100 I_{\text{normal}}}{X_{\text{short}}}\right), \tag{154}$$

and the share of A and B in this short circuit will be, respectively,

$$A\text{'s share} = \frac{X_b}{X_{AB}} \cdot I_{\text{short}} \tag{155}$$
$$B\text{'s share} = \frac{X_a}{X_{AB}} \cdot I_{\text{short.}} \tag{156}$$

[1] *A.I.E.E. Trans.*, February, 1924.

The curves of Fig. 140[1] show the reduced short-circuit kilovolt-amperes for the double-winding ring bus as compared with the kilovolt-amperes for a corresponding single-winding ring and single-winding star bus, for various number of generators up to 10. The percentages of the reactances used are shown in the diagrams of connections. Each section is supplied by a conventional type 100,000-kva. alternator. It is to be noted that for

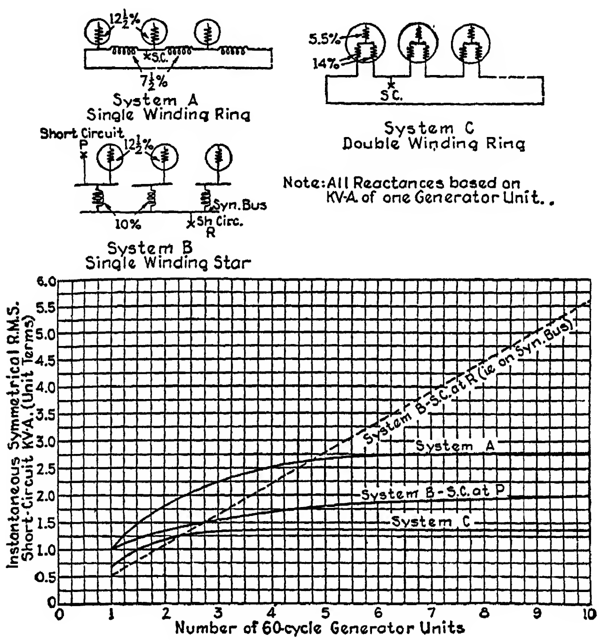


FIG. 140.—Diagrams of connections and short-circuit kilovolt-amperes for three typical bus systems.

system A the short-circuit kilovolt-amperes increase up to a total of six alternators on the bus. In system B, the short-circuit kilovolt-amperes on the feeder sections are less than in system A, and they increase gradually with an increase in the number of alternators on the bus. The short-circuit kilovolt-amperes on the synchronizing bus increase rapidly with addition of alternators. System C has much lower short-circuit kilovolt-amperes than either system A or B, and they increase only up

[1] From BARTON, T. F., The Double-winding Generator, *Gen. Elec. Rev.*, June, 1929.

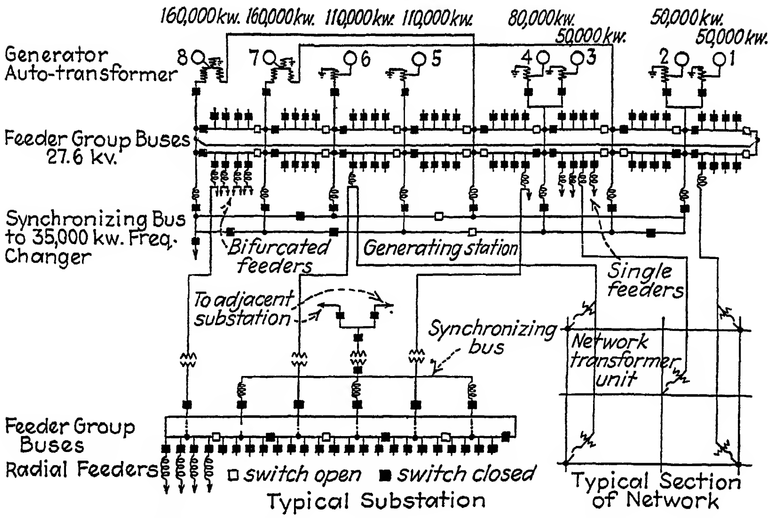


FIG. 141.—Single-line diagram showing Hudson Avenue Station, Brooklyn Edison Company, with typical substation and section of the network. (*Customer Service Determines System Development*, *H. R. Woodrow*, *Elec. World*, *May* 21, 1932.)

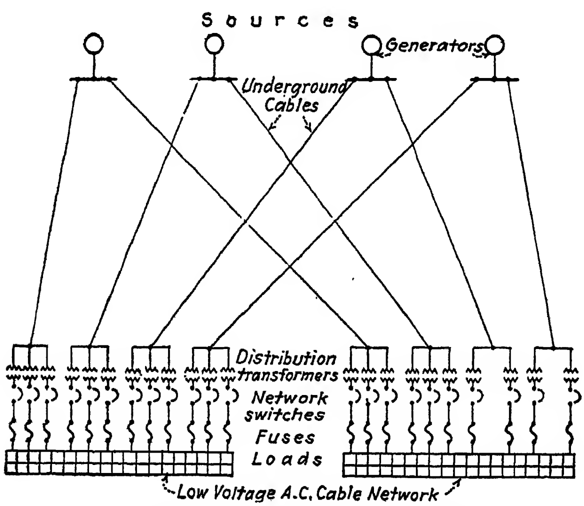


FIG. 142.—Schematic diagram of system with generators synchronized at the load. (*Jour. A.I.E.E.*, *August*, 1929.)

to four alternators on the bus. In addition, whereas synchronizing must be done through the included reactance of the bus in systems A and B, there is no included reactance for synchronizing in system C. Generators 7 and 8 at Hudson Avenue Station, Fig. 141, typify the use of the double winding in large plants.

121. Synchronization at the Load.[1]—This new type of system has been developed for the New York 60-cycle metropolitan load. A schematic diagram of the system in use with the low-voltage distribution network is shown in Fig. 142. The arrangement provides high reactance for the synchronizing flow between the generators but normal reactance for the energy flow between the generators and the loads. Where the distribution is entirely networked, the $^{120}\!/_{208}$-volt underground mains are used as a short-circuit-proof paralleling bus for all the units that are synchronized at the load. Where sufficient capacity in networks is not available, the low-voltage buses of step-down transformer substations are added in parallel. Thorough reliability of system supply is obtained by having sufficient generating sources synchronized at the load with ample synchronizing power between the units. The latter must be able to maintain stable operation between all sources for any single fault without respect to location, provided the fault is eliminated promptly by the protective equipment. If enough reserve capacity is in operation, the loss due to a single fault will not cause an interruption, since one fault cannot affect all the generating capacity. The system offers a material reduction in the magnitude of fault currents and in the voltage disturbances due to system faults. The disturbances caused by faults on a high-capacity feeder have shown an average voltage drop of only 9 per cent for the system synchronized at the load as compared with 20.7 per cent for the old system synchronized at the generating-station busses.

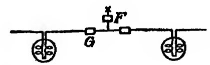
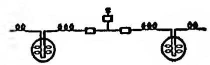
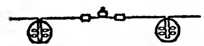
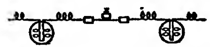
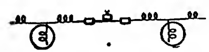
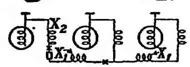
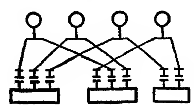
A comparison of the kilovolt-ampere concentrations for this system and other standard systems is given in Table 27.[2]

[1] See Kehoe, Griscom, Seering, and Milne, Synchronized at the Load, *A.I.E.E., Trans.*, October, 1929; Powers and Kilgore, Developments in Generators and Systems, *Elec. Jour.*, October, 1929.

[2] From Powers and Kilgore, Generator and System Design, *Elec. Jour.*, October, 1929.

TABLE 27.—DATA FROM SHORT-CIRCUIT STUDIES MADE WITH DIFFERENT TYPES OF WINDINGS

Three-phase fault on bus section; instantaneous short-circuit values based on 9-100,000 kva.—13,800-volt generators, 13.4 per cent leakage reactance

System and constants	Single line diagrams	Comments	Interrupting capacity of breaker instantaneous short-circuit kva.		Percentage voltage maintained in first instant	
			Feeder	Generator	First bus	Second bus
1. Double-winding generator, no reactors		Saturation neglected	922,000	461,000	90.0	99.6
2. Same		Saturation considered	1,966,000	983,000	41.3	78.9
3. Double-winding generators with 10 per cent reactors		Saturation neglected	627,000	314,000	92.8	99.7
4. Same		Saturation considered	766,000	383,000	81.3	98.7
5. Double-winding generators alternate pole windings in series		Full pitch ⅔ pitch	2,320,000 1,060,000	1,160,000 530,000	48.0 97.5	74.0 99.5
6. Same as 5 with 10 per cent reactors		Full pitch ⅗ pitch	900,000 690,000	450,000 345,000	72.5 98.3	92.0 99.9
7. Ordinary generators 10 per cent reactors			1,040,000	520,000	68.4	90.0
8. Ordinary generators with 20 per cent bus reactors			1,433,000 Synchronizing bus	746,000 Main bus	68.6	90.2
9. Standard generators with synchronizing bus		$X_1 = 0, X_2 = 10$ $X_1 = 5, X_2 = 10$ $X_1 = 20, X_2 = 20$	3,500,000 2,350,000 1,500,000	1,500,000 1,375,000 1,000,000		
10. Standard generator synchronized at the load. No machine reactors; reactance from internal voltage of one unit to all other units in parallel = 60 per cent, Case 1; 40 per cent, Case 2		Case 1 60 per cent Case 2 40 per cent	982,000 1,140,000	770,000 770,000	96.4 93.9	96.4 93.9
11. Same as 10 except 5 per cent machine reactors		Case 1 60 per cent Case 2 40 per cent	788,000 1,005,000	550,000 550,000	96.0 92.5	96.0 92.5

121A. Survey of the Various Bus Designs.—The student may ask why there are such differences in design practice among power stations that perform practically equivalent functions. Electrical engineers have themselves asked the same question, and technical committees have studied the problem. An N.E.L.A. committee[1] circulated a questionnaire and asked for fundamental design data and the reasons for the practices followed in three types of stations: (1) stations supplying residential loads, (2) indoor switching stations for generating stations, and (3) outdoor switching stations for generating plants. The A.I.E.E. held a symposium on Electric Power Switching at the Winter Convention in 1934, the five papers presented covering modern large generating plants at Hudson Avenue Station (Brooklyn, 1922), Richmond Station (Philadelphia, 1925), Long Beach No. 3 Station (California, 1928), State Line Station (Chicago, 1928), and Essex Station (Detroit, 1932).[2] Figure 141 gives a one-line diagram of the bus system of Hudson Avenue Station. It is gratifying to note that the later design, Fig. 143, shows a marked improvement in simplification, having a simple single bus with one circuit breaker for each individual feeder.

The committee reported that the data from the questionnaire and the papers show a wide range in practice, some designers prefer metal-clad equipment, some use air-insulated equipment in cells, and others favor isolated phase construction or segregated phase arrangements. On one system, future residential stations will be entirely of outdoor construction, other engineers did not believe this was economical, and a third group intended to eliminate this class of station. Certain designers carefully segregated oil-filled equipment, whereas others did not.

Since each plan is expected to provide equally good service, the committee concludes that the reasons for the variations are: (1) the effect of local conditions and the fact that the station must fit into the plan of the system, (2) company policy, and (3) past experience. Of course, labor costs will differ widely so that factory-built equipment may be very attractive in some places but may not have that favorable margin in others. The committee appreciates the difficulty of overcoming the natural tendency of organizations to continue plans that have grown

[1] Station Design Subcommittee, October, 1932.

[2] See *Elec. Eng.*, January, 1934, p. 147.

out of past experience and local conditions, but points out that standardization of design and simplification of installation offer an opportunity for reduction of investment costs.

Upon the completion of several large new hydroelectric stations with their associated high-voltage transmission lines, the exten-

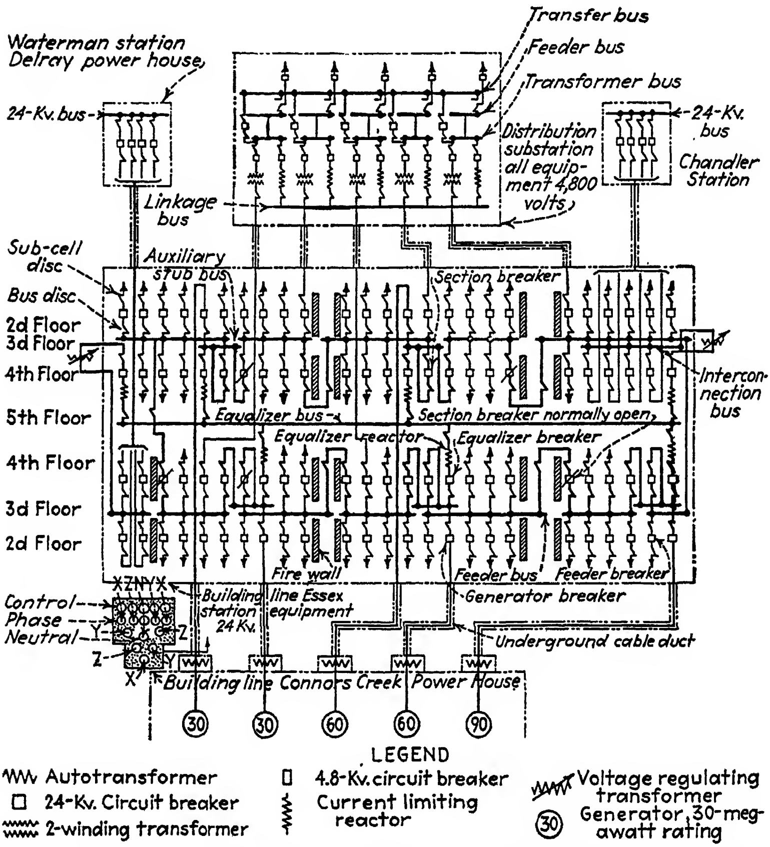


FIG. 143.—Single-line diagram of Essex Switching Station, Detroit Edison Company, with typical distribution substation. (*Switching at the Connor's Creek Plant, A. P. Fugill, Elec. Eng., January,* 1934.)

sion of many of the steam plants, and further interconnection of systems, new emphasis has been placed upon the switching. Old designs have become inadequate for the increased capacities in some cases, and several major interruptions of service have shown the need for reconstruction and modernization. The A.I.E.E. Committee on Power Generation held a session in January, 1939,

on the rehabilitation of the various stations of the Consolidated Edison Company (New York), Essex Station (Public Service, N. J.), L Street Station (Boston) together with a paper on the general topic. The papers described the problems in the stations, the aim of the reconstruction, and the methods to be used. The present trends in design were presented as follows:[1]

1. Complete relay protection.
2. Improved arrangement of equipment.
3. Limitation of capacity connected to one bus section, so that a fault will not remove too great a proportion of the station capacity.
4. Better physical segregation of bus sections and oil circuit breakers, by means of smoke and fire barriers, coupled with forced ventilation.
5. Reduction of oil content of breakers.
6. Use of improved fire-fighting equipment.
7. Improvement as to control, segregation, and grounding.

Since smoke and soot deposits on insulators and busses were responsible for prolonging the time and extent of recent interruptions, the new designs tend to reduce the oil content of circuit breakers, to provide barriers between indoor sections of the switch house and expansion space for the dissipation of explosions, and to furnish large-capacity ventilating systems to clear out fumes. Modern fire-fighting equipment of both the carbon-dioxide type and the fine-water-spray, or mist, type have been installed.

Metal-enclosed bus structures with metal-breaker cubicles have been used in a number of the new designs, and the Consolidated Edison Company has developed a nonmagnetic steel tube bus which is gastight.

122. Tie-line Reactors.—Reactors in tie lines between power stations or power systems are practically bus-sectionalizing reactors because the tie lines are extensions of the station busses. Since some of the present tie lines have transmitting capacities of 50,000 and 90,000 kva., the reactors may require a large current-carrying capacity. On account of the considerable length of the tie line, it is most likely that any faults which develop will be in the section between stations which will entail

[1] See Progress in Power Generation, *Elec. Eng.*, January, 1940.

power feed from each station to the fault. Hence the tie-line reactance should be in two parts, each half being placed as close to the station bus at each end as is permissible. With such an arrangement, the voltage drop over the reactors at rated current and normal power factor should not exceed about 5 per cent. For tie lines of overhead-line construction, the inherent reactance of the lines themselves may be sufficient.

123. Feeder Reactors.—Most of the faults on a power system occur on the feeders owing to their great length and exposure. Some idea of the number of these may be had by examination of *E.E.I. Publication* G-5, January, 1940, entitled "Cable Operation, 1938." Trouble rates per 100 miles of cable were 6.3 for all high-voltage cable, 1.7 for all high-voltage joints, and 0.6 for potheads. Approximately half of the cable troubles were assignable to four causes: initially defective sheath, deterioration in cable over 10 years old, sheath corrosion, and external mechanical damage. A small feeder reactor, in general not to exceed 3 per cent, will give great protection to the system in case of a fault. Since the reactor is rated on the maximum continuous capacity of the feeder, say 5,000 kva., the energy loss and voltage drop are generally not important, particularly as the reactive drop is displaced 90 deg. from the line current.

If the alternator capacity is 50,000 kva., the feeder reactor will represent $\frac{50{,}000}{5{,}000} \times$ say $3.0 = 30$ per cent reactance on the alternator base. This with the alternator reactance will curtail the current and localize the disturbance due to a short circuit on the feeder.

Figure 144[1] gives the equivalent short-circuit kilovolt-amperes resulting from feeder short circuits with various sizes of feeder reactors and for different station capacities. For this case, it is assumed that all the generators have 12 per cent reactance, that no bus reactors are used, and that the feeders are rated as of 5,000-kva. capacity.

Figure 145[1] compares the equivalent short-circuit kilovolt-amperes produced by feeder faults on a system without bus reactors and then with 12 per cent bus reactors. The generators

[1] LYMAN, PERRY, and ROSSMAN, Protective Reactors for Feeder Circuits of Large City Power Systems, *A.I.E.E. Trans.*, Vol. 33, Part 2.

are assumed to have 10 per cent reactance, and 3 per cent feeder reactors are used in the 5,000-kva. feeders. It is to be noted that with the bus reactors the equivalent short-circuit kilovolt-amperes for a feeder short circuit become constant at 125,000-

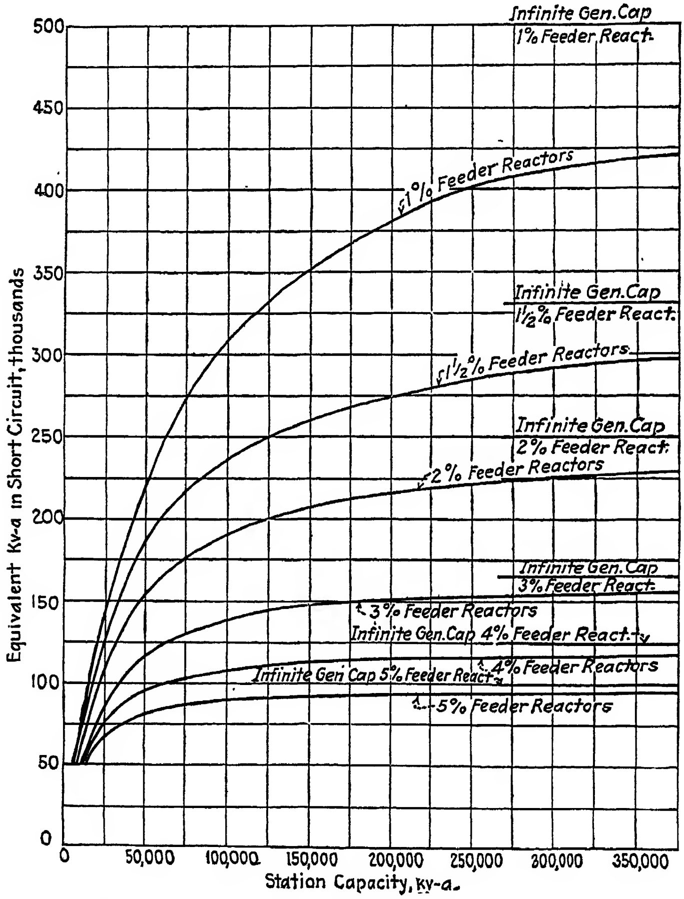


FIG. 144.—Short-circuit kilovolt-amperes on feeders with various percentages of feeder reactors and various station capacities. Inherent generator reactance 12 per cent without bus reactors; 5,000-kva. feeders.

kva. station capacity and do not increase for a growth in station capacity. Without the bus reactors, the short-circuit kilovolt-amperes increase to 167,000 kva.

Naturally, the addition of a reactor to the feeder circuit will have an effect on the voltage regulation, but for a good power

factor the change in the circuit regulation is surprisingly small. Suppose an extreme amount of 10 per cent reactance were to be

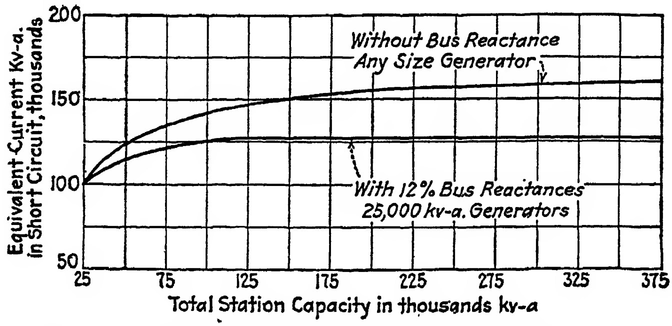


FIG. 145.—Short-circuit kilovolt-amperes with 3 per cent feeder reactors and 10 per cent generator reactance, with and without 12 per cent bus reactors; 5,000-kva. feeders.

added to a circuit of 90 per cent power factor. Let the load voltage be E_1 and $Z_1 = 1$ ohm; then $R = 0.9$ ohm and $X = 0.436$ ohm, as shown in Fig. 146. If we now add 10 per cent external reactance, then $X' = 0.1Z_2$. For the original circuit,

$$E_1 = IZ_1. \qquad (157)$$

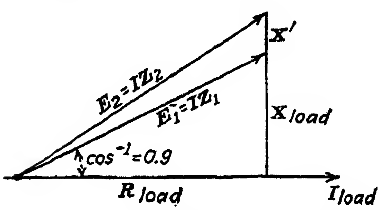


FIG. 146.—Voltage triangle for variation of feeder regulation for addition of external reactance, X'.

From the triangle of Fig. 146,

$$Z_2 = \sqrt{R^2 + (X + X')^2} \qquad (158)$$

or

$$Z_2 = \sqrt{0.9^2 + (0.436 + 0.1Z_2)^2}, \qquad (159)$$

i.e.,

$$Z_2^2 = 0.81 + 0.19 + 0.087Z_2 + 0.01Z_2^2 \qquad (160)$$

so that

$$Z_2 = 1.05, \text{ and } E_2 = IZ_2. \qquad (161)$$

$$\text{Then the variation of regulation} = \frac{E_2 - E_1}{E_1} \qquad (162)$$

$$= \frac{IZ_1(1.05 - 1)}{IZ_1} \qquad (163)$$

$$= 5\%.$$

Figure 147[1] shows the variation in the regulation of a feeder circuit of various power factors for varying per cents of feeder reactors.

124. Most Economic Reactor Losses.—Since the current-limiting reactors are connected in their circuits almost continuously, their losses must be considered. These will be important in the cases of feeder- and tie-line reactors carrying fairly heavy loads but not so much so for bus reactors since the section loads and the generator capacity will be kept approximately balanced. With the design pointed to keep the eddy-current losses low, the losses are but little more than the I^2R watts and average about 3 per cent of the rating of 60-cycle reactors and about 5 per cent for 25-cycle units. If it is desired to lower the

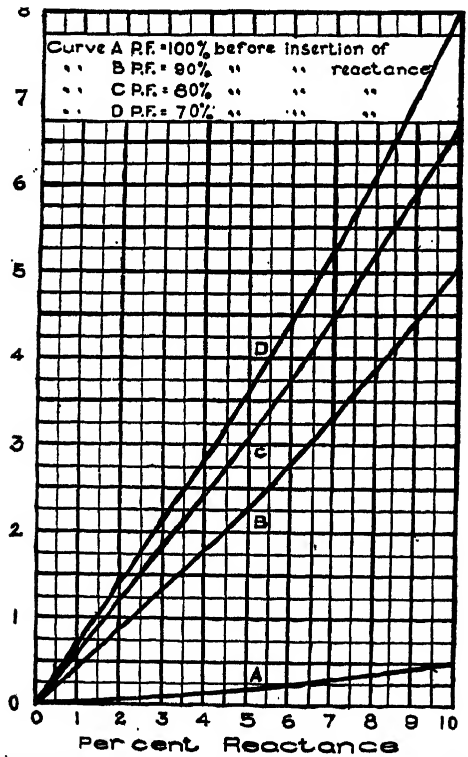


FIG. 147.—Percentage variation of the regulation of a feeder circuit with percentage external reactance.

[1] From *Gen. Elec. Co. Bull.* A-1116, Current Limiting Reactors.

reactor losses, it can be done by using larger conductors or putting more conductors in parallel. As against the decrease in operating cost for such procedure, there will be the increased fixed charges on the additional investment in conductors. The cost of supplying the kilowatt-hour losses in a reactor for the year may be evaluated by taking the reactor root-mean-square current for the year and the cost of energy per kilowatt-hour, or the load factor may be approximated as perhaps full load for 25 per cent of the year.

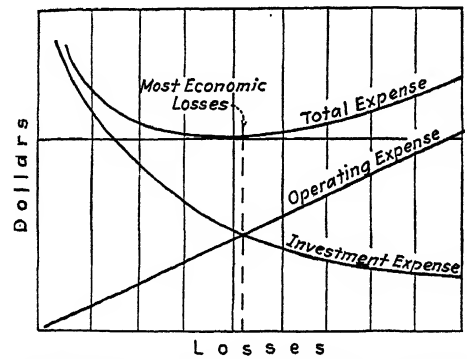


Fig. 148.—Curves showing most economical value of losses for reactors.

Just as our study of economic conductor section in Chap. V showed, we have here two conflicting factors, investment charges and operating expenses. These are shown in Fig. 148[1] together with the curve of total expense per year which is the sum of the ordinates of the other two. The minimum point on the total expense curve will determine the economical reactor. Outside of its effect on the amount of short-circuit protection provided, the question of conductor size, and hence reactor losses, may be analyzed as follows:

$$\text{Let the cost of the reactor, } \$ = A + B \text{ cir. mils} \tag{164}$$

where A = that part of the cost independent of size of the conductors.

B = the marginal cost per circular mil.

For f per cent annual charges on the investment,

$$\text{then the investment charge } \$ = f(A + B \text{ cir. mils}). \tag{165}$$

[1] See Kierstead and Stephens, Current-limiting Reactors, *A.I.E.E. Trans.*, June, 1924.

The cost of the annual operating losses,

$$\$ = \frac{kI_{\sqrt{\text{m.s.}}}^2}{\text{cir. mils}}. \tag{166}$$

Then total expense per year

$$\$ = f(A + B \text{ cir. mils}) + \frac{kI_{\sqrt{\text{m.s.}}}^2}{\text{cir. mils}}. \tag{167}$$

For a minimum annual total cost, take the first derivative with respect to circular mils

$$\frac{d\$}{d \text{ cir. mils}} = fB - \frac{kI_{\sqrt{\text{m.s.}}}^2}{\text{cir. mils}^2} = 0, \tag{168}$$

then

$$fB = \frac{kI_{\sqrt{\text{m.s.}}}^2}{\text{cir. mils}^2} \tag{169}$$

or multiplying each side by cir. mils,

$$fB \text{ cir. mils} = \frac{kI_{\sqrt{\text{m.s.}}}^2}{\text{cir. mils}}, \tag{170}$$

i.e. (Kelvin's law), for a minimum cost the investment charges on the marginal cost of the conductors will equal the cost of the losses.

125. Short Circuits for Power Stations in Parallel.—In addition to the determination of short-circuit kilovolt-ampere concentrations for given generator and reactor arrangements in individual power stations, as in the previous paragraphs, we must consider those resulting from a group of stations working in parallel as is customary in any large-sized public-utility system. In such a case, the reactance of transmission and tie lines connecting the stations and the reactance of their associated apparatus will all enter the problem. Section VIII, "System Fault Calculations," of the N.E.L.A. "Relay Handbook" gives a very complete treatment of this subject. R. F. Gooding, also, discusses the subject in detail in an article entitled, "Calculating Short-circuits on Power Systems," in the *Electrical World* for Oct. 18 and Nov. 15, 1919, based on a system of three stations, as shown in Fig. 149.

By application of the method illustrated in the solution for the single straight bus station of Fig. 133 in Sec. 117, for three-

phase symmetrical short circuits, the individual stations of Fig. 149 have the following reactance characteristics:

For a short circuit on Station A alone,

On bus section X = 100,000 kva. at 10% X. (171)
On bus section Y = 100,000 kva. at 8% X. (172)
On bus section Z = 100,000 kva. at 10% X. (173)

For a short-circuit on Station B alone,

On bus section M = 100,000 kva. at 8.8% X. (174)
On bus section N = 100,000 kva. at 10% X. (175)

For a short-circuit on Station C alone,

On bus section L = 100,000 kva. at 10.5% X. (176)
On bus section R = 100,000 kva. at 10.5% X. (177)

For the short-circuit characteristics of two power stations in parallel, say Stations A and B, it will be necessary to consider the

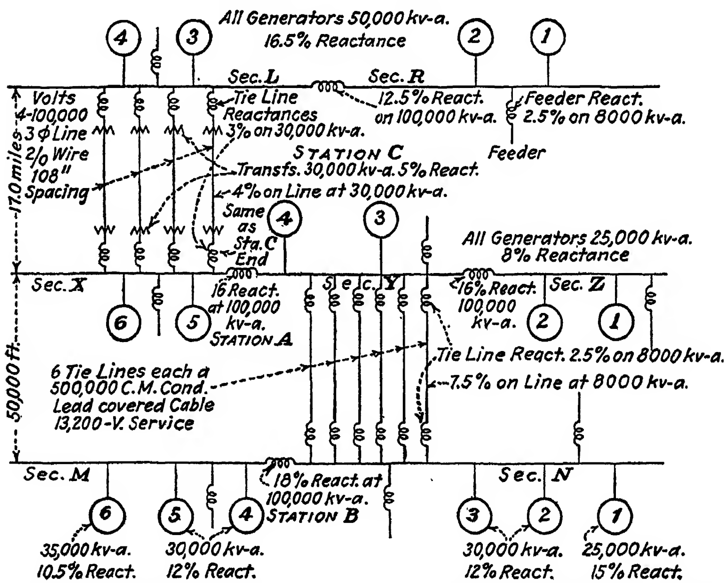


FIG. 149.—Diagram for short-circuit study of three power stations operating in parallel.

reactance of the six cable tie lines from bus section N of Station B to bus section Y of Station A since the entire contribution of

either station to a short on the other must pass over these lines. On the 8,000 kva. capacity shown, each tie has two 2.5 per cent tie-line reactors in series with 7.5 per cent reactance for the cable itself, making a total of 12.5 per cent per cable. Had the per

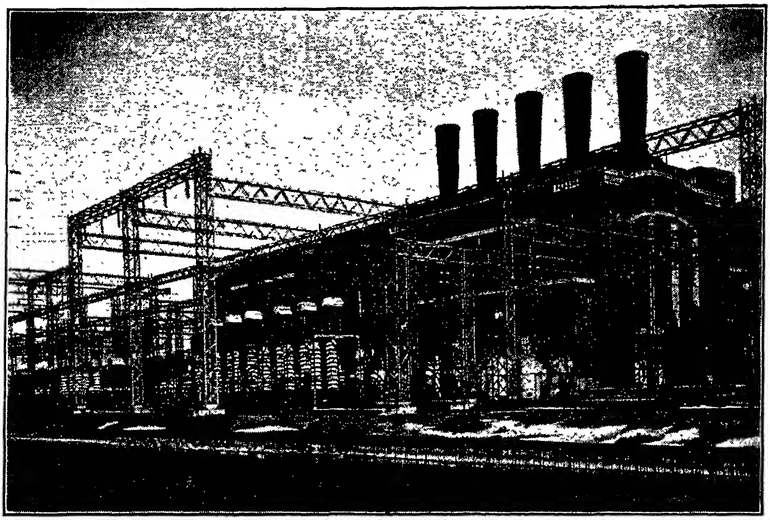


FIG. 150.—Trenton Channel Power Station, transformer and switchyard. Three single-phase, 21,000-kva., 12- to 120-kv. step-up transformers per generator. Duplicate 120-kv. bus. Oil circuit breakers are rated 132 kv., 400-amp. interrupting capacity, 1,500,000 kva. Plant output transmitted over six 120-kv. lines. (*Courtesy of Detroit Edison Company.*)

cent reactance not been given, it would have to be determined from the physical constants of the cable, as follows:

Assuming three-conductor paper and lead cable for 15,000 volts, the spacing center to center of the 500,000-cir. mil conductors would approximate 1.149 in. From General Electric Company Table 28, the single-phase inductive reactance per 1,000 ft. of cable conductors at 60 cycles would be

For 1-in. spacing, 500,000 cir. mils = 0.0286 ohm.
For 2-in. spacing, 500,000 cir. mils = 0.0445 ohm.

Interpolating, then for 1.149-in. spacing = 0.031 ohm.

On the basis of 8,000 kva. per tie line, the normal three-phase current is $8{,}000/(\sqrt{3} \times 13.2) = 350$ amp. For the total length then, the XI drop per conductor will be 350×0.031 ohm $\times\ 50 = 542.5$ volts, and the per cent reactance per cable will be

$$\frac{542.5}{(13{,}200/\sqrt{3})} = 7.1 \text{ per cent, approximately.}$$

If the short circuit is on bus section X of Station A with Stations A and B in parallel,

The total of Station B on Section N from Eq. (175) is		100,000 kva. at	10% X.	
The six tie lines in parallel, *i.e.*, 48,000 kva. at 12.5% X	=	100,000 kva. at	26% X.	(178)
The sum of these in series	=	100,000 kva. at	36% X.	(179)
or the power from Sta. B on to Sec. Y	=	22,000 kva. at	8% X.	(180)
From Sec. Z, gens. 1 and 2	=	50,000 kva. at	8% X.	(181)
Through bus reactor 100,000 at 16%	=	50,000 kva. at	8% X.	(182)
Or Sec. Z on to Y	=	50,000 kva. at	16% X.	(183)
	=	25,000 kva. at	8% X.	
Gens. 3 and 4 on Sec. Y	=	50,000 kva. at	8% X.	(184)
from Sta. B on Sec. Y, Eq. (180)	=	22,000 kva. at	8% X.	
Total on Sec. Y	=	97,000 kva. at	8% X.	(185)
	=	100,000 kva. at	8.25% X.	
Through bus reactor	=	100,000 kva. at	16 % X.	
On to Sec. X.	=	100,000 kva. at	24.25% X.	(186)
	=	32,900 kva. at	8% X.	
Gens. 5 and 6 on Sec. X in parallel 50,000 kva. at 8% X	=	50,000 kva. at	8% X.	(187)
Total on Sec. X,	=	82,900 kva. at	8% X.	
Stations A and B in parallel	=	100,000 kva. at	9.65% X.	(188)

Since the concentration on Section X for Station A alone was 100,000 kva. at 10 per cent X, from Eq. (171), there has been no appreciable increase in short-circuit capacity due to adding Station B so far as an increase of transmitted energy from Station A to Station C is concerned.

For Stations A and C in multiple, each connecting transmission line has two 3 per cent tie-line reactors, step-up and step-down

transformer reactances of 5 per cent, and its own line reactance of 4 per cent, or a total of 20 per cent per line. To calculate the per cent reactance from the line characteristics, the inductive reactance for a standard No. 00 conductor spaced 108 in. at 60 cycles is 0.798 ohm per mile.[1] The normal three-phase current for the 30,000-kva. rating of the line is

$$\frac{30{,}000}{\sqrt{3} \times 100} = 173 \text{ amp.}$$

$$\text{Then the } XI \text{ drop per conductor} = 17 \times 173 \times 0.798 = 2{,}350 \text{ volts.}$$

$$\text{And the per cent reactance} = \frac{2{,}350}{100{,}000/\sqrt{3}} = 4.07.$$

If the short-circuit is on bus section Y of Station A with Stations A and C in parallel,

Sta. C alone on Sec. L, from Eq. (176) is		100,000 kva. at 10.5% X.	
The 4 transmission lines in parallel, *i.e.*, 120,000 kva. at 20% X	=	100,000 kva. at 16.7% X.	(189)
The sum of these in series	=	100,000 kva. at 27.2% X.	(190)
Or the power from Sta. C on to Sec. X	=	29,400 kva. at 8% X.	(191)
Gens. 5 and 6, Sec. X, in parallel	=	50,000 kva. at 8% X.	
Total on Sec. X.	=	79,400 kva. at 8% X.	(192)
	=	100,000 kva. at 10.06% X.	
Through bus reactor	=	100,000 kva. at 16 % X.	
On to Sec. Y from X.	=	100,000 kva. at 26 % X.	(193)
	=	30,700 kva. at 8% X.	
Gens. 3 and 4 on Sec. Y	=	50,000 kva. at 8% X.	
Gens. 1 and 2 on Sec. Z through bus reactor on to Y	=	25,000 kva. at 8% X.	(194)
Total on Sec. Y	=	105,700 kva. at 8% X.	(195)
Sta. A and C in parallel	=	100,000 kva. at 7.6% X.	(196)

[1] From "Standard Handbook for Electrical Engineers," Sec. 14-58A.

Since the concentration on Section Y for Station A alone was 100,000 kva. at 8 per cent X, from Eq. (172), there has been no appreciable increase in short-circuit capacity due to adding Station C so far as an increase of transmitted energy from Station A to Station B is concerned.

For Stations A, B, and C in parallel, by application of the method used above, the short-circuit concentrations on the various bus sections are found to be as follows:

For a fault on Station A:

On bus Sec. X = 100,000 kva. at 7.1% X. (197)
On bus Sec. Y = 100,000 kva. at 6.25% X. (198)
On bus Sec. Z = 100,000 kva. at 9.55% X. (199)

It will be noted that these capacities represent a considerable increase for Station A over the values for the station alone, as given in Eqs. (171), (172), and (173), or for the values for Station A in parallel with Station B or Station C, as given in Eqs. (188) and (196), respectively.

Similarly, for all three stations operating in parallel, the other bus sections would have short-circuit concentration values

Sta. B:

On Sec. N = 100,000 kva. at 7.7% X. (200)
On Sec. M = 100,000 kva. at 8.4% X. (201)

Sta. C:

On Sec. L = 100,000 kva. at 7.5% X. (202)
On Sec. R = 100,000 kva. at 9.55% X. (203)

Figure 151 shows the foregoing results for the entire system in parallel and represents a key-reactance diagram. This shows plainly where the heaviest short-circuit concentrations occur and hence will be a determining factor in selecting the oil circuit breakers and in designing the bus systems.

126. Forces on Bus Bars.—From a consideration of Section Y in Fig. 151, it is seen that if a bus short circuit were mechanically possible it would give 70,000 amp. at 13,200 volts as a symmetrical three-phase short-circuit from Eq. (198). However, as will be developed in the discussion of alternator short circuits, in Sec. 141, for the first few cycles the short may be entirely

unsymmetrical and reach a value twice that for symmetrical short circuits. For the bus design, the double value means four times the stress on supports that would result from a symmetrical short. Therefore, the bus should be designed on a basis of 140,000 amp. With a spacing of 18 in. between busses

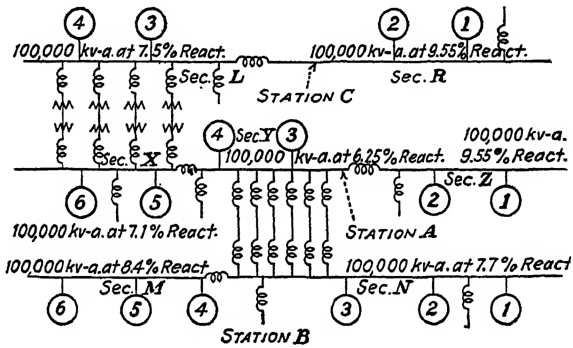


FIG. 151.—Reactance diagram showing short-circuit concentrations, three stations in parallel. (*Gooding, "Short Circuits on Power Systems," Elec. World, Nov.* 15 *and* 22, 1919.)

arranged in a plane, the force per foot of bus on an outer bus would be

$$F = \frac{140{,}000^2 \times 10^{-7} \times 4.04}{18} = 440 \text{ lb.} \tag{204}$$

For, considering the field intensity at a distance of A cm. around a long straight conductor carrying a current of I absolute units, if H is the field intensity, the work done in moving a unit magnetic pole around the wire at a distance of A centimeters from it against the force H is

$$\text{Work} = 2\pi AH \text{ ergs.} \tag{205}$$

The work done is equal to the product of the current and the flux cut, therefore,

$$2\pi AH = 4\pi I \tag{206}$$

and

$$H = \frac{2I}{A} \text{ dynes.}^1 \tag{207}$$

[1] See CHRISTIE, C. V., "Electrical Engineering," Eq. (91), McGraw-Hill Book Company, Inc.

In air, the flux density at distance A is $B = H = (2I)/A$ lines per square centimeter. Now for parallel conductors carrying currents I_1 and I_2, the flux density at wire 2 (Fig. 152a), produced by current I_1, is

$$B_1 = \frac{2I_1}{A}. \tag{208}$$

This field acts on wire 2 with a force

$$f_{1-2} = B_1 I_2 = \frac{2I_1 I_2}{A} \text{ dynes per centimeter length.} \tag{209}$$

If the currents are in the same direction the force will be attraction, if they are in opposite directions the force will be repulsion.

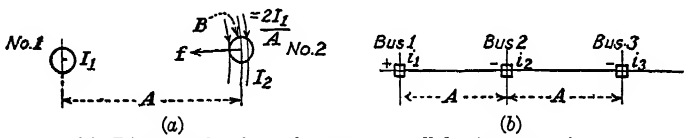


FIG. 152.—(a) Diagram for force between parallel wires carrying current; (b) diagram for three-phase bus in one plane.

For a three-phase bus-bar system with the busses all in one plane as in Fig. 152b, the force on bus 1, for a symmetrical three-phase short for the instant when bus 1 is, say, maximum positive and busses 2 and 3 are negative, would be a repulsion away from the center bus, and would be the sum of

$$f_{1-2} = \frac{2i_1 i_2}{A} \text{ and } f_{1-3} = \frac{2i_1 i_3}{2A} = \frac{i_1 i_3}{A}. \tag{210}$$

But

$$i_1 = I_m \sin \omega t,\ i_2 = I_m \sin\left(\omega t + \frac{2\pi}{3}\right),\ i_3 = I_m \sin\left(\omega t + \frac{4\pi}{3}\right) \tag{211}$$

and owing to the inertia of the bus system it will be affected by the average force during a cycle. Then the average force

$$F_1(\text{dynes}) = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{2i_1 i_2}{A} + \frac{i_1 i_3}{A}\right) d\omega t. \tag{212}$$

Substituting from Eq. (211),

$$F_1 = \frac{I_m^2}{2\pi A}\int_0^{2\pi}\left[2 \sin \omega t \sin\left(\omega t + \frac{2\pi}{3}\right) + \sin \omega t \sin\left(\omega t + \frac{4\pi}{3}\right)\right] d\omega t. \tag{213}$$

Expanding

$$F_1 = \frac{I_m^2}{2\pi A}\int_0^{2\pi}\left[2\sin\omega t\left(\sin\omega t\cos\frac{2\pi}{3} + \cos\omega t\sin\frac{2\pi}{3}\right)\right.$$
$$\left. + \sin\omega t\left(\sin\omega t\cos\frac{4\pi}{3} + \cos\omega t\sin\frac{4\pi}{3}\right)\right] d\omega t \quad (214)$$

$$F_1 = \frac{I_m^2}{2\pi A}\int_0^{2\pi}\left[2\sin\omega t\left(-\frac{1}{2}\sin\omega t + \frac{\sqrt{3}}{2}\cos\omega t\right)\right.$$
$$\left. + \sin\omega t\left(-\frac{1}{2}\sin\omega t - \frac{\sqrt{3}}{2}\cos\omega t\right)\right] d\omega t \quad (215)$$

$$F_1 = \frac{I_m^2}{2\pi A}\int_0^{2\pi}\left(-\frac{3}{2}\sin^2\omega t + \frac{\sqrt{3}}{2}\sin\omega t\cos\omega t\right) d\omega t \quad (216)$$

$$F_1 = \frac{I_m^2}{2\pi A}\left[-\frac{3}{2}\left(\frac{\omega t}{2} - \frac{1}{4}\sin 2\,\omega t\right) + \frac{\sqrt{3}}{2}\left(\frac{1}{2}\sin^2\omega t\right)\right]_0^{2\pi} \quad (217)$$

$$F_1 = \frac{I_m^2}{2\pi A}\left(-\frac{3\pi}{2}\right) = -\frac{3}{4}\frac{I_m^2}{A}\text{ dynes} = -\frac{3}{2}\frac{I^2}{A}\text{ r.m.s. dynes.} \quad (218)$$

When F is desired in pounds per foot of bus with I in amperes and A in inches, then the conversion constant

$$k = \frac{30.48}{4.448 \times 10^5 \times 2.54 \times 100} = 2.7 \times 10^{-7} \quad (219)$$

then,

$$F_1(\text{pounds}) = 4.04\,\frac{I^2}{A} \times 10^{-7}\text{ per foot of bus.} \quad (220)$$

Thus for the short on Section Y, Fig. 151, with 140,000 amp., $F_1 = 440$ lb. per foot of bus. Thus on such a short circuit, the bus bar will give a heave of about half a ton against its support, repeated each cycle but diminishing in intensity with the decrement characteristic of the current wave. The flat oval springs mounted between the bus clamps and the supporting insulators are very effective in taking up these mechanical surges.

Since bus 3 is similarly located with regard to bus spacing with bus 1, and current conditions during a cycle are the same, it will experience the same average force as found in Eq. (220) for bus 1. Considering the forces on bus 2, we can see that they must total zero on the average owing to the similarity of its position with respect to the outside busses, and that the currents in all busses are assumed equal.

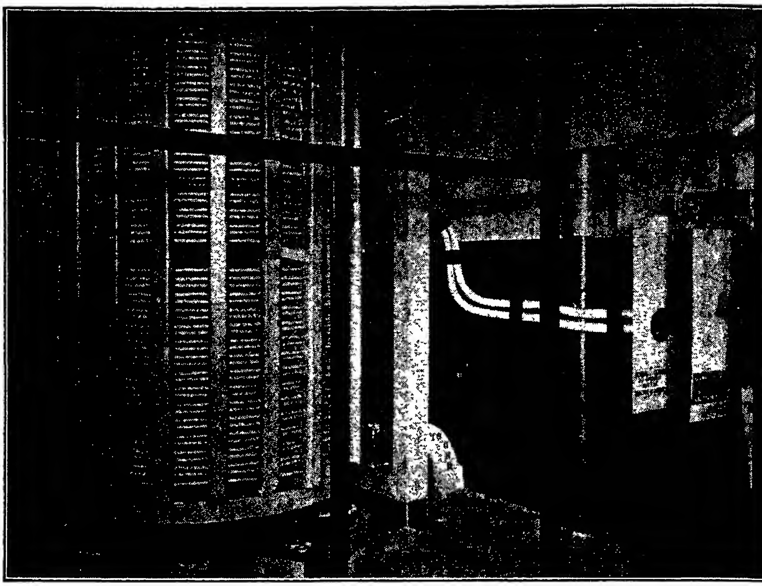


FIG. 153.—General Electric type CLS-25, 80,000-kva., 8,000-volt, 10,000-amp. neutral-grounding reactor in system of Consolidated Gas, Electric Light and Power Co., Baltimore, Md.

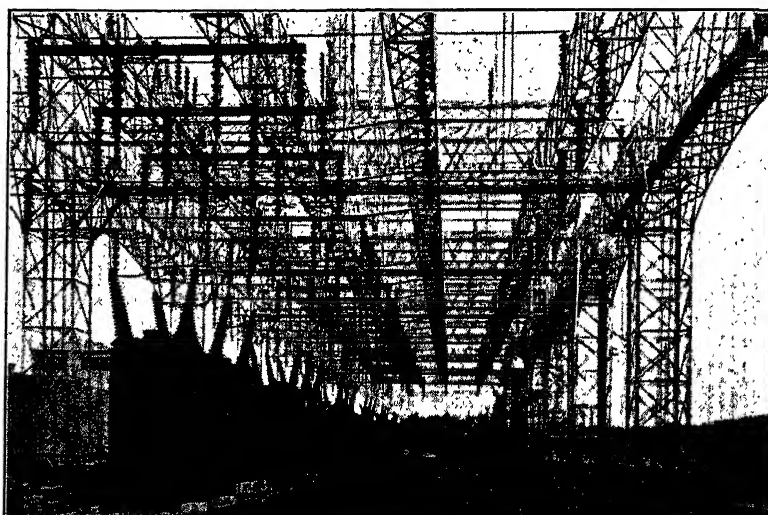


FIG. 154.—Conowingo Hydro-electric Station, general view of 220-kv. installation on roof. Two generators tie together for an 80,000-kva. transformer bank, 13.8 to 220 kv. Plant paralleled on 220-kv. bus. Reserve breakers provided for lines. (*Courtesy of Philadelphia Electric Co.*)

127. Problems.

1. In Fig. 155, the generators are three-phase, 20,000-kva., 11,000 volts, 60 cycles, each of 20 per cent reactance. A, B, and C bus reactors are each 10 per cent on 20,000 kva. The step-up transformers raise the voltage to 66,000. D has 10 per cent reactance on 20,000 kva., E has 10 per cent

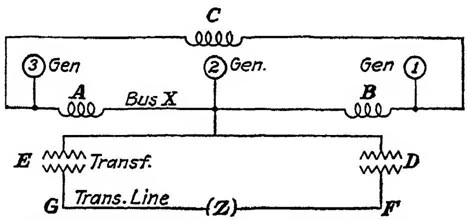


FIG. 155.—Diagram of generator bus and transmission system. Fault at Z, Problem 1.

reactance on 10,000 kva. The transmission lines F and G, 30 miles long, have each 21.75 ohms reactance.

There is a three-phase short circuit at the substation at Z.

Find the amperes delivered there.[1]

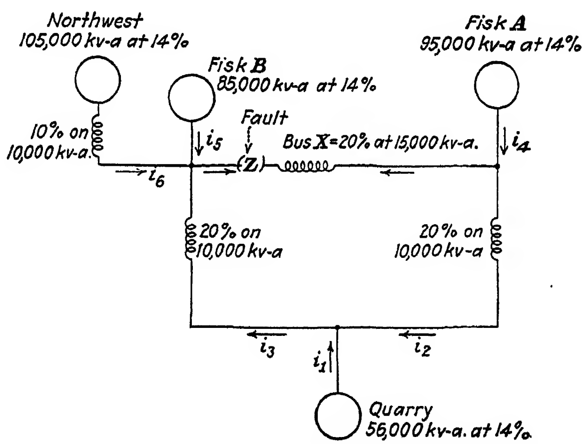


FIG. 156.—Diagram of generator buses and tie lines. Fault at Z, Problem 2.

2. Suppose the layout of Fig. 156 represents a 25-cycle power system with total kilovolt-amperes of 341,000. The system is three-phase at 11,000 volts. In case there is a three-phase fault on the bus of Fisk B at Z, what will be the kilovolt-amperes at the fault, assuming that full voltage is maintained?[2] The circuit may be simplified by using a mesh-star transformation.

[1] From N.E.L.A. "Relay Handbook," Fig. 711.

[2] From *Elec. World*, June 19, 1926.

3. In the network of Fig. 157, the reactances of generators and transformers are given in per cent on their own kilovolt-ampere base. The

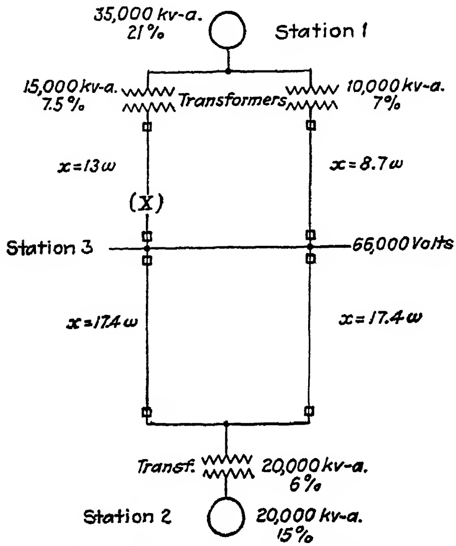


FIG. 157.—Diagram of network with fault at X, Problem 3.

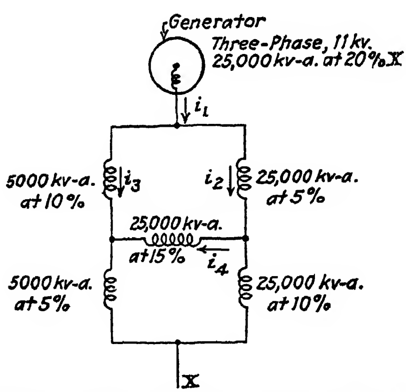


FIG. 158.—Diagram of simple bridge network with fault at X, Problem 4.

reactances of the transmission lines are ohms per conductor. Assume a three-phase short circuit at X:

a. Reduce all the reactances to a basis of 20,000 kva.

b. Find the total short-circuit kilovolt-amperes at X.[1]

[1] From LEWIS, W. W., "Transmission Line Engineering," p. 138, McGraw-Hill Book Company, Inc.

4. Solve the network of Fig. 158 for the total amperes and short-circuit kilovolt-amperes for a three-phase short circuit at the fault X.[1] Transform mesh to star.

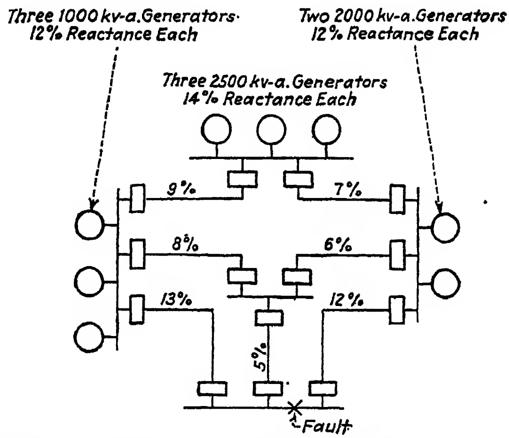


FIG. 159.—Diagram of a system network with fault at X. Three power stations in parallel, Problem 5.

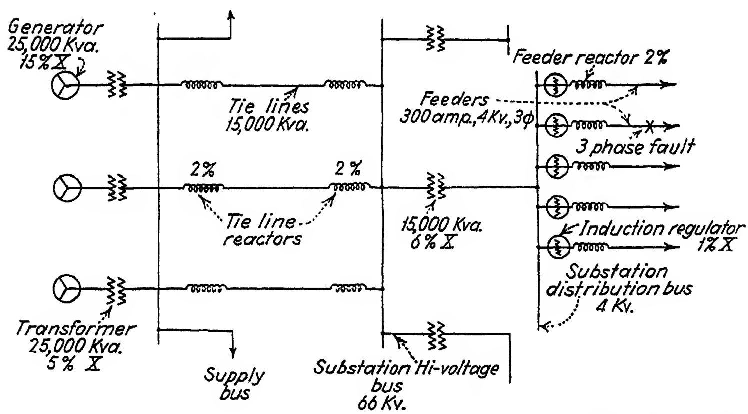


FIG. 160.—Diagram for fault on feeder circuit, Problem 6. (*Based on Gen. Elec. Bull. A*-1116.)

5. Solve the system network of Fig. 159 for the amperes and short-circuit kilovolt-amperes for a three-phase fault at X. All lines are rated on 14,500 kva. capacity. The system is three-phase at 13.2 kv.[2] Use successive transformations.

6. For a three-phase fault on the feeder as shown in Fig. 160:

a. Find the symmetrical r.m.s. amperes and kilovolt-amperes.

[1] Based on Fig. 703, N.E.L.A. "Relay Handbook."

[2] From the Question Box, *Elec. Jour.*, February, 1924.

b. If normal voltage is maintained on the substation high bus, find the per cent voltage on the distribution bus during the fault.

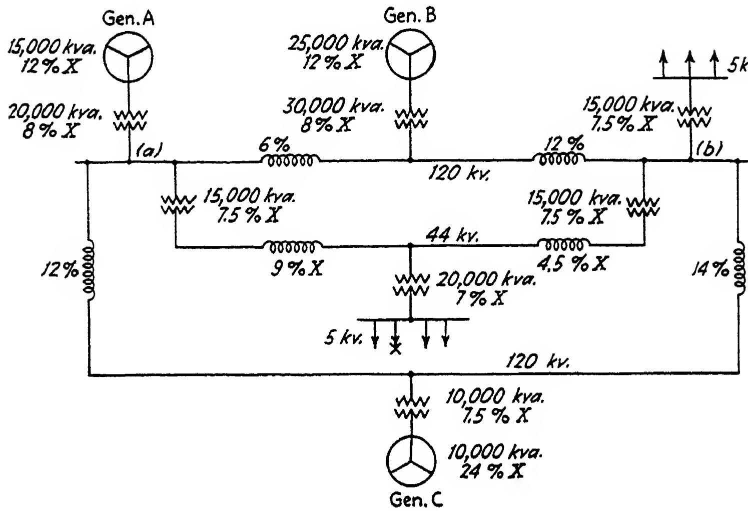


FIG. 161.—Diagram for network, Problem 7.

c. If normal voltage is maintained on the supply bus, find the per cent voltage on the distribution bus during the fault.

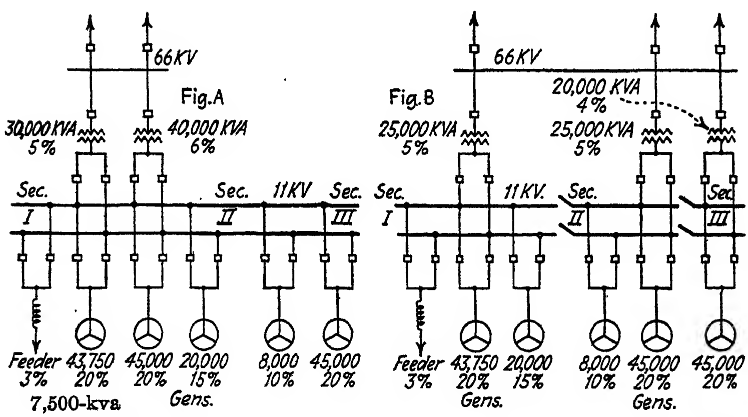


FIG. 162.—Bus layout, old and new arrangement, Problem 8. (*Elec. World*, *Dec.* 12, 1931, *p.* 1044.)

7. Solve the system network of Fig. 161 for the short-circuit kilovolt-amperes for a three-phase fault at X. The per cent reactance of all transmission lines is based on 50,000 kva. Use successive transformations.

8. Figure 162*A* shows the original bus at a steam station. Figure 162*B* shows the revised arrangement. Compare the short-circuit kilovolt-ampere concentrations (back-feed eliminated) in the two layouts, for

a. A three-phase fault on both busses, Sec. I.

b. A three-phase fault on an 11-kv., 7,500-kva. feeder, Sec. I.

c. A three-phase fault on the 66-kv. bus.

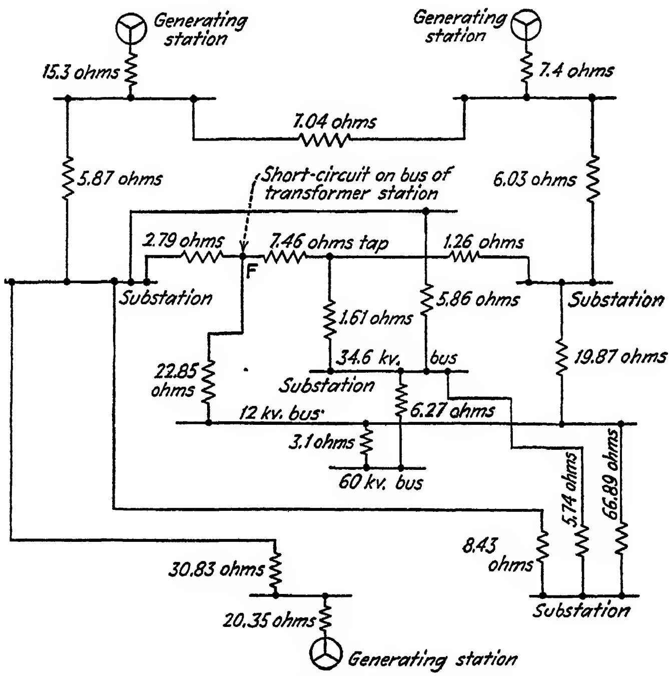


FIG. 163.—Diagram of network, fault at *F*, Problem 9. (*Elec. Jour.*, *October*, 1929, *p.* 492.)

9. Figure 163 shows a simplified positive phase-sequence reactance diagram for a high-voltage network. Calculate the short-circuit kilovolt-amperes for a three-phase fault at *F*. Use successive transformations. (From *Elec. Jour.*, October, 1929, p. 492.) All reactances are in ohms for 20-kv. line to neutral.

CHAPTER VIII

CIRCUIT BREAKERS AND THEIR APPLICATIONS

128. Definition.—Section 19-50 of the Standards of the A.I.E.E. defines a circuit-breaker as "a device for interrupting a circuit between separable contacts under normal or abnormal conditions. Ordinarily circuit breakers are required to operate only infrequently, although some classes of breakers are suitable for frequent operation." An oil circuit breaker is a circuit breaker "in which the interruption occurs in oil."

A circuit breaker may be (19–121) automatic tripping, *i.e.*, it may open under predetermined or other conditions without the intervention of an operator. This may be accomplished by

1. Low-voltage tripping from a trip coil connected in shunt to the main circuit and responsive to a decrease in the main circuit voltage.
2. Overvoltage tripping from a trip coil connected in shunt to the main circuit and responsive to an increase in the main circuit voltage.
3. Reverse power tripping upon reversal of power in the main circuit.
4. Series overcurrent tripping from a trip coil in series with the main circuit, responsive to an increase in the main circuit current.
5. Series undercurrent tripping from a trip coil in series with the main circuit, responsive to a decrease in the main circuit current.
6. Shunt tripping by a trip coil energized from the same or a separate shunt circuit or source of power, the trip coil circuit being closed through a relay, switch, or other means.
7. Transformer overcurrent tripping from a trip coil in series with the secondary winding of a current transformer whose primary winding is in series with the main circuit thus making the trip coil responsive to an increase in the main circuit current.
8. Transformer undercurrent tripping by a trip coil in series with the secondary winding of a current transformer whose

primary winding is in series with the main circuit thus making the trip coil responsive to a decrease in the main circuit current.

9. Transformer underload tripping (see 8).

129. The Necessity for a Circuit-interrupting Device.—Because of the constant increase in the capacity of modern power systems with centralization of the generation at large stations, the responsibility placed upon the switching elements is ever increasing, since the continuity of the major portion of the service and the safety of the connected apparatus depend absolutely upon their ability to open the circuits whenever it becomes necessary.

Under normal operating conditions, facilities must be provided, of course, to give complete control of the energy flow in the various parts of the power system. It may be desired to take a feeder line or generator off the bus or to change it from the main bus to the transfer bus, to drop off a transformer bank or open a transmission line, tie line, or bus section.

Under abnormal conditions, the excessive currents or overvoltage, etc., present in the circuits will operate interrupting devices directly or else flow through the relay transformers. In the latter event, the transformer currents or voltages will operate the protective relays, which will in turn energize the tripping coils of circuit-interrupting devices. When the latter perform, they will isolate some portion of the system on which a fault has occurred. The speed and efficiency with which this isolation is effected are important in preventing the spread of the trouble to other sections of the system adjoining the faulty section. This opening of the circuit at appropriate points will clear the trouble, and by confining the cutout to the minimum amount of line and apparatus possible, the tendency will be to hold the interruption to service to a minor part of the load. Thus complete resumption of supply is made easier and more rapid.

A recent development which offers material improvement in the continuity of service furnished on feeder lines is the ability to reclose immediately breakers that have opened because of a fault on the circuit. If instead of waiting the former customary period of 15 sec. or more, the breaker is reclosed as fast as the round trip from closed to open to closed positions can be made, approximately ½ sec., it has been the experience of operating companies that 80 to 90 per cent of the first reclosures are successful. In

such cases, motors will not shut down and the lighting may only blink; hence production will not be interrupted.[1] Treat and Verwoert give the time in cycles, on a 60-cycle basis for a closed-opened-closed cycle at 25 to 100 per cent interrupting rating of the breaker, utilizing a direct-current solenoid-operated mechanism, for standard breakers and relays as follows:[2]

	Type of oil circuit-breaker		
	FKO-60-C	FKR-255	FHKO-239
	Cycles	Cycles	Cycles
Protective relay time (occurrence of fault to energizing trip coil)......	1	1	1
Trip coil energized to interruption of current.....................	8	8	8
Interruption of current to reclosure of breaker.....................	21 to 26	21 to 26	31 to 36
Total time, from occurrence of fault to reclosure of breaker..........	30 to 35	30 to 35	40 to 45

As an example of this development, the duty cycle used by the Georgia Power Company is 0–15–120 sec. and Electric Bond and Share Company uses 0–45–120 sec.

The same idea has been extended to ultra-high-speed closing of important high-voltage lines because of the performance of eight breakers on 132-kv. lines in Indiana where 13 out of 15 reclosures were made without untoward incident.[3] Figure 164 shows an automatic reclosing breaker for outdoor high-voltage service.

130. Major Requirements of an Interrupting Device.—From the operating point of view, the interrupting device must open the circuit with safety to the operators and equipment and with no visible arcing which might ignite vapor to cause fire or explosion. It is important that the device occupy only minimum space, that it be constructed of simple mechanical elements easily repaired, and that it be easy of adjustment and control.

[1] See LOGAN, J. T., Faster Reclosing Breakers Needed, *Elec. World*, Apr. 21, 1934, p. 571, also *Gen. Elec. Rev.*, April, 1934, p. 162.

[2] See Service Continuity Promoted by Rapid Breaker Reclosure, *Elec. World*, Aug. 6, 1932, p. 172.

[3] See Experience with Ultra-high Speed Reclosing on High-voltage Transmission Lines, by Sporn and Muller, *A.I.E.E.*, January, 1939.

131. How the Oil Circuit Breaker Fulfills These Requirements. The oil circuit breaker has usually been used to open large amounts of alternating-current power and to control high-voltage alternating-current circuits. The distinctive feature of the oil circuit breaker is that it terminates the alternating-current wave at its normal zero value, at which time the magnetically stored energy of the system is a minimum. This

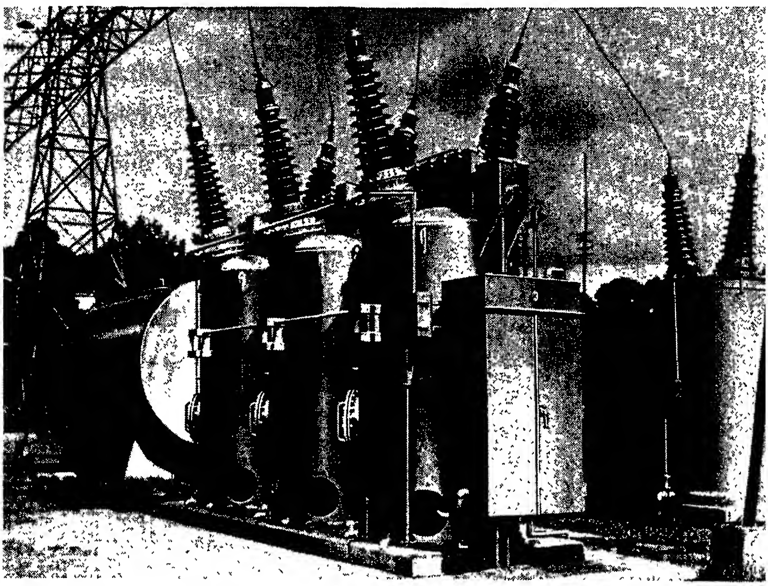


FIG. 164.—Allis-Chalmers automatic reclosing oil circuit breakers with ruptors, type BZO-60-115F, 600 amp., 115 kv.

termination, therefore, does not cause excessive surges in the connected circuits. Physically, it embodies to a high degree the desirable construction features and the requisites of small space along with elimination of fire and noise. It has distinct advantages in these points over an interrupter working in the air, where unfavorable air currents may carry a high-voltage arc many feet and permit it to jump to other lines.

When the contacts separate under oil,[1] the resultant arc plays in a gas bubble formed by the decomposing oil. The size of the bubble depends upon the number of amperes flowing at the instant the contacts part and on the length of time that the arc is

[1] Circuit breaker oil is substantially decane, $C_{10}H_{22}$.

maintained. Although the arc thus appears to be playing in a chamber of gas of considerable size, and therefore should not be very different from arcs in the open, nevertheless, the volts per centimeter which can be interrupted are very much greater than for arcs in the open, amounting to several hundred volts per centimeter.[1] The principal cause of this great arc-interrupting

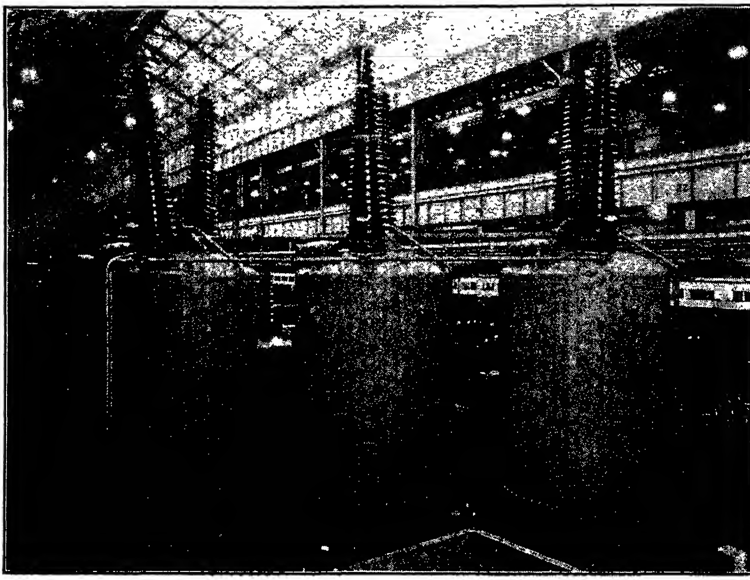


FIG. 165.—Westinghouse 220-kv., 600-amp., three-pole, solenoid-operated, outdoor oil circuit breaker set up for test.

capacity in the oil circuit breaker lies in the fact that drops of oil, or carbonized residues of such drops, float in the arc space and act as deionizing centers for the ions. These deionizing centers or nuclei consist of volumes of relatively cool, un-ionized gas, which came from the decomposition of the oil and mixed turbulently into the arc space. Thus the oil circuit breaker is a gas-blast switch, the gas blast arising from the decomposing oil. Therefore, the greater the rate of formation of gas and its thorough mixture with the ionized gas carrying the arc, the better the breaker.

The blowout magnetic field in the oil circuit breaker also increases the volts per centimeter which can be interrupted.

[1] See SLEPIAN, J., Extinction of a Long A-C Arc, *A.I.E.E. Trans.*, April, 1930.

It seems that the field may drive the greatest current density to one side of the bubble close up against the oil wall, thus increasing the rate of new gas formation to mix with the ionized gas and accelerate the extinction of the arc.

The growth of the gas bubble in the breaker works against the extinction of the arc because it puts the oil boundary away from most of the arc section and reduces the rate of oil decomposition.

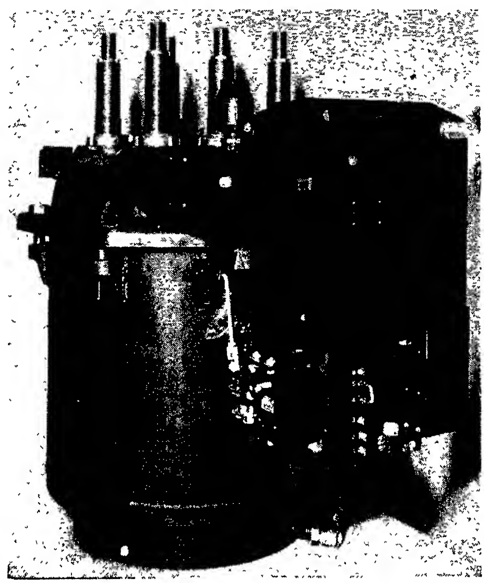


FIG. 166.—External view Westinghouse type B-28-B oil circuit breaker, 1,200 amp.

Any means, then, that will prevent the oil-gas bubble surface from receding from the arc will aid in the arc extinction.

132. General Description of Typical Oil Circuit Breakers. 1. *Pipe- or Frame-mounted Type.*—The Westinghouse type B circuit breaker, shown in Fig. 166, is a typical example of this class. It is designed for indoor service on circuits of moderate interrupting capacity, 100,000 to 500,000 kva. for the various sizes, and is especially suited to power plants, substations, and industrial plants with minimum space available for switching equipment. The B-28-BS and B-28-B are rated up to 2,000 amp. at 15,000 volts. Type B breakers may be operated either manually or electrically, with or without auxiliary features.

The essential feature of the breaker is the enclosing of all the phases in a single circular die-pressed or formed and welded-steel shell. The dome-shaped top allows space for an expansion chamber proportionate to the body of oil used. The maintenance is simplified in that only one tank is handled in inspections or in renewing the oil. The advantages of locating the

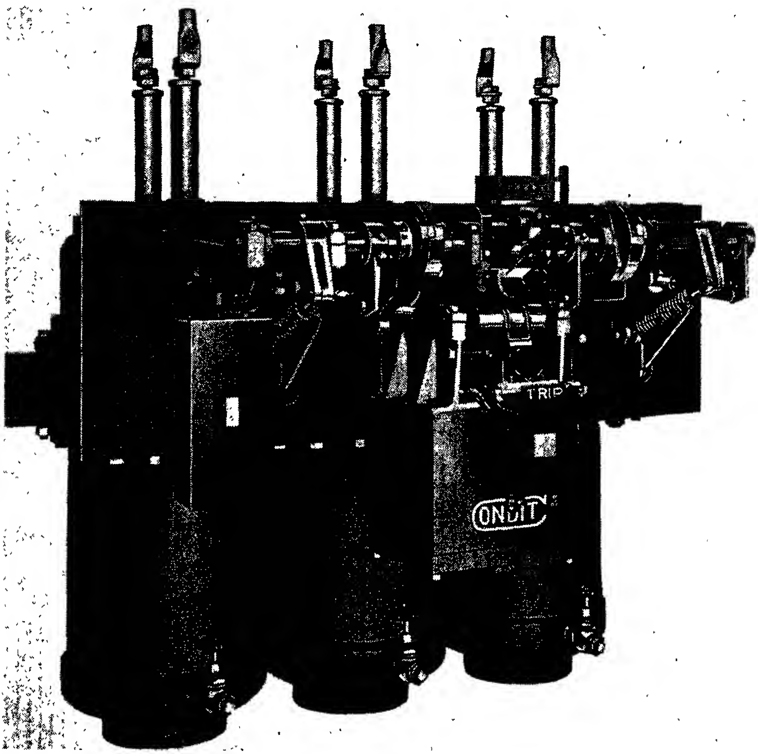


FIG. 167*a*.—Allis-Chalmers type FZ-220 oil circuit breaker, 8 cycles. Rated up to 4,000 amp., 34.5 kv., 2,500,000 kva. interrupting capacity.

operating levers inside the breaker chamber lie in the increase in clearance to grounded parts outside the breaker, as well as the elimination of an oiltight joint on the lifting rods and of the unbalanced forces on the rod which might retard the opening movement. All these breakers operate in 8 cycles.

2. *Cell Type.*—The Allis-Chalmers FZ-220 oil circuit breaker, as shown in Figs. 167*a* and *b*, is an example of this type. It is

intended for heavy-duty indoor service in central stations and on large distribution systems. It is rated according to E.E.I. standard steps for indoor breakers from 15 to 34.5 kv. The cylindrical steel tanks have rounded bottoms and are reinforced at the top by a deep-extending web on the frame. This breaker

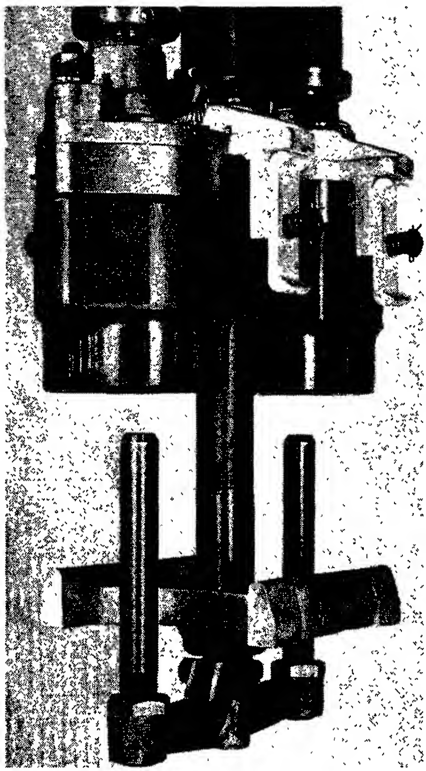


FIG. 167*b*.—Internal view of Allis-Chalmers type FZ-220 oil circuit breaker showing Ruptor contacts.

uses special Ruptor contacts in which the stationary element is of the tulip type consisting of four segments backed by springs to ensure ample contact pressure and mounted in a floating ring to provide self-alignment for the movable bayonet. Figure 167*b* shows the special contacts for this interrupting device.

3. *Outdoor High Voltage.*—The General Electric FHKO-239, shown in Fig. 168, illustrates this type. These breakers are

constructed of boiler-plate steel with domed tops and bottoms on round tanks. The high-interrupting-capacity units have oil-blast explosion chambers. The ratings range from 600 to 2,000 amp. at voltages from 7,500 to 230,000. They are solenoid operated. Above 73 kv., the breakers are floor mounted, owing to their great weight.

4. *Impulse Breaker.*—These breakers, like those of the FHKO type, are for outdoor, high-voltage service but are for faster

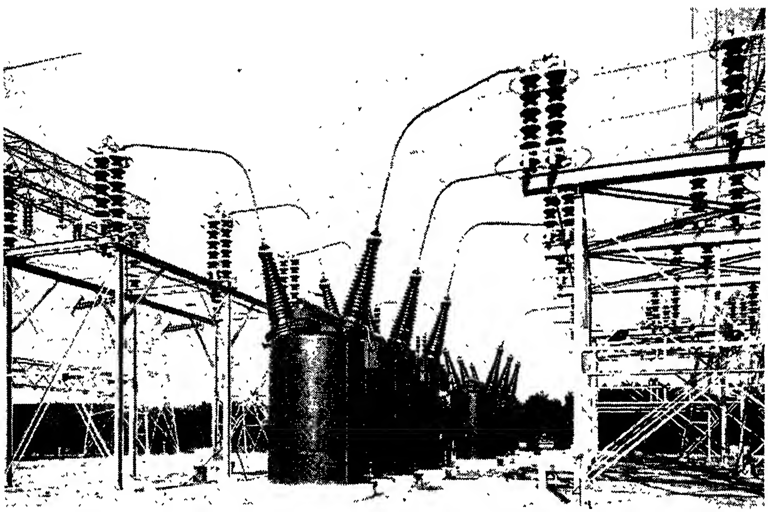


Fig. 168.—General Electric type FHKO-239-108CY-F7, 230-kv. oil circuit breakers at Tewksbury substation, New England Power Company.

operation at three cycles instead of eight cycles; also they save in weight and oil, since at 287.5 kv. the FG-30 weighs one-third and contains only one-tenth of the oil as compared with a corresponding FHKO-339 breaker. The general arrangement of a single-pole unit is illustrated by Fig. 169*a*.

Each of the two horizontal members consists essentially of a herkolite cylinder inside an oil-filled porcelain housing. The contacts and interrupting elements are mounted inside these cylinders. The vertical members or supporting columns consist of herkolite cylinders protected from the weather by oil-filled porcelain shells. Each of the end columns contains a current transformer. A capacitance transformer tap for use with a bushing potential device could be included if desired. The

force for opening and closing is transmitted from the operating mechanism to the contacts by a wooden operating rod located in the central column.

In order to protect the porcelain from all possible strèsses, the weight of the horizontal members and all operating forces are transmitted to the herkolite cylinders of the vertical members. The porcelain shells support only their own weight and that of a portion of the oil. Expan-

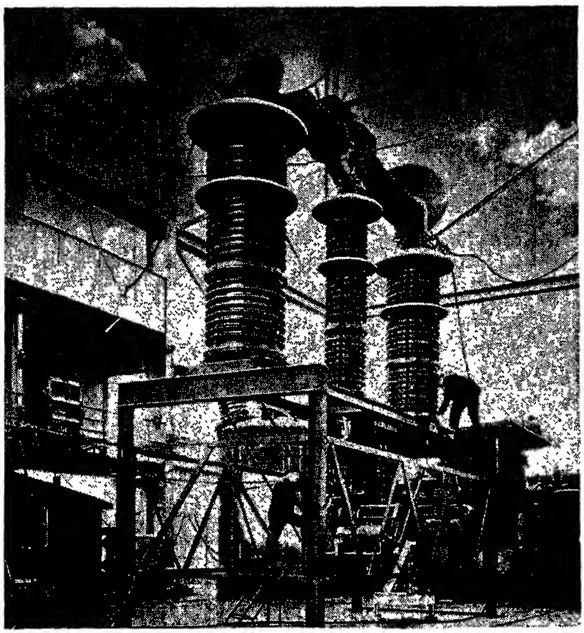


Fig. 169a.—General Electric single-pole type FG-30 impulse oil circuit breaker for Boulder Dam transmission line, 287 kv. Compressed springs release a force of 24,000 lb. in 0.01 sec., when breaker is tripped.

sion joints are provided at the upper ends of the vertical columns and in the center of the horizontal members. In order to provide further flexibility in the structure, the ends of the horizontal members are mounted on rollers which allow horizontal motion.

The interrupting element, Fig. 169b, consists of a herkolite tube with the moving and stationary contact assemblies for one horizontal member. Each single-pole unit will have two interrupting elements which are separated above the central vertical column by the mechanisms that transmit the motion from the operating mechanism to the contacts. Each interrupting element is enclosed in a porcelain housing, which is not subjected to pressures generated during arc interruption. Clamps at each end of the herkolite tube assure gas- and oil-tight joints

of sufficient strength to withstand the stresses to which the tube may be subjected. The oil between the porcelain shell and herkolite tube is entirely segregated from the oil used for circuit interruption. The moving and stationary contacts are carried on a baffle plate and so arranged that the complete contact assembly of each interrupting element may be removed for inspection and maintenance. There are removable covers at the ends of the horizontal porcelain housings for this purpose.

The contacts are of the butt type with eight breaks per phase, four in each interrupting unit. Each break consists of a stationary and a movable element, the connection from the stationary element of one

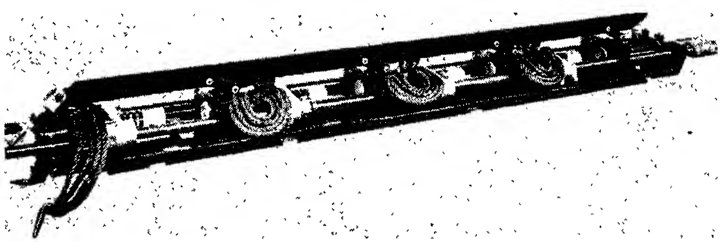


Fig. 169b.—General Electric contact and interrupting element for half single-pole unit, for impulse oil circuit breaker type FG-30, 1,200 amp., 287 kv.

break to the movable element of the next break being made through flexible braid. The movable contacts are actuated by two operating rods made from treated wood. Compression springs, isolated from arcs, are provided on each movable contact to maintain the proper contact pressure.

The arc interruption is obtained by forcing a blast of cool oil, at the proper velocity, across the arc path and through the adjacent ports in the baffle plate. When the breaker opens, the breaker operating rod moves downward, driving the oil piston down. This produces a flow of oil through the flapper valve past the contacts and out through the ports. At a predetermined point in the piston stroke, when the oil has reached the proper velocity, the contacts are separated and the oil is driven across the arc path at each "break."

Each single-pole unit contains two operating mechanisms transmitting force to the contacts and piston, one mechanism for the contacts of each interrupting element. The oil piston is provided with valves which allow the oil used during the opening stroke to return during the closing stroke. Each single-pole unit

is provided with an emergency vent and a pebble-filled separating chamber.[1]

133. Oilless Circuit Breakers.—In accordance with the tendency to decrease the oil content of the circuit breakers as one of the important elements in the rehabilitation of switch houses, discussed in Sec. 121A, new designs and developments for generator

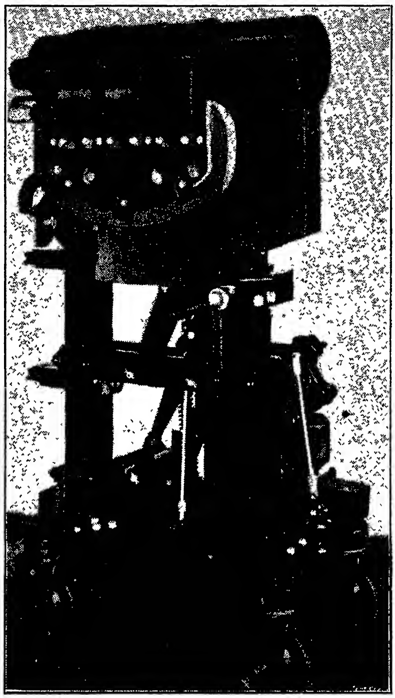


FIG. 170.—Westinghouse De-ion circuit breaker, side view.

voltages have been made which provide air De-ion breakers up to 750,000 kva. interrupting capacity, hydro-blast units of 500,000 kva. in service and up to 1,500,000 kva. on order, and compressed-air breakers with similar ratings.

A more detailed discussion of these breaker types follows.

1. *Air Breakers.*—The De-ion breaker[2] is shown in Fig. 170, and Fig. 171 shows a three-phase unit with its subcell disconnecting switches in a metal housing in the Essex Station of the Detroit

[1] See *Gen. Elec. Service Letter*, O.P.S.-247-A.

[2] See SLEPIAN, J., Theory of De-ion Circuit-breaker, *A.I.E.E. Trans.*, April, 1929.

Edison Company. As developed by J. Slepian and his associates, it consists of a stack of copper plates, each plate being $\frac{1}{16}$ in. thick ($\frac{3}{32}$ in. thick for the heavy-current units) and separated $\frac{1}{16}$ in. from each other. When an arc is drawn between the contacts under the plates, it is blown by a magnetic field into the plates,

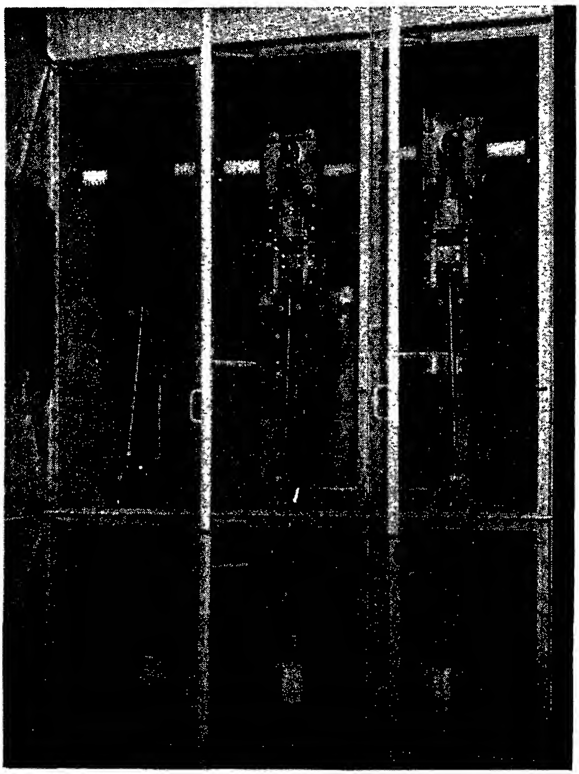


Fig. 171.—Westinghouse De-ion breaker and line disconnect-switch cells in Essex Station. Breaker 600 amp., 24 kv., 1,600,000 kva., operates in nine cycles. *(Courtesy of Detroit Edison Company.)*

where it is broken up into a series of short arcs each $\frac{1}{16}$ in. in length. Thus in every inch of plate structure there are eight cathodes each with its immediately adjacent, rapidly deionizing gas layer. Immediately after the current passes through the zero value in its normal cycle, each cathode layer is almost instantly deionized. Thus in part of a microsecond the cathode layer becomes nonconducting and therefore acquires the ability to withstand 250 volts much faster than any practical power circuit

of corresponding voltage can supply the 250 volts. Thus the voltage necessary to reignite the arcs after the current zero is eight times 250, or 2,000, peak volts per inch of structure. The structure will therefore interrupt circuits whose voltage is not over $2{,}000/\sqrt{2}$ or 1,414 volts r.m.s. per inch length.

Experiment showed that $\frac{3}{32}$-in. spacing still left sufficiently free motion of the arc, and on this spacing there would be 21.3 plates per inch which would give the structure an interrupting value of 3,760 volts r.m.s. per inch.

In the De-ion breaker, the circuit volts per plate are less than 130 volts, whereas the theoretical limit is 175 volts r.m.s. This is partly to provide a factor of safety and partly because when the voltage is impressed upon a long stack of plates insulated from one another the potential does not divide among the plates in a uniform manner. This lack of uniformity in the voltage distribution is approximately compensated for by the static shield which fits over the stack.

To prevent the electrodes being melted when carrying heavy currents, sufficient magnetic field is provided to make the arcs move over the surface of the electrode over and over again on an annular path.

In recent tests for the Detroit Edison Company on a three-pole De-ion breaker rated at 600 amp., 24 kv., the breaker handled successfully a maximum current of 53,000 amp. on one pole.

Although based upon a different theory from the De-ion air breaker, so-called "De-ion grids" have been developed and are now standard equipment on the contacts of Westinghouse oil circuit breakers for 88 kv. and above. They are discussed in Sec. 137.

The Air-blast breaker uses the prestored energy of compressed air for both circuit-breaker operation and circuit interruption, producing arcing times practically constant at one-half cycle or less. A typical design of the indoor axial-blast type is shown in Fig. 172.

In opening the breaker,

> . . . electrical or manual actuation of the control valve opens the air blast valve admitting air at 135 or 215 psi into the interrupting chambers. The movable arcing contact, normally held closed by the action of heavy springs, is attached to a piston. As air pressure builds up within the interrupting chamber, the piston is depressed, withdrawing

the contact the required distance away from the stationary arcing contact and air blast orifice, drawing an arc between the contacts. As the tubular stationary contact is the only outlet for the compressed air, a blast of high velocity air is simultaneously impelled across the initial arcing zone, resulting in limitation of arc energy release and preventing reignition after the first current zero.

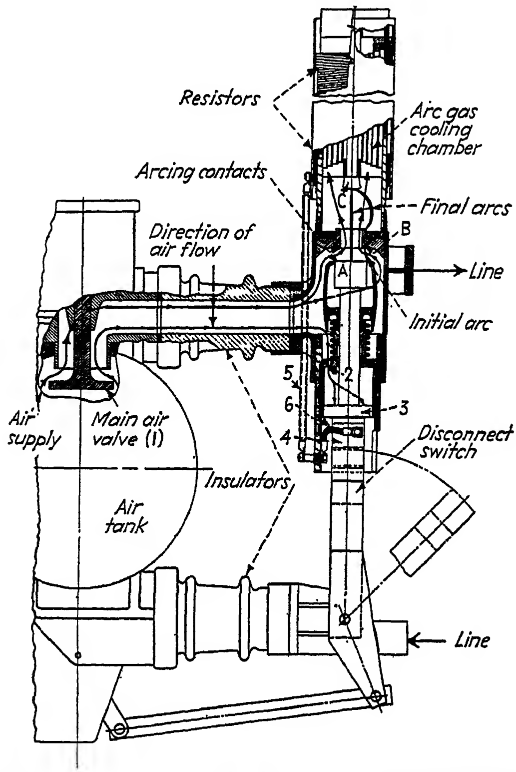


FIG. 172.—Typical section of Allis-Chalmers indoor axial air-blast interrupting device with paralleling resistor.

Simultaneously with the admission of air to the interrupting chamber, air pressure is applied to a piston resulting in opening of the disconnect contacts, under no load, as soon as arc interruption is complete.

Immediately the disconnecting contacts open, an auxiliary contact interrupts the circuit to the control valve, closing the valve. This cuts off the air supply to the air blast valve and permits the air remaining on the reverse side of this valve to escape. The main valve then closes under the influence of a heavy spring and the pressure of air

remaining in the tank; pressure in the arcing chambers falls and results in reclosing of the arcing contact due to spring pressure.

The air required for arc interruption causes pressure in the breaker air tank to fall. Since the tank is permanently connected to a compressed

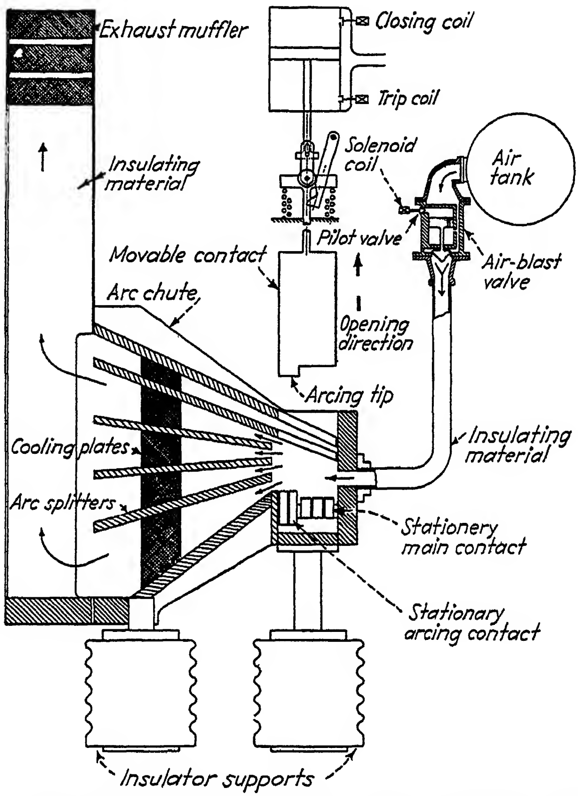


FIG. 173.—Cross section of interrupting unit of General Electric cross-air-blast circuit breaker, type AR-20.

air storage system, the supply of air within the tank is quickly replenished.[1]

Figure 173 is a cross section of the interrupting unit of the cross-blast design.

This consists of a pair of contacts located in an arc chute of insulating material. Air is introduced at one side of this chute. The air velocity

[1] See Circuit Interruption by Air Blast, by Edsall and Stubbs, *A.I.E.E.* Convention, January, 1940, *Paper* 40–66.

is, therefore, at right angles to the arc stream and blows the arc against a number of barriers also of insulating material. In effect the arc is caught in the grip of a multiple shear having a movable member consisting of jets of air and stationary members of insulating material. At current zero the arc is cut up into sections and the arc products blown away into a muffling structure through the arc chute. In opening the breaker an electrical impulse to the air-blast pilot valve causes the pilot to open, and the initial movement of the pilot closes a switch to the opening trip coil. This operation of switches in series assures air for

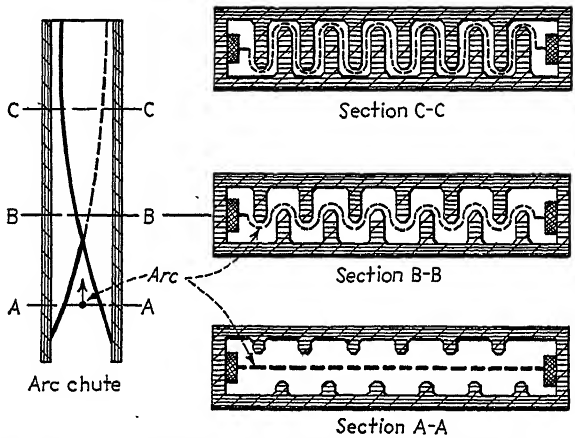


FIG. 174.—A schematic diagram showing the treatment of an arc in the General Electric Magne-blast interruptor. At section *A-A* the arc is struck in a line between arc runners. As it is forced into the chute the fins constrict the arc and force it into a serpentine path *B-B*. Section *C-C* shows its final shape under extreme conditions.

the blast action before the contacts part. The initial motion of the piston trips the mechanical latch. The breaker then opens by the action of the piston, aided by opening springs. After the contacts have parted a predetermined distance, a timing switch cuts off the air blast. At the end of the opening stroke another cut-off switch shuts off air to the cylinder.[1]

The Magne-blast breaker has been developed for 5,000-volt service for use in vertical-lift metal-clad switchgear with 600, 1,200, and 2,000-amp. ratings and interrupting capacities of 75,000, 100,000, and 150,000 kva. As shown in Fig. 174, the

[1] See The Cross-air-blast Circuit Breaker, by Prince, Henly, and Rankin, and Design and Construction of High Capacity Air Blast Circuit Breakers, by Strang and Boisseau, *A.I.E.E.* Convention, January, 1940; also Medium-Capacity Air-blast Circuit Breakers for Metal-clad Switchgear, by Bennett and Wyman, *A.I.E.E.*, June, 1940, *Paper* 40–125.

breaker extinguishes arcs by means of arc chutes designed for fast cooling. The chute consists of a system of projecting fins on one side interleaving with corresponding fins on the opposite side, producing a serpentine path (Section C-C), lengthening and cooling the arc, and causing early interruption.[1]

2. *Water Breakers.*—"Expansion-breaker" type interrupters using distilled water for the extinguishing medium have been developed in which the arc is drawn in a confined chamber where the generation of steam and gas builds up a high pressure. Then vents are opened to a condensing chamber, the arc is subjected to a turbulent blast of steam and gas, and the deionization of the arc space is greatly accelerated by the cooling effect accompanying the sudden expansion of the steam. Figure 175 shows a cross section of the interrupting chamber.

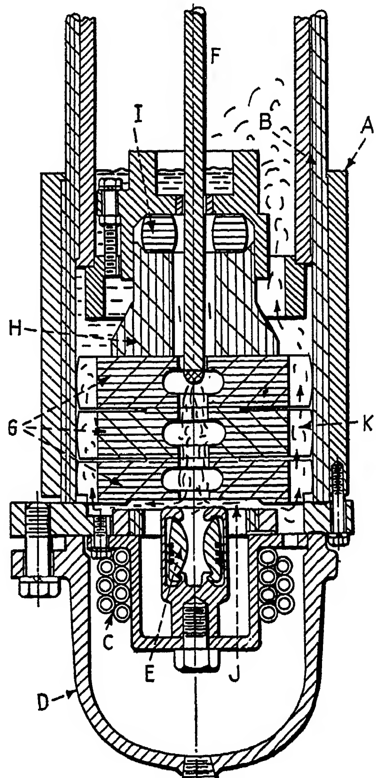


FIG. 175.—Cross section of interrupting chamber for Westinghouse water breaker. *A*, bronze cylinders; *B*, micarta tube; *C*, air-filled rubber; *D*, bottom bell cover; *E*, finger contacts; *F*, arcing contact rod; *G*, micarta blocks; *H*, hollow spacer; *I*, rubber ring; *J*, arcing-chamber vent.

Overlapping inside and outside bronze cylinders *A* hold a heavy insulating tube *B* in compression and shear to develop maximum strength. Special attention has been given to the problem of shock absorption by providing a large air volume above the water level and also by utilizing the compressibility of air pockets sealed in rubber *C* located in the removable bottom bronze bell cover *D*.

The stationary arcing contact assembly consists of a cluster of fingers *E* while the arcing contact itself is made of a strong rust-proof alloy

[1] See Magne-blast Air Circuit Breaker, by Bochne and Linde, *A.I.E.E.*, June, 1939, *Paper* 39–128.

rod F with a strip of arc resisting material. This contact moves in and out of a chamber formed by three large blocks G of special insulating material with a centrally located water trapping pocket in each. A hollow spacer H above these blocks engages a rubber ring I. Pressure under the lowest block raises the whole arcing chamber assembly, compressing this rubber ring and at the same time opening the vent J. . . . The de-ionization produced by the turbulent flow of the relatively cool water vapor and steam through the arc space is so effective that at the first or second current zero after the arc is drawn into the bottom of the arcing chamber, the dielectric strength builds up with sufficient rapidity that re-ignition cannot take place and the arc is extinguished. . . . The steam and gas from the vents are directed through a condensing labyrinth and the permanent gases such as hydrogen and carbon monoxide that remain after the water vapor is removed pass out of a small port to the atmosphere.[1]

Tests covering a range of 50 to 61,000 amp. at 13.2 kv. gave an average arcing time close to 1½ cycles for all currents.

The Hydro-blast type interrupter is completely interchangeable with the standard "H" type oil circuit breaker but eliminates all inflammable material. Figure 176 shows a unit for 500,000-kva. rating.

When the contact rod rises an arc is drawn from the bottom of the rod to the arcing bell. As the rod continues to rise and is withdrawn into the central hole of the baffle, the metal gates at the lower end of this hole close under the influence of their compression springs, blocking off any direct flow of liquid or gas up through this hole and dividing the arc into two portions in series, one below the baffle and one in its central hole. The arc below the baffle generates pressure which forces the liquid through the cross-blast passage as entering the baffle from the bottom at the right, then passing horizontally over to the left side of the baffle and finally leaving the baffle from the top on this side. The horizontal passage intersects the central hole of the baffle, so that the liquid forced through this passage by the pressure below the baffle crosses the path of the arc.[2]

134. Isolated Phase Breakers.—Some of the large stations with specially heavy power concentrations have arranged their

[1] See A High-power Oilless Circuit Interrupter Using Water, by W. M. Leeds, *A.I.E.E.*, June, 1940, *Paper* 40–85.

[2] See High-capacity Hydro-blast Circuit Breaker for Central-station Service, by Skeats and Saylor, *A.I.E.E.*, June, 1939, *Paper* 39–132.

bus systems so as to have a separate floor or room for each phase of the bus, thus eliminating the danger of a phase-to-phase short circuit and reducing the hazard to that of faults from one phase to ground. This arrangement necessitates placing the individual phase breakers at a considerable distance apart either horizontally or vertically, depending upon which plan of isolation is adopted.

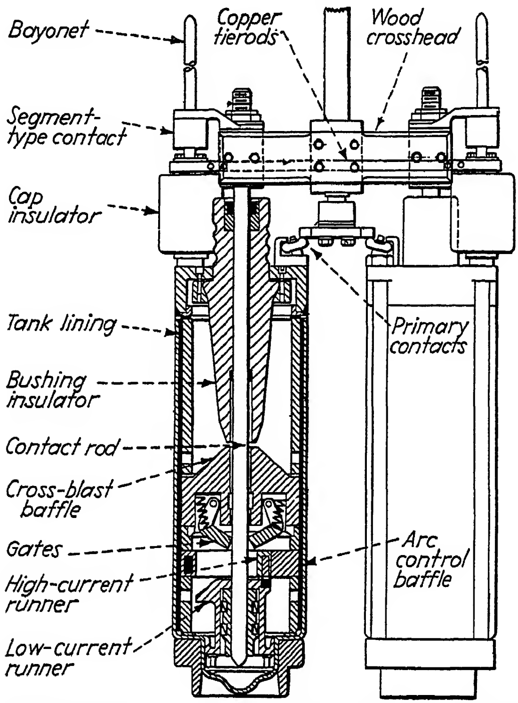


FIG. 176.—General Electric Hydro-blast circuit breaker, 500,000-kva. rating.

Although the breaker elements may be of standard construction, the control and operating mechanism to secure simultaneous operation of all phases will be special in these cases. Figure 177 shows an arrangement for horizontal isolation of the phases.

The vertical plan of isolation is of great value where ground space is severely restricted. Thus one triple-pole 28-kv. breaker in the Hudson Avenue Station of the Brooklyn Edison Company occupies only 5 sq. ft., but attains a height of 58 ft. 6 in. The unit breakers for each phase are placed one above the other, one phase to a floor, with the operating mechanism located on the fourth floor.

135. Truck-type Breakers.—These have been developed for the auxiliary and feeder circuits in large central stations and at distribution substations. As is shown in Fig. 178, the entire unit of oil circuit breaker, instruments, panel, bus connectors, etc., is factory assembled and shipped as an assembled unit. The

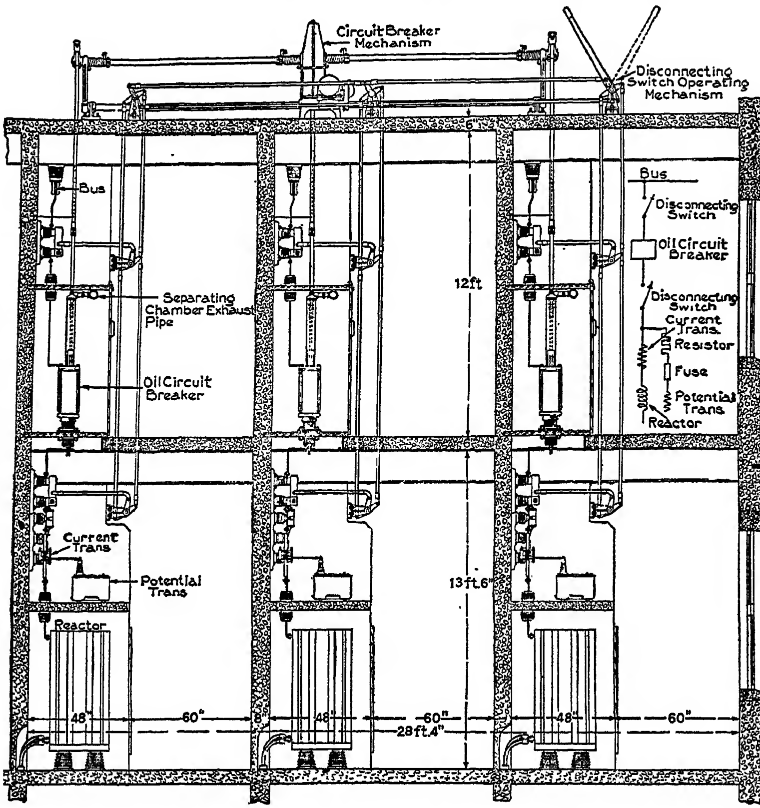


FIG. 177.—Isolated phase arrangement of General Electric FH oil circuit breakers with horizontal separation of phases.

installation work then consists of installing the bus bars, attaching the main and control cables, and filling the circuit-breaker tanks with oil. These steel-housing and panel units therefore save considerably in space requirements and in installation time and provide better protection to the operating and maintenance crews. The truck mounting facilitates inspection and repair, and interchangeable units permit one or two spares to assure maintenance of service for an entire station. Interlocking fea-

tures, which keep the disconnect switches closed except when the breaker is open, are provided to guard the operation of inserting or removing the trucks from the housings.

For moderate duty, the removable truck-type switchgear has been replaced by metal-clad equipment except where additions are made to existing installations.

136. Metal-clad Switchgear.—This type of equipment has been installed for large indoor and outdoor structures at the

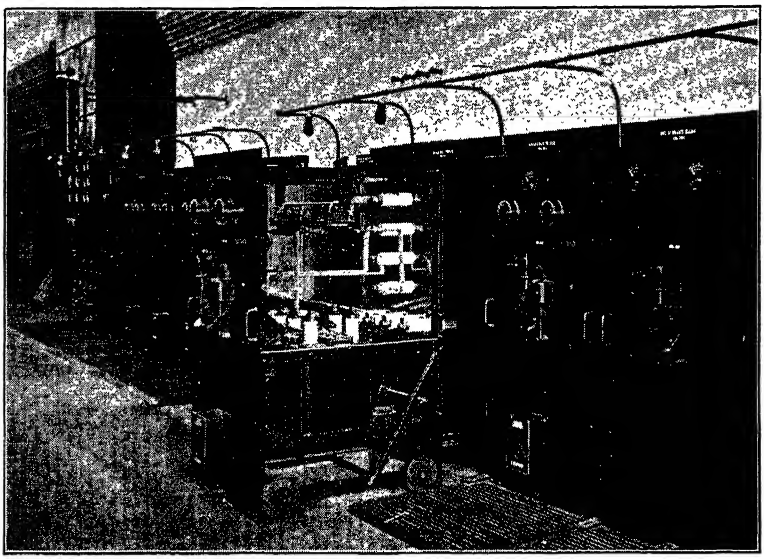


FIG. 178.—Pacific Gas and Electric Co., San Francisco, Calif. Interior station "A" manual General Electric truck panels with one truck pulled out.

Waukegan[1] and Joliet Stations of the Public Service Company of Northern Illinois, the Leaside[2] Station of the Hydro-electric Power Commission of Ontario, etc. As an operating advantage, the live leads for busses and all connections are entirely surrounded by a compound or oil filling inside metal, so that accidental contact by an operator with any conductor is prevented, and any fault must be limited to a failure of phase to ground. Each unit contains the oil circuit breaker, bus, instrument transformers, disconnecting devices, supporting structure, interlocking equipment, and all necessary small wiring. Because

[1] See *Elec. Eng.*, June, 1932, p. 393.
[2] See *Elec. Jour.*, January, 1931, p. 16.

of this unit arrangement, the switchgear is very economical of space, it can be completely built at the factory and be installed with a minimum of labor in the field. Complete mechanical and electrical interlocks can be readily provided. Because of the high insulation and the strength of the supporting structure and equipment used, it brings considerable simplicity in the switchgear design. Installations are in service indoors with

FIG. 179.—State Line Station, switchyard equipment. Outdoor 22-kv. generator double-ring bus in three sections, one generator per section, with reactors and breakers between sections. The insulated conductors are enclosed in oil-filled grounded metal pipes and junction boxes. Disconnecting switches are eliminated. Energy is transmitted at 33, 66, and 132 kv. (*Courtesy of State Line Generating Co.*)

440-, 2,400-, and 12,000-volt busses, and outdoors at 13,000, 33,000, and 132,000 volts, with breakers reaching 2,500,000 kva. interrupting capacity. Figure 179 shows the installation of oil-filled bronze pipe and oil-filled bronze junction boxes, with isolated phases, for the outdoor generator bus and breakers for the State Line Generating Station.

137. Devices for Arc Control in Oil Circuit Breakers.—The extensive researches of the past few years have thrown much light on the detailed mechanism of the extinction of an arc in an alternating-current circuit.[1] When such an arc plays, it takes a

[1] See SLEPIAN, Extinction of an A-C Arc, *A.I.E.E. Trans.*, October 1928; SLEPIAN, Extinction of a Long A-C Arc, and BAKER and WILCOX, Use of Oil in Arc Rupture, *A.I.E.E. Trans.*, April, 1930.

voltage that is generally less than the voltage generated in the circuit and which influences the course of the current only in a minor way. Following its natural cycle, the current comes to a zero value, and at such a moment the arc extinguishes. Thus, in the few microseconds of the zero current, the medium containing the arc changes from its momentary condition of a comparatively good conductor, carrying current at a low voltage, to its normal state of a comparatively good insulator withstanding the full-generated voltage of the circuit. It is this rapid transition at the moment of zero current, from the state of the highly conducting arc to the state of an insulating nonionized gas, which is important for the extinction of the arc in an alternating-current interrupting device. This transition must be made sufficiently rapidly if the arc is not to reestablish. In practical power circuits, the time available for the extinction process in the arc will depend upon the transient characteristic of the external circuit and may be, in general, about 10 microsec. Therefore, under certain conditions the interrupting capacity of an alternating-current switch may be greatly affected by the nature of the circuit in which it operates. The foregoing time represents the building up of the voltage at the arc terminals by the external circuit tending to reignite the arc. Opposed to this is the rate at which the arc space recovers dielectric strength resulting from the disappearance of ions.

In the extinction of a short alternating-current arc instantly after the current zero, a layer of gas immediately adjacent to the cold cathode is almost deionized and thus acquires dielectric strength. Even the thinnest layers of an un-ionized gas require several hundred volts to break them down. The rest of the arc space loses ionization more slowly, and as its deionization proceeds, the deionized cathode layer increases in thickness and in dielectric strength.

In the extinction of a long alternating-current arc, the greater part of the recovered dielectric strength is in the space outside the cathode layer. Following the current zero, the ionization of this space disappears in a relatively gradual manner. This is due to loss of ions by direct recombination of ions of opposite sign within the space and by the passing of ions outside the boundaries of the space due to diffusion, electric fields, air blast, etc. If the impressed electric gradient remains less than the critical gradient

which will make up the ionization being lost in the arc space, then the space loses conductivity and the arc extinguishes. If the impressed gradient equals or exceeds the critical gradient, the arc does not extinguish.

1. *Plain-break Contacts.*—Early breakers used a wedge-and-finger-type contact with removable arcing tips. The moving

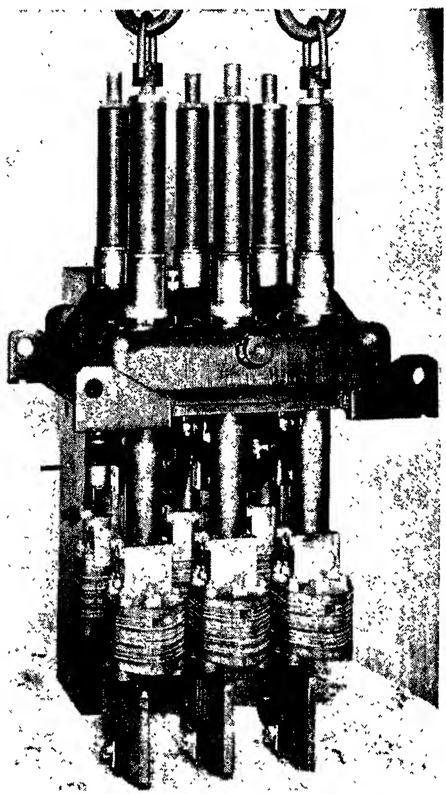


FIG. 180.—Westinghouse type F-100 breaker, with De-ion grids, 600 amp.

wedges were heavy extruded copper, and the arcing tips were of extruded copper of high thermal capacity. The individual fingers of the stationary contacts were arranged in pairs facing each other, permitting the wedges to be drawn up between them to close the circuit. The fingers had forged or extruded copper contact tips which aligned themselves automatically with the inclined surfaces of the wedges because of the pressure of flat steel springs. The current was conducted from each contact tip to

the contact block by a laminated copper shunt. The outermost pair of contacts always opened or closed the circuit so that the arc was drawn by this pair of fingers.

More recently, however, advantage has been taken of the newer interrupting elements which have replaced the wedge-and-finger type, in this case with the De-ion grid, as in Fig. 180.

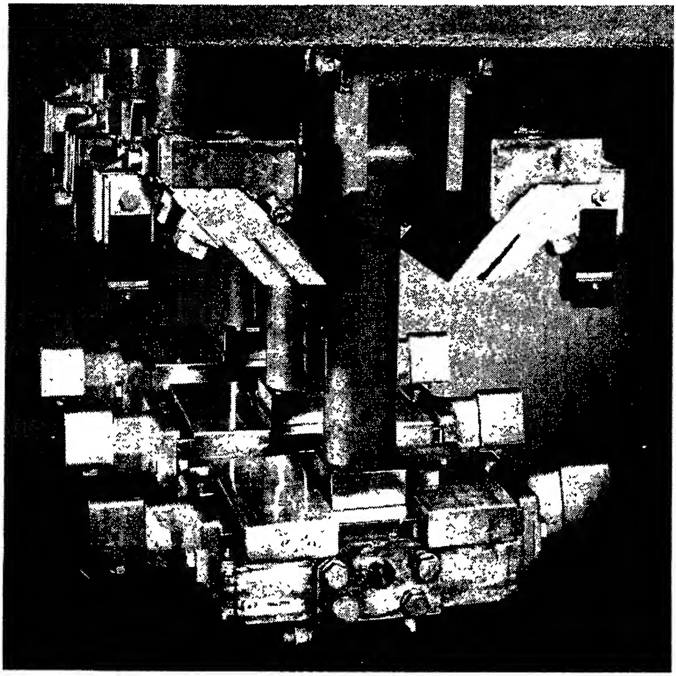


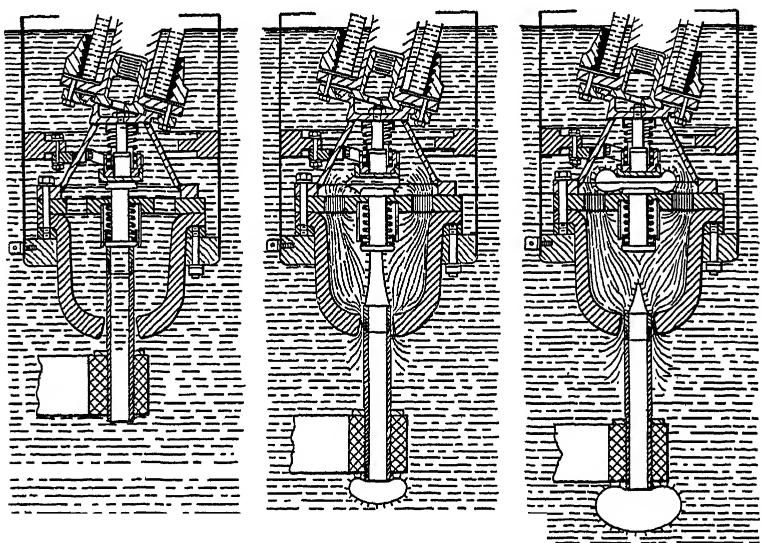
FIG. 181.—Westinghouse plain breaker for heavy currents.

For heavier current-carrying capacity, the contacts illustrated in Fig. 181 are used. The main contacts consist of reverse-brush-type stationary contacts with a butt-type moving contact. Heavy cast-copper contact feet are screwed and sweated to the lower end of the terminal rod. The laminated brush which is bolted to this casting slants inward toward the center line of the breaker unit. The path of the current passing through the breaker has no sharp turns, and the effect of the magnetic forces due to the current tends to increase the contact pressure.

The moving contact consists of a heavy copper casting with suitable adjustable means for attaching to the lift rod.

The arcing contacts are located outside the main contacts and so arranged that the arc will always be formed on them in opening. This leaves the main contacts clean for current-carrying purposes.

2. The oil-blast explosion chamber of Fig. 182 is an important feature of the General Electric breakers for high interrupting capacities. The steel chambers at the lower voltages are changed



(a) (b) (c)

FIG. 182.—General Electric oil-blast explosion chamber for circuit breaker. (*a*) Closed; (*b*) before interruption; (*c*) interruption.

at 110 kv. to a molded structure mounted on the lower end of the breaker bushing underneath the surface of the oil. As shown in the figure, the chamber contains an upper butt contact, an intermediate contact, and a bottom hollow contact. In the closed position, the current is carried by all three members firmly pressed together.

After the breaker has been tripped the intermediate and bottom contacts start down, driven by the opening springs, drawing a short arc between the two upper contacts. This upper arc generates gas, which puts the oil in the explosion chamber under considerable pressure, which pressure tends to collapse the walls of the lower elongated gas bubble maintained by the lower arc. As current zero approaches, the

current and rate of gas generation decrease and the walls of the gas bubble, surrounding the lower arc, close in. At current zero the bubble is forced through the hollow bottom contact by the converging walls of oil, and a solid wall of oil is forced between the contacts effectively preventing reestablishment of the arc.[1]

Figure 183 shows an isolated phase unit on the oil-blast principle of 1,500,000 kva. interrupting capacity.[2]

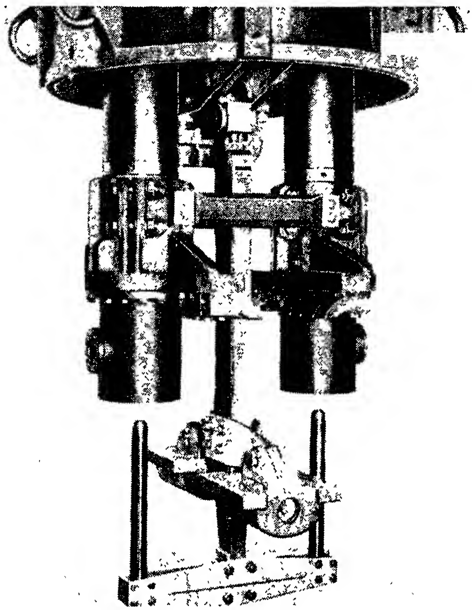


Fig. 183.—General Electric type FHK-330-328 oil circuit breaker, single-pole unit in open position with explosion chambers; 15,000 volts, 5,000 amp.

Figure 184 shows the special contacts used in the Allis-Chalmers type DZ-200A oil circuit breakers for indoor station application.

3. *Multiple-break Contacts.*—Interrupters. A new multi-breaker interrupter to clear short circuits in less than five cycles has been developed for conventional high-voltage tank-type oil circuit breakers, 115 to 230 kv. Six breaks per terminal, three pressure generating and three interrupting, are adequate for 230 kv., and four breaks are used for 138-kv. service.

[1] See *Gen. Elec. Co. Bull.* A-1560.

[2] See Poitras, Kuehni, and Skeats, Oil Circuit Breaker and Voltage Recovery Tests, *Elec. Eng.*, February, 1935, p. 170.

Figures 185*a* and *b* show a six-break interrupter for 230 kv.

The main housing of the interrupter is a thick-walled cylinder of insulating material having an unusually high dielectric and mechanical strength. Stationary members of the conducting parts are supported on 4 vertical insulating posts, each fastened at the top to the upper cover

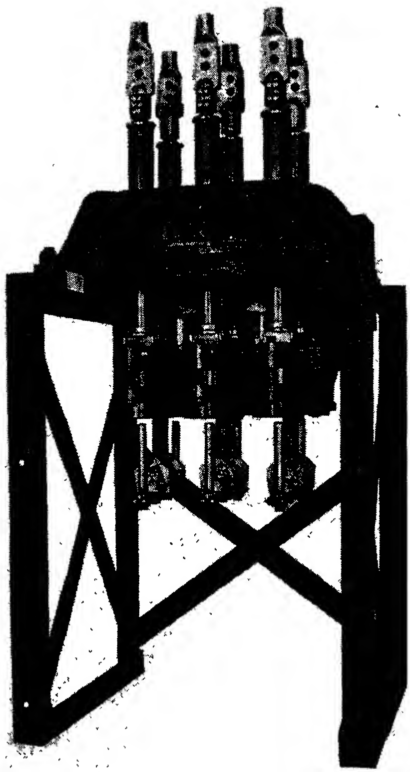


Fig. 184.—Contacts of Allis-Chalmers type DZ breakers, 1200 amp., 15 kv., 500,000 kva.

of the interrupter, and at the bottom to a retainer. Extending through the center of the interrupter is an insulating rod which carries the moving contacts. This insulating rod, which projects out of the lower end of the interrupter, terminates in a metal cap which is the contact engaged by the crosshead. A flexible lead from this cap carries the current from the cap to the main current path of the interrupter. Heavy pressure for each contact is obtained by inserting a spring between each contact and its fastening to the center operating rod. Interrupting contact

faces are of Elkonite, which is outstanding in its ability to withstand severe and repeated arcing with minimum loss of material.

When the breaker opens, the crosshead moves downward, thus permitting a spring inside the interrupter to force the center rod supporting the interrupting contacts downward and part the contacts inside the

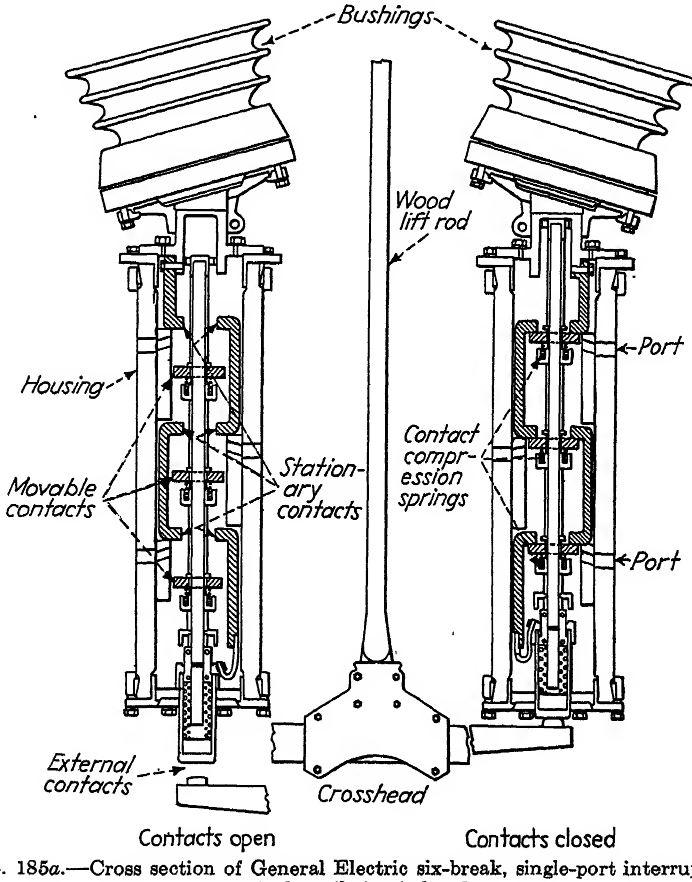


FIG. 185*a*.—Cross section of General Electric six-break, single-port interrupters for 230-kv. oil circuit breaker.

interrupter. The arcs formed in the interrupter decompose oil into gas which creates pressure and forces oil out through the ports. The ports are so located in relation to the contacts that the flow of oil is directed between the contacts. Thus, during the time current is flowing, both oil and gas are flowing out of the port. As current zero is approached, less gas and more oil are carried over the contacts and out the port. It is important to bring out that the bleeding of gas from the interrupter

takes place at the contacts adjacent to the port. Gas created at the contacts which have no port adjacent to them is stored in the chamber so that at current zero when no gas is generated, the stored pressure inside of the chamber is sufficient to drive oil into the space between the contacts adjacent to the port. Only a small contact separation is required, therefore, to allow sufficient oil insulation between the contacts to prevent the recovery voltage from breaking down the dielectric between the contacts. This interrupting action is entirely in accordance with the oil-blast principle of interruption.

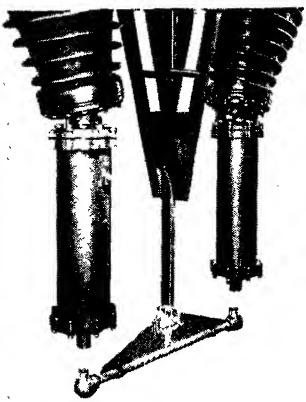


FIG. 185b.—General Electric six-break, single-port interrupters installed on a 230-kv. oil circuit breaker.

Proper arrangement of the port or ports in relation to the contacts results in an effective device which interrupts the current at an early current zero after the contacts part. Interruption of the circuit generally occurs before the crosshead contact parts from the exterior contact at the bottom of the interrupter. After interruption, the crosshead moves downward to the full stroke of the breaker, a distance sufficient to withstand the required insulation tests with the breaker in the open position.[1]

On field tests[2] at Philo Station of the American Gas & Electric Service Corporation, at 132 kv. and duties from 1,400,000 to 2,000,000 kva., the breaker time varied from the maximum of 3.5 cycles to a minimum of less than 2 cycles. Tests were made also at Saugus substation of the Southern California Edison Co., Ltd., at 220 kv. 50 cycles. The interrupting time ranged from 1.37 to 3.1 cycles for a duty of 1,700,000 kva.

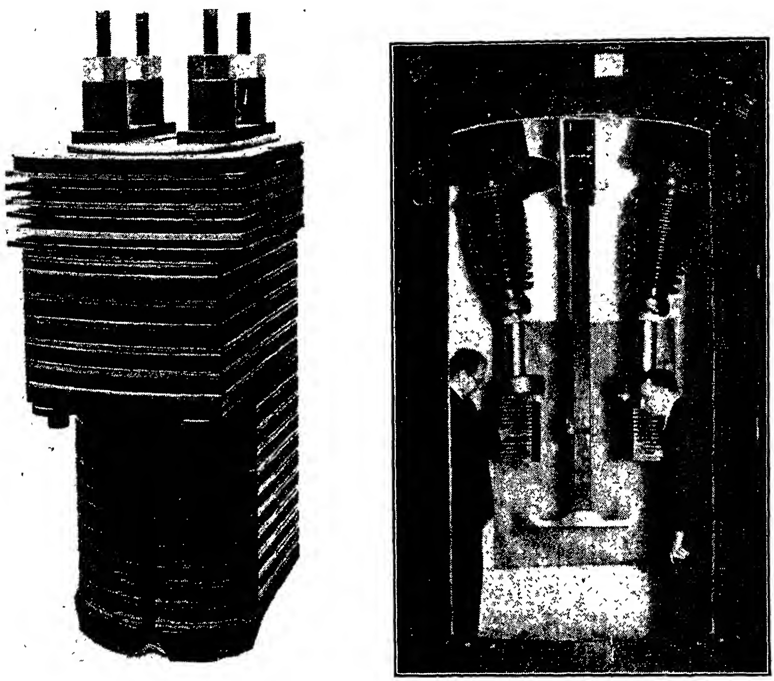
4. *De-ion Grid Contacts.*—A very impressive improvement in circuit breakers resulted from the substitution of De-ion grids,[3] shown in Figs. 186*a* and *b*, for the usual stationary contacts. The gain is due to a more efficient use of the oil in the arc rupture.

[1] See A New Multibreak Interrupter for Fast-clearing Oil Circuit Breakers, by Spurck and Strang, *Trans. A.I.E.E.*, December, 1938.

[2] See Tests on and Performance of a High-speed Multibreak 138-kv Oil Circuit Breaker, by Sporn and St. Clair, *Trans. A.I.E.E.*, December, 1938.

[3] See Deion Grids for Oil Breakers, *Elec. World*, Feb. 1, 1930; BAKER and WILCOX, Use of Oil in Arc Rupturing, A.I.E.E. Convention, January, 1930.

The arc is drawn between the parting contacts in the vertical, narrow, deep groove formed by the slots in the iron, fiber, and fullerboard layers of the grid and closed at the outer end. Magnetic plates throughout the grid provide a magnetic field when an arc is drawn through the groove. This magnetic field moves the arc steadily toward the closed end of the groove. Since the



(*a*) (*b*)

FIG. 186.—(*a*) Westinghouse De-ion grid, type A, 230 kv., showing the assembly. (*b*) The Westinghouse De-ion grids assembled in a circuit-breaker structure, the side of the tank cut away to show the interior mechanism. The grids are supported from the terminal bushings.

groove is filled with oil and the arc fills practically the entire open end of the groove, in moving toward the closed end, the arc is forced against a solid wall of oil. This gives a high rate of decomposition of the oil with an accompanying continuous and adequate supply of fresh un-ionized gas. The open end of the groove being filled with the arc stream, the gas is forced through the arc stream, diluting it with small volumes of un-ionized gas. These act as deionizing surfaces after the current zero

is reached. An additional deionizing effect is obtained from the sides of the groove where the arc impinges as it moves.

In order to develop breakers for voltage classes 138 to 230 kv. which can operate in five cycles, a multiple-unit De-ion grid assembly has been designed. The units are arranged to operate

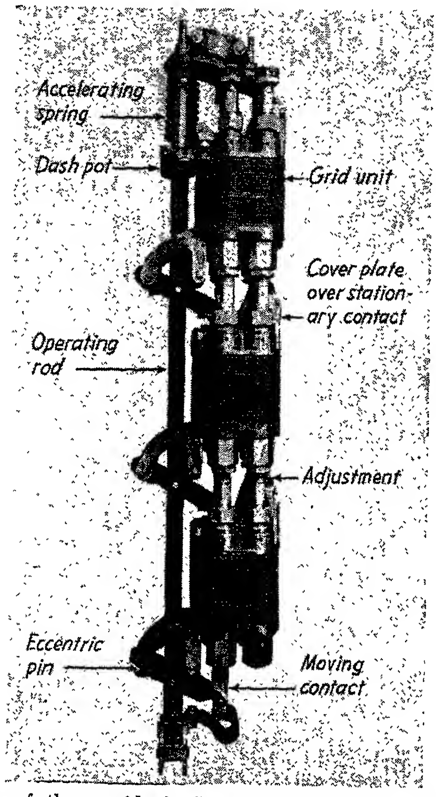


FIG. 187*a*.—View of three grids in Westinghouse 220-kv., six-grid breaker.

simultaneously, giving at relatively high speed a series of low-voltage arcs. Figures 187*a* and *b*, show the arrangement of a six-grid breaker for 220 kv. Four grids can be used for 138-kv. apparatus.[1]

5. *High Operating Speeds.*—Standard oil circuit breakers are expected to perform the service of interrupting faults within

[1] See Multiple-grid Breakers for High-voltage Service, by MacNeill and Hill, *Trans. A.I.E.E.*, August, 1939.

their rated capacity, 3,500,000 kva. at 287.5 kv. in the largest size, with a satisfactory degree of surety. However, in recent years the requirements of power-system stability have demanded in addition to surety a great increase in speed of operation of the breaker. Fortunately there has been a marked improvement in the equipment within the last few years so that opening speeds of 8 cycles are now standard, and breaker speeds of 5 and 3 cycles may be obtained in special cases. Extensive tests on the oil circuit breakers with modern arc-extinguishing devices have demonstrated that the breakers are more reliable both electrically and mechanically. It has been possible to increase the interrupting capacity and to improve the speed of operation of many of the old breakers by equipping them with the new arc-extinguishing devices and the higher speed mechanisms.[1] At the Harding Street Generating Station of the Indianapolis Power and Light Company, tests were made on a system composed of the 87,500-kva. generating station, a 50-mile, 132-kv. transmission ring, and four 30,000-kva. high-voltage substations. The equipment consisted of De-ion grid breakers, bushing-type current transformers, potential network units, capacitors, and high-speed relays. All sorts of short circuits and grounds were placed on the system, a total of 600 short-circuit tests being made. Two breakers each cleared 122 short circuits, and others cleared as many as 64. The breakers showed an average clearing time of 7.5 cycles, from energizing shunt-trip coil to arc extinction, on short circuits of 700,000 to 1,200,000

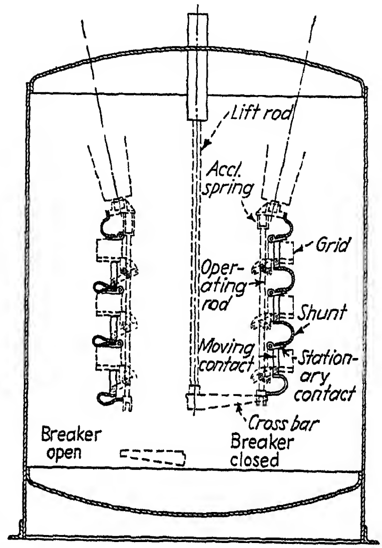


FIG. 187b.—Schematic arrangement of contacts, 220-kv., six-grid, Westinghouse breaker, in open and closed positions.

[1] See Progress in Power Generation, *A.I.E.E.*, Committee on Electrical Engineering, January, 1940.

kva., three-phase equivalent. All faults were cleared with negligible system disturbance.[1]

138. Oil Circuit-breaker Operating Mechanisms. 1. *Manual.* Breakers arranged for regular manual operation are closed by a hand lever pivoted on a face plate and mounted on a control panel. Only low-voltage light-capacity breakers should be permitted on the rear of the panel, all others being set some dis-

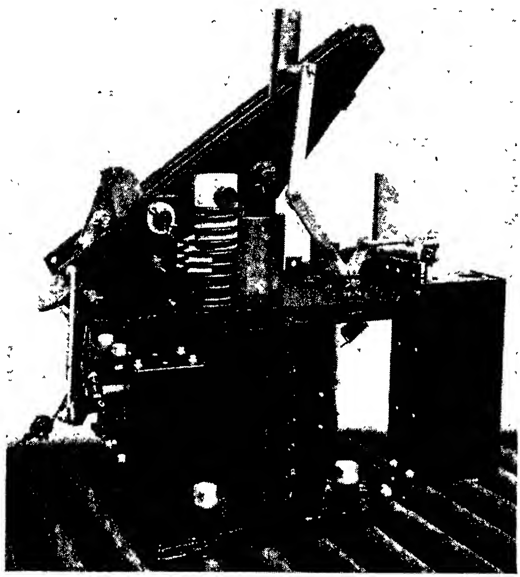


Fig. 188.—Solenoid mechanism type SAF-4 for Westinghouse type O breakers.

tance away from the control point and coupled to the handle by rods and bell cranks. In general, a horizontal distance of 35 ft. or a vertical distance of 20 ft. should not be exceeded.

Electrically operated breakers may be arranged for manual operation in an emergency, or to check the adjustment.

2. *Solenoid.*—Where a very reliable supply of direct current is available, the solenoid is widely used for operating the breakers. Each mechanism consists of one or more coils and plungers for the closing operation, one coil and plunger for the tripping operation, a crank for transmitting the force from the closing plunger or

[1] See *Elec. World,* Jan. 21, Feb. 11, 1933, and *Elec. Jour.* February, March, 1933.

plungers to the breaker mechanism, and a toggle for holding the crank in the closed position until tripped.

The solenoid mechanism may also be equipped with an accelerating device to speed up the opening operation of the breaker. This is a powerful spring in a cylinder which is compressed when

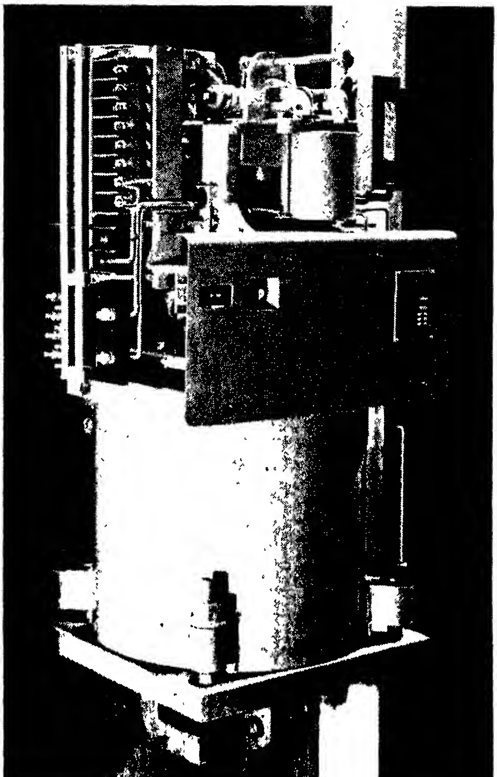


FIG. 189.—Allis-Chalmers type SO-500 solenoid operator with type Q-78 auxiliary switch and circuit breaker.

the breaker is closed. The device may also have an air dashpot and regulator to retard the operation at the end of the stroke and reduce the slam of the moving parts. Hydraulic bumpers may be used instead of the dashpot.

The nature of the design features of the modern solenoid operators is such that they are more universal in their application, since by changing the operating coil the solenoid can handle several types of breakers. The use of the solenoid permits the

concentration of the control and the operation of the heavy high-capacity breakers. Figure 188 shows the solenoid mechanism for the Westinghouse type O and other breakers.

Where only alternating current is available for closing the circuit breakers, a rectifier, Tungar, copper oxide, etc., may be used in conjunction with the solenoid operator. Such equipment is

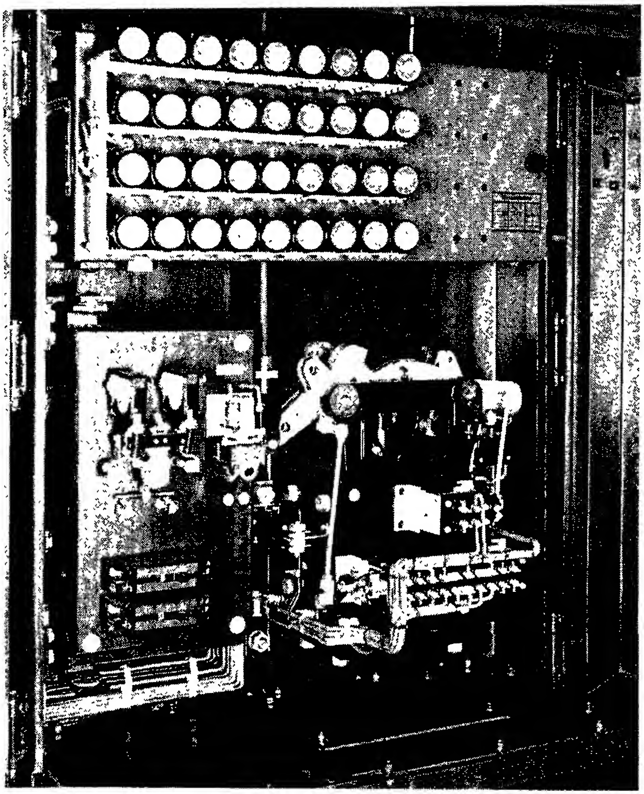


Fig. 190.—Westinghouse solenoid unit showing mechanism, control panel, and Rectox.

standard for the isolated self-contained cubicle units. The cost of the rectifier and solenoid operator is probably about equal to that of the equivalent motor-operating mechanism.

Figure 189 illustrates the Allis-Chalmers type SO-500 solenoid operator, and Fig. 190 shows a Westinghouse solenoid unit energized through a Rectox which has replaced the type CF motor-operated mechanism.

3. *Motor.*—Several types of breaker-operating equipment have been developed with the motor, one type using the motor associated with compression springs, another having the motor operate centrifugal flyballs, and a third working through a cam-type mechanism.

Figure 191 shows the cam-type operating mechanism designed for closing the breaker in about one-half second, *i.e.*, faster than the solenoid or centrifugal motor-type mechanisms. The driving

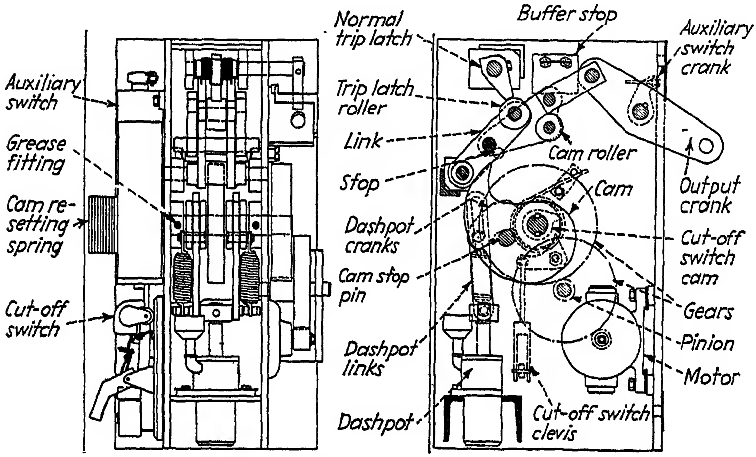


FIG. 191.—Diagram of General Electric motor-operated cam-type mechanism for oil circuit breakers, type ME-4.

element consists of a specially designed high-starting torque motor arranged to rotate a cam approximately 270 deg. through a spur-gear reduction. This driving unit is substituted for the solenoid coil and plunger of solenoid mechanisms, thereby utilizing the same simple, reliable linkage and being mechanically trip free in all positions. It eliminates the slow starting of solenoids caused by the gradual current build up and reduces slam and over-travel at the end of the closing stroke.

4. *Hydraulic and Air Mechanisms.*—For some of the large breakers used with the isolated-phase type of bus construction, breaker-operating mechanisms of the hydraulic and compressed-air type have been used.

139. Control Features of Breaker-operating Mechanisms. 1. *Mechanically Trip-free Mechanism.*—All automatic circuit breakers should be free to trip on any abnormal condition pro-

vided for, regardless of the actuation of the closing mechanism. This is necessary so that, if the breaker were closed manually, the trip-free feature would allow it to open as soon as the trip coil is energized, without injury to the operator or without waiting for him to release the operating handle. Also if the breaker is closed by an operating mechanism or manually against a fault, it will be free to open without being hindered by the mechanism or the operator. Thus the CF motor-operating mechanism operates the breaker through a toggle, which may be forced above center by any one of the tripping devices acting on the trip lever and thus allow the slip-off trigger to release. The mechanism can be tripped, therefore, in any position between open and closed.

2. *Electrically Trip-free Mechanism.*—All automatic circuit breakers should be electrically trip free in the closed position if they are not mechanically trip free, to prevent the breaker reclosing on a fault. This is necessary in order that the breaker be free to trip on any abnormal condition provided for, regardless of the position of the control switch, and that it does not reclose even though the control switch is in the closing position. This is provided for in the CF mechanism in the form of a control relay and a trip-free relay. If the mechanism is closed against a fault and the control switch is held closed after the breaker trips, the trip-free relay will operate to open the motor circuit until the control switch is returned to normal. Thus the mechanism is electrically trip free.

140. Oil Circuit-breaker Rating.[1]

19-201 *Rated Voltage.*—The rated voltage of an oil circuit breaker is the highest rms voltage at which it is designed to operate.

19-203 *Rated Continuous Current.*—The rated continuous current of an oil circuit breaker is the designated limit of current in rms amperes which it will carry continuously without exceeding the limit of observable temperature rise.

19-204 *Rated Short-time Current.*—The rated short-time current of an oil circuit breaker is the highest current including the d-c component that the breaker shall be required to carry without injury for specified short-time intervals. The ratings shall recognize the limitations imposed by both thermal and electro-magnetic effects. The standards for short-time ratings shall be:

[1] A.I.E.E. Standards, No. 19, 1938.

(*a*) The rated momentary current is the maximum rms total current which the breaker shall be required to carry for any time however small up to one second. The current shall be the rms value including the d-c component during the maximum cycle as determined from the envelope of the current wave.

For determination of current see Appendix.

(*b*) The rated five-second current is the rms total current including the d-c component which the breaker shall be required to carry for five seconds. For practical purposes this current shall be measured at the end of the first second.

19-205 *Rated Making Current.*—The rated making current of an oil circuit breaker is the maximum rms current including the d-c component against which the breaker must be capable of closing, without the welding of or undue damage to the breaker or contacts, with rated control voltage at the closing mechanism.

19-206 *Rated Latching Current.*—The rated latching current of an oil circuit breaker is the maximum rms current including the d-c component against which the breaker must be capable of closing and latching, with rated control voltage at the closing mechanism.

19-207 *Rated Interrupting Current* (*Rated Interrupting Capacity*).—The rated interrupting current of an oil circuit breaker is the highest rms current at a specified operating voltage which the breaker shall be required to interrupt under the operating duty specified, and with a normal frequency recovery voltage equal to the specified operating voltage. Where limited by testing equipment, the maximum tolerance for normal frequency recovery voltage shall be 15 per cent of the specified operating voltage. The current shall be the rms value including the d-c component at the instant of contact separation as determined from the envelope of the current wave. For determination of current and normal frequency recovery voltage, see Appendix.

19-208 *Operating Duty* (*Duty Cycle*) *of an Oil Circuit Breaker.*—The operating duty of an oil circuit breaker shall consist of a specified number of unit operations at stated intervals.

19-209 *Unit Operation of an Oil Circuit Breaker.*—The unit operation of an oil circuit breaker shall consist of a closing followed immediately by its opening without purposely delayed action, the letters CO signifying the operations of the breaker: Closing-Opening.

19-210 *Standard Operating Duty of an Oil Circuit Breaker.* (Duty Cycle of Oil Circuit Breakers).—The standard operating duty of an oil circuit breaker shall be two unit operations (CO) with a 15-second interval between operations. For circuit breakers of the non-oil-tight construction having interrupting ratings of 50,000 kva and below, the interval shall be two (2) minutes.

19-211 *Standard Interrupting Rating.*—The standard interrupting rating shall be based on the standard operating duty. (See also 19-207.)

19-212 *Rated Interrupting Kva.*—The rated interrupting kva at a specified voltage is the product of rated interrupting current, voltage and proper phase factor, i.e. 1.73 for three phase; two for 2 phase; one for single phase. (See also 19-207.)

19-213 *Interrupting Performance of Oil Circuit Breakers.*

(*a*) An oil circuit breaker shall perform at or within its rated interrupting capacity without emitting flame or an appreciable quantity of oil. For breakers of the non-oil-tight construction only, the emission of limited quantities of oil may be permitted.

(*b*) At the end of any performance at or within its interrupting rating the circuit breaker shall be in the following condition:

(1) Mechanical—

The breaker shall be substantially in the same mechanical condition as at the beginning.

(2) Electrical—

The breaker shall be capable of withstanding rated voltage in the open position and of carrying rated continuous current at rated voltage, for limited time, but not necessarily without exceeding rated temperature rise.

After a performance at or near its interrupting rating, it is not to be inferred that the breaker can meet its interrupting rating without inspecting and, if necessary, making repairs.

141. Characteristics of an Alternator Short Circuit.[1]—When a short circuit occurs on an alternating-current power system, the current in the alternator starts out at a value that is limited by the generator voltage expended over the alternator effective resistance and leakage reactance, together with the external impedance of transformers, lines, reactors, etc., which are in circuit up to the fault. As the armature reaction becomes effective, the current will decrease gradually to the sustained short-circuit value. With the very large armature currents often prevalent in faults, the path of the armature-reactance flux will become saturated and the reactance be reduced below its normal value. The reaction replaces a magnetomotive force and is retarded by the mutual induction between the windings of the armature and the field and by the hysteresis and eddy currents in the poles. Since in the majority of cases in

[1] See Dalziel, Charles F., Decrement Curves for Power Systems, *A.I.E.E. Trans.*, February, 1934, p. 33.

power circuits the resistance is small compared with the reactance, the short-circuit current will lag almost 90 deg. behind the induced e.m.f. The decrease of the current is shown in Fig. 192*a*, *b*, and *c*, the wave of *a* being initially completely offset, that of *b* being initially partly offset, and that of *c* being

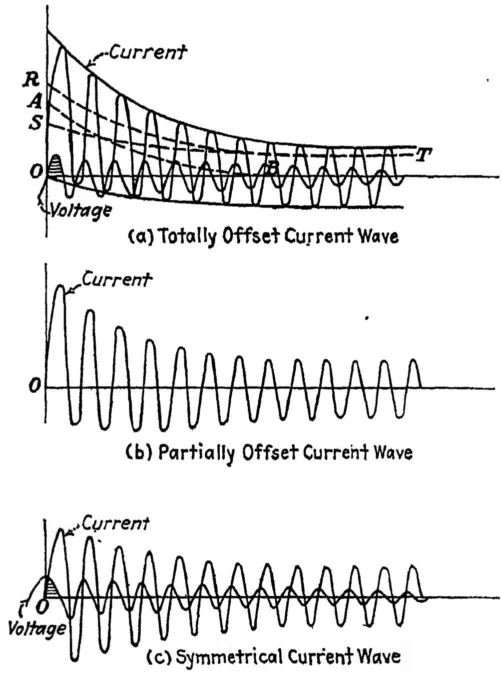


FIG. 192.—Alternator short-circuit current wave.

not offset at all, depending upon the point of the voltage wave at which the short circuit occurs.

If we consider the fault to occur very close to the alternator running with normal field so that the impedance of the load circuit is reduced to zero, then the initial wave of current will be limited only by the transient impedance of the generator and will be

$$I_1 = \frac{E_0}{\sqrt{r^2 + x^2}} = \frac{E_0}{x}^* \qquad (221)$$

where E_0 is the open-circuit voltage. The magnitude of the

* See CHRISTIE, C. V., "Electrical Engineering," p. 337, McGraw-Hill Book Company, Inc.

initial current will depend upon the time of the voltage cycle at which the short circuit occurs. If the short circuit is established at the time of the maximum point of the generated e.m.f. wave of a phase, then the current will be as given in Eq. (221). For as shown in Fig. 192*c*, if the armature resistance is neglected, the generated e.m.f. is consumed by the counter e.m.f. of armature inductance $= -L\frac{di}{dt}$. The crosshatched area under the e.m.f. wave is

$$\int_0^{\sqrt{2}E_0} e dt \text{ and is } = \int_{\sqrt{2}I_1}^{0} -L\frac{di}{dt}dt = \sqrt{2}\,I_1L,$$

and it causes the current to increase from zero to $\sqrt{2}\,I_1$. It is noticed that the current is practically symmetrical about the zero line, and its amplitude decreases as the armature reaction becomes effective.

If, however, the short circuit is established at the zero point of the generated e.m.f. wave, as illustrated in Fig. 192*a*, the crosshatched area under the wave is twice as great as before and the current builds up to double the value shown in Fig. 192*c*, *i.e.*,

$$I_2 = 2\sqrt{2}\,I_1 \tag{222}$$

As this current decreases, it alternates about the line *AB* in Fig. 192*a*, which line starts above the zero by $\sqrt{2}\,I_1$ amp. Thus the wave of total current may be considered as being made up of two components: a direct transient current *A-B*, which produces an offset of the wave and whose magnitude is dependent upon the point of the e.m.f. wave at which the short circuit occurs; and an alternating current *S-T*, which is independent of the point of the e.m.f. wave at which the short circuit occurs. The effective value of the total current *R-T* at any instant is equal to the square root of the sum of the squares of the direct component and the effective value of the alternating component at that instant.

The transient direct current may be explained by considering that in such a reactive circuit we perceive the physical limitation that a short-circuit current cannot reach its maximum value instantaneously, but that at time *t* equal to zero *i* must also equal zero. Therefore, for any instant of closing the short circuit on the system, there must be a transient current that

will just equal and offset the steady value of the current flowing in the circuit at that particular instant. Thus, if the short circuit is established at the point where the e.m.f. wave rises through zero, the current will be nearly 90 deg. behind and the transient will have to be equal to the current maximum and positive, as shown in Fig. 192*a*. For the e.m.f. wave falling through zero, the transient would equal the current maximum and be negative. For the e.m.f. wave at maximum, either positive or negative, the current would be nearly zero, and this unique point of short circuit would give no transient as in Fig. 192*c*.

The asymmetrical first-cycle value of the current in Eq. (222) is of extreme importance since, as was developed in Sec. 126 of Chap. VII, the electromagnetic stress and the heating in the breaker and connected structures are proportional to the square of the current. Therefore, for the low-voltage heavy-current breakers, any simplifying assumptions or inaccuracies which affect the peak ampere values must be critically examined. Even for high-voltage units with lower current-carrying capacity, the greater clearances and longer creepage distances necessary reduce the mechanical strength of the breaker. The heating is directly proportional to the square of the current and the time with a given cross section but it is inversely proportional to the mass to be heated. For a given loss then, the heat is a squared function of the current density of the current-carrying parts. Unless the circuit breaker opens the short circuit very rapidly, therefore, the heating may be important in breakers of any voltage. H. R. Woodrow[1] noted that these two functions comprise the usual breaker troubles where the electromagnetic stresses reduce the contact area of the movable parts and thereby increase the current density with resultant heat damage to the breaker parts.

142. The Time Element in Breaker Action and Methods of Tripping.—The decrement curves of short-circuit current in Fig. 192 show very plainly what effect a variation in the time of opening the contacts will have on the amount of current to be interrupted. The time of the opening will depend in part upon the method of control which is used to trip the circuit breaker. For nonautomatic operation, theoretically the time will be

[1] See Short-time Rating of Oil Breakers, *Elec. World*, Nov. 9, 1929, p. 944.

entirely controlled by the operator, but there will be the practical limitation of some 2 sec. delay before he could physically operate the breaker after an unexpected short circuit.

For automatic control of breakers installed in large-current-capacity or high-voltage circuits, current transformers and relays will be used. Hence, time is required to energize the magnetic circuits and to overcome the inertia and friction of the moving parts. Section 137 gives operating times of 3, 5, and 8 cycles for field tests on modern breakers, whereas the older units may take 15 to 30 cycles depending upon the type of control. The shortest time may be obtained for a breaker tripped directly by a cam or series trip coil which releases the locking mechanism. The breaker then opens by gravity, or spring tension may accelerate it at a rate greater than the action of gravity. A longer time corresponds to a shunt-trip coil where a current transformer, in series with the load current, passes its secondary current to operate a relay which energizes the shunt-trip circuit. The trip coil may be designed for either direct or alternating current. Since it is of vital importance that this control voltage be available at all times in order to operate the circuit breakers, the trip coils are generally supplied from a storage battery. The same device lends itself admirably to remote control of the circuit breakers.

With the use of the high-speed relays (see Chap. X) and their very fast operation, the decrement curves are not necessary. They may be used, however, for the slower overcurrent relays.

The time of operation will not be important, so far as variation of the short-circuit current is concerned, for a breaker on a large system at considerable distance from the generators. On account of the large amount of reactance which will be in circuit, representing the lines and apparatus between the location of the fault and the generators, the short-circuit current will be approximately constant and much smaller than would be involved for a fault closer to the generators.

143. Calculation of Initial Short-circuit Current.—Of the various alternator reactances defined in Sec. 112, we are here interested only in the transient reactance, since this reactance and the external impedance of the circuit determine the initial short-circuit current. In order to simplify the calculations, the resistance is generally neglected, thus avoiding the addition of

impedances at various angles and leaving only the reactances, with voltage drops all in the same phase. On this basis, with all reactances expressed in per cent of the circuit voltage from line to neutral, *i.e.*, $E = 100$ per cent xI normal, the amperes or kilovolt-amperes at the point of short circuit are found similarly to the method developed in Sec. 117, and the following sections of Chap. VII, by the equation

$$I_1 = \frac{100}{X_{\text{transient}} + X_{\text{external}}} \cdot I_{\text{normal}}. \tag{223}$$

Assuming that the current wave is completely offset, as in Fig. 192*a*, then the direct-current component is equal to the amplitude of the alternating component, and the effective value of initial short-circuit current is

$$\sqrt{(\sqrt{2}\,I_1)^2 + I_1^2} = \sqrt{3}\,I_1. \tag{224}$$

Thus if the alternator transient reactance is 25 per cent and the circuit reactance to the fault is 50 per cent, then from Eq. (223) the maximum effective value of the alternating-current component of initial short-circuit current

$$I_1 = \frac{100}{25 + 50} I_n = 1.33 I_n, \tag{225}$$

and for completely offset total current the effective value of initial short-circuit current is

$$I_1 = \sqrt{3} \times 1.33 I_n = 2.30 I_n. \tag{226}$$

The selection of the proper circuit breaker for this point of the circuit, then, will be governed by the value of the greatest transient disturbance which can occur here on short circuit. Because of the impracticability of determining the short-circuit transient by actual test on the system, it will be found by analysis on the system short-circuit test board or, as has been outlined in Chap. VII, by computation based on the approximate method of using only the reactances of the circuit.

144. Application of Oil Circuit Breakers.—After the determination of the maximum short-circuit current at the system point where the circuit breaker is desired, the breaker which is selected must have sufficient current, voltage, short-time and interrupting-capacity ratings at the prescribed service voltage to meet the

requirements. Also the following factors which affect the choice of the device must be considered.

The size of breaker to install must be thought of in terms of the possible future increase in short-circuit duty at the point of the system where the breaker is located. New generating capacity, new lines and transformers coming into service will surely make breakers hopelessly inadequate unless they have been installed with large capacity in the first place. A station with 20,000 kw. on the bus now may even grow to 100,000 kw. within five years or so.

The method of system neutral connection will have a bearing on the choice of breaker equipment. For ground faults on a system, a neutral impedance will increase the stability by reducing the short-circuit currents and preventing demagnetization, but this will not be effective for line-to-line or for three-phase faults. Switching surges, lightning discharges, and arcing grounds must be considered. A more liberal voltage rating on the breaker will be required for an ungrounded system than for one that is grounded.

It must be remembered that a certain kilovolt rating is simply a manufacturer's classification, and that for main transmission lines with their attendant surges and oscillations, the engineer's judgment may call for the use of a breaker with a considerably higher than normal kilovolt rating. Thus, for example, all the outside breakers on a certain metropolitan 24-kv. transmission system are rated at 37 kv.

Any special-duty cycle which might be prescribed for the breaker, such as automatic reclosing, would be a factor in the breaker selection.

The availability of the oil circuit breaker for inspection and maintenance is important. If the breaker is in a customer's installation at the end of a line, it may get only monthly inspection. If an air switch is shunted around the breaker or a transfer bus is installed, the maintenance facilities are improved, and some reduction in the breaker rating may be permitted.

145. Effect of Automatic Voltage Regulators.—The automatic voltage regulator works on the principle of vibrating contacts closing a shunt around the exciter field rheostat at low voltage on the alternator, or inserting the field rheostat for high voltage on the alternator. These actions raise or lower the exciter voltage

applied to the alternator field so as to maintain the alternator voltage at the determined constant value. In the event, then, of a short circuit on the alternator leads with the attendant drop in voltage, the regulator will increase the exciter voltage in an endeavor to hold the machine voltage up to normal. Such maximum exciter voltage will be approximately 50 per cent greater than that assumed for the decrement curve for full load, 80 per cent power factor on the alternator. If the short circuit has reduced the machine voltage, the resultant flux density in the machine has reduced, and a 50 per cent increase in excitation will

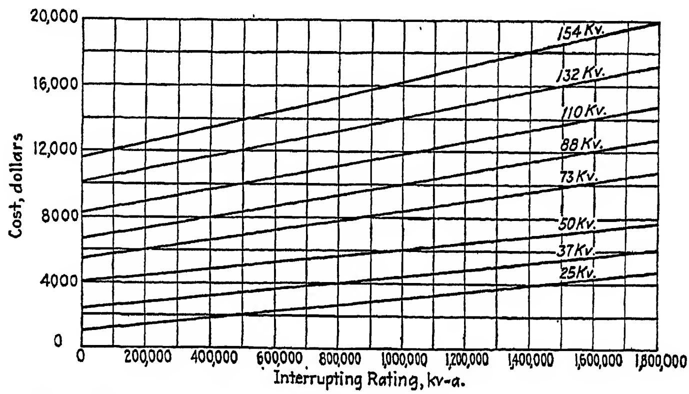


FIG. 193.—Cost versus interrupting rating for oil circuit breakers, 25 kv. and above.

at the end of 2 to 3 sec. result in an increase of 50 per cent in the sustained short-circuit current.

There would be no such increase in the sustained current in the case where the total reactance in the circuit is so large that it can limit the current to a value at which the regulator can maintain the alternator normal terminal voltage.

146. Cost of Oil Circuit Breakers.—Figure 193 shows the comparative costs of oil circuit breakers as presented by E. C. Stone in a paper entitled "The Oil Circuit Breaker Situation from an Operator's Viewpoint."[1] Brown's "Electrical Equipment"[2] quotes three-pole breakers, indoor, \$0.4 to \$1.5 per 100 kva. of interrupting capacity; outdoor, \$0.5 to \$2.0 per 100 kva. of interrupting capacity.

[1] *A.I.E.E. Jour.*, July, 1925.

[2] McGraw-Hill Book Company, Inc.

Although the cost of an ample equipment in oil circuit breakers is undoubtedly high, the value of extreme continuity of service is such that it justifies any reasonable gain in security, control, and flexibility.

147. Examples in Application, Three-phase Short Circuits.—To select the proper oil circuit breakers and to set the control relays to operate them will require the determination of the values of the short-circuit currents in the system. It will be remembered that the solutions are based on the assumption that

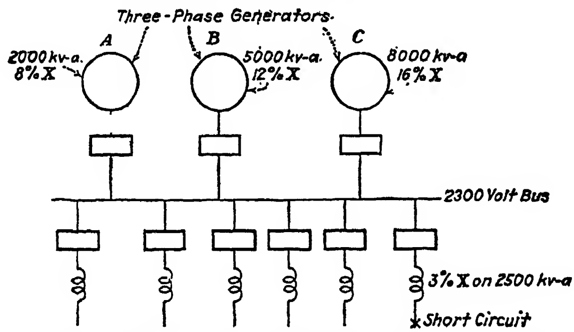


FIG. 194.—Diagram for three-phase short circuit on feeder.

resistance and capacitance have been neglected, hence the results will be approximate. Also for a symmetrical three-phase short circuit on all phases, we shall have the simple case of balanced currents.

In Fig. 194,[1] there is a three-phase short circuit on one of the feeders, outside the feeder reactor, as shown. Since the generators are all in parallel, the combined reactance, as was developed in Sec. 117, based on say 12 per cent, will be

$$\text{For generator A} = 12\%\ X \text{ on } 3{,}000 \text{ kva.} \tag{227}$$
$$\text{For generator B} = 12\%\ X \text{ on } 5{,}000 \text{ kva.} \tag{228}$$
$$\text{For generator C} = \underline{12\%\ X \text{ on } 6{,}000 \text{ kva.}} \tag{229}$$

or

$$\text{Total equivalent} = 12\%\ X \text{ on } 14{,}000 \text{ kva} \tag{230}$$

But the total generator capacity is 15,000 kva.; hence transferring the equivalent reactance of the generators to 15,000 kva. gives

$$12.85 \text{ per cent } X \text{ on } 15{,}000 \text{ kva.} \tag{231}$$

[1] From HEWLETT, MAHONEY, and BURNHAM, Oil Circuit Breakers, *A.I.E.E. Trans.*, February, 1918.

Now we have the feeder reactance of 3 per cent on 2,500 kva. in series with the combined generators giving

$$\begin{aligned} &\text{For combined generators} &= 12.85\%\ X \text{ on } 15{,}000 \text{ kva.} &\quad (232)\\ &\text{For feeder reactor} &= \underline{18.00\%\ X \text{ on } 15{,}000 \text{ kva.}} &\quad (233)\\ &\quad\text{Total to fault} &= 30.85\%\ X \text{ on } 15{,}000 \text{ kva.} &\quad (234) \end{aligned}$$

The normal three-phase current for 15,000 kva. at

$$2{,}300 \text{ volts} = \frac{15{,}000}{\sqrt{3} \times 2.3} = 3{,}760 \text{ amp.} \qquad (235)$$

The effective value of the alternating-current component of short-circuit current, based on the transient reactance, is then

$$\frac{100}{30.85} \times 3{,}760 = 12{,}200 \text{ amp. at } 2.3 \text{ kv.} \qquad (236)$$

(48,500 kva.)

and the peak value for the first half cycle of a totally offset

$$\text{wave} = 2\sqrt{2} \times 12{,}200 = 34{,}400 \text{ amp.} \qquad (237)$$

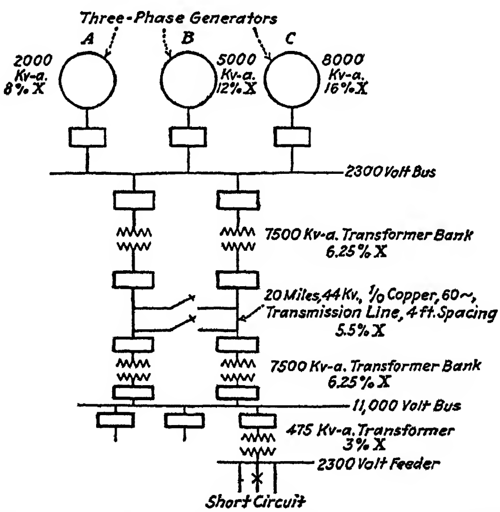


FIG. 195.—Diagram for three-phase short circuit on the low-voltage distribution.

In Fig. 195,[1] there is a three-phase short circuit on one of the 2,300-volt feeders below the 475-kva. transformer. From Eq.

[1] From HEWLETT, MAHONEY, and BURNHAM, Oil Circuit Breakers, *A.I.E.E. Trans.*, February, 1918.

(231), we have the generators represented as 12.85 per cent X on 15,000 kva. From the generator bus to the 11,000-volt bus, there are two symmetrical systems in parallel each of the following reactances:

$$\begin{array}{lrl} \text{Step-up transformer} & = & 6.25\%\ X \text{ on } 7{,}500 \text{ kva.} \quad (238) \\ \text{Transmission line} & = & 5.50\%\ X \text{ on } 7{,}500 \text{ kva.} \quad (239) \\ \text{Step-down transformer} & = & \underline{6.25\%\ X \text{ on } 7{,}500 \text{ kva.}} \quad (240) \\ \text{Series total per system} & = & 18.00\%\ X \text{ on } 7{,}500 \text{ kva.} \quad (241) \end{array}$$

Then the two systems in parallel will be

$$18.00\%\ X \text{ on } 15{,}000 \text{ kva.} \tag{242}$$

and adding the generators in series

$$\underline{12.85\%\ X \text{ on } 15{,}000 \text{ kva.}} \tag{243}$$

$$\text{The total to 11-kv. bus} = 30.85\%\ X \text{ on } 15{,}000 \text{ kva.} \tag{244}$$

Now the 3% on 475-kva. transformer

$$\text{when based on 15,000 kva.} = \underline{94.70\%\ X \text{ on } 15{,}000 \text{ kva.}} \tag{245}$$

And the total reactance to fault

$$= 125.55\%\ X \text{ on } 15{,}000 \text{ kva.} \tag{246}$$

The normal three-phase current for 15,000 kva. at 2,300 volts is

$$\frac{15{,}000}{\sqrt{3} \times 2.3} = 3{,}760 \text{ amp.} \tag{247}$$

The effective value of the alternating-current component of short-circuit current, based on the transient reactance, is then

$$\frac{100}{125.55} \times 3{,}760 = 2{,}990 \text{ amp. at } 2.3 \text{ kv.} \tag{248}$$

(11,900 kva.)

and the peak value for the first half cycle of a totally offset wave

$$= 2\sqrt{2} \times 2{,}990 = 8{,}440 \text{ amp.} \tag{249}$$

Similarly determine the necessary kilovolt-amperes, voltage, and ampere capacity of all the oil circuit breakers shown, for the particular system fault and connections that would give the maximum interrupting duty on the breaker.

148. Use of Short-circuit Calculating Board.—The numerical values of the short-circuit currents at any desired location may be determined without too much labor, for the circuits shown in Chaps. VII and VIII, by computation with the slide rule. For the complicated networks of large systems, however, the problem may be set up on a short-circuit calculating board and the desired values obtained by instrument readings at the required points.

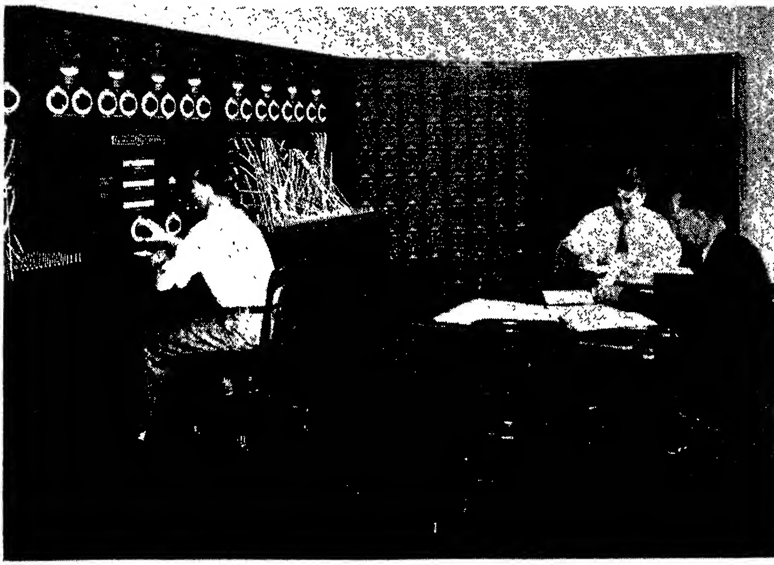


Fig. 196.—General Electric 480-cycle, alternating-current network analyzer. Operator at centralized metering system board.

Figure 196 shows a recent alternating-current network analyzer[1] assembled with a central instrument and control cabinet, then on either side a circuit-connecting cabinet, and next the wings of four cabinets containing the adjustable circuit-element units. Five different types of circuit units are installed, as follows:

1. One hundred fifty line-impedance units of adjustable resistors and reactors connected in series.
2. Fifty load-impedance units of adjustable resistors and reactors which may be connected either in series or in parallel.
3. Thirty capacitor units, adjustable.

[1] See A New A-C Network Analyzer, by Kuehni and Lorraine, *A.I.E.E. Trans.*, February, 1938.

4. Twelve autotransformer units to step up or down the voltage at points in the network.

5. Fifteen mutual-transformer units to simulate magnetic coupling between circuits.

Twelve single-phase generator units, adjustable for phase angle and voltage, are installed in compartments at the top of the two plugging cabinets and the central instrument cabinet. The nominal voltage is 50 volts, and the nominal current is 50 milliamp. Power is supplied to the generator units by a 5-kva. motor generator at three-phase, 480 cycles, 440 volts, through an autotransformer. There are 300 key switches in the analyzer, one for each circuit.

149. Problems.

1. A three-phase feeder of No. 0 copper wire is 4 miles long delivering 25-cycle power at 7,500 volts with 18 in. between wires. For No. 0 wire, R is 0.5327 ohm per mile and X is 0.251 ohm per mile. The feeder is supplied from a large system where a short in the feeder would not pull down the system voltage. What is the maximum possible current for a three-phase short circuit at delivery end of line? Select the circuit breaker for the feeder.[1]

2. A three-phase transformer bank is made up of three single-phase 200-kva. transformers having 3.5 per cent reactance and rated at 11,000 volts high tension and 2,200 volts low tension. The bank feeds a 2,200-volt line from a high-capacity power line. Select the circuit breaker for the feeder line to be located just beyond the transformer bank.[1]

3. Two 18,000-kva. three-phase 8,800-volt alternators of 6 per cent reactance have a bus section from which a 22,000-volt transformer bank of 2,000 kva. per phase and 3 per cent reactance feeds out to a circuit breaker. Select the breaker.[1]

4. For the system shown in Fig. 197, find the current that a breaker will have to interrupt. Bus reactances are based on 100,000 kva. The common bus voltage is 12 kv. Find the currents and kilovolt-amperes.[2]

a. For a three-phase short at X.

b. For a three-phase short at Y.

5. A station is connected as shown in Fig. 198. What is the reactance of the station at the 12,000-volt bus?

How much short-circuit current must be interrupted by breakers, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.[3]

[1] From MacNeill, J. B., Oil Circuit Breakers and Their Application, *Elec. Jour.*, August, 1916.

[2] From Woodward, W. R., Application of Decrement Factors in Short-circuit Studies, *Elec. Jour.*, May, 1924.

[3] Westinghouse *Problem* 2013.

6. Figure 199 shows the power-system supply for a steel mill,[1] the total power (stations 1 + 2 + 3) being 58,000 kva., 6,600 volts, three-phase. All

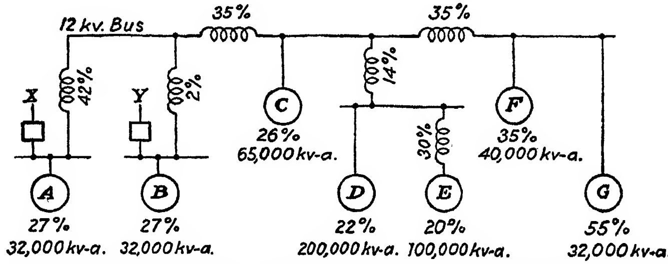


FIG. 197.—Three-phase short circuit on a network of dissimilar generators, Problem 4.

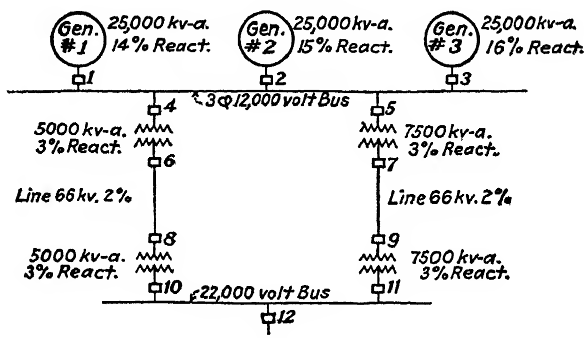


FIG. 198.—Diagram for circuit-breaker application in network, Problem 5.

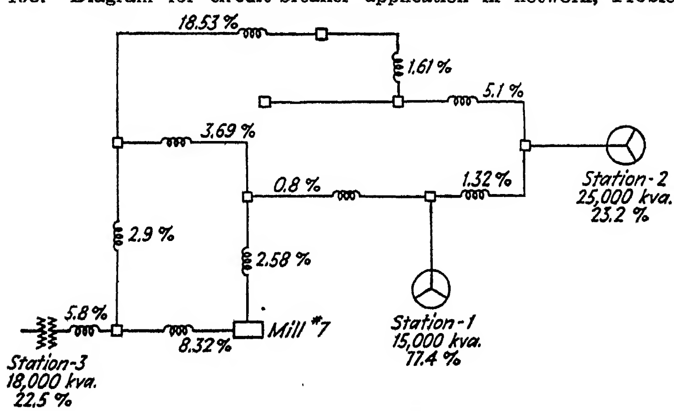


FIG. 199.—Power-system supply for a steel mill, Problem 6.

reactances, including those at stations 1, 2, and 3, are based on 58,000 kva. Find the short-circuit amperes for a three-phase bus fault at Mill 7. Use delta Y transformations.

[1] From *Elec. Jour.*, September, 1924, p. 412.

7. In the loop-distribution system[1] of Fig. 200, all the reactances are rated on full line to line voltage. The power factor of each load is 80 per cent lagging. The generator bus is maintained at 7 kv., and all transformers are

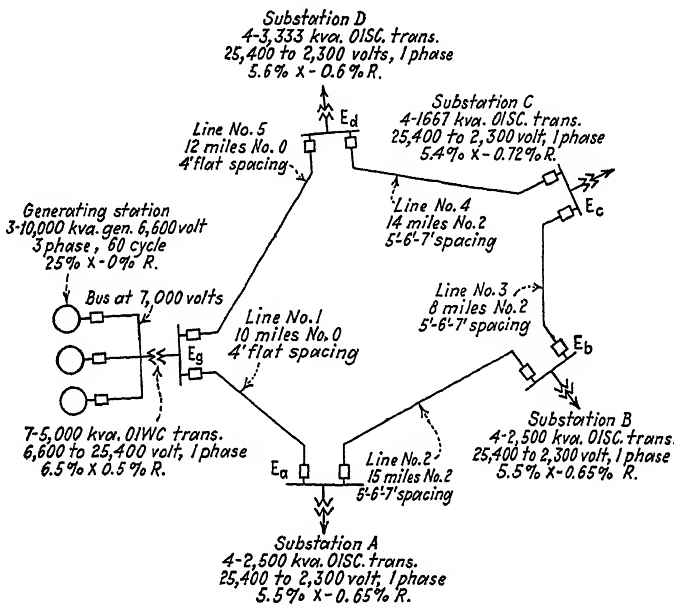


FIG. 200.—Loop-distribution system, Problem 7. (*Westinghouse Prob. Series* 5, *No.* 2.)

connected delta on low-voltage side and star on high-voltage side. One transformer is used as a spare in each bank.

a. Compute the maximum short-circuit currents all breakers must handle for normal feed, line 1 open, line 5 open.

b. Compute voltage regulation at each load for normal feed, line 1 open, line 5 open.

[1] *Westinghouse Problem* 2, series 5.

CHAPTER IX

INTRODUCTION TO SINGLE-PHASE SHORT CIRCUITS

150. Single-phase Short Circuits.—The symmetrical balanced currents for a full three-phase fault, as discussed in Chap. VIII, are of rare occurrence in practice. More often, the short circuit will involve only a single phase, or a conductor will ground on a grounded neutral system giving extremely unbalanced currents. The solution of the system network for these currents will therefore be much more difficult than for the balanced currents. Naturally, most of the faults occur on the transmission and distribution systems, which, because of their large mileage of exposed conductors, are affected by lightning and extremely high winds. On 7,140 miles of line (right of way) of which 3,890 were of two-circuit, steel-tower construction, 1,725 of single-circuit, steel construction, and 1,526 of single-circuit, wood construction, with transmission voltages of 110–165 kv., the average outages because of lightning per 100 miles of line per year were reported[1] as 30 for lines with no ground wire, 9.8 with one ground wire, and 7.0 with two ground wires.

151. Generator Reactances on Single-phase Short Circuits.—For a single-phase short circuit in a three-phase alternator, the resulting armature reaction may be considered as made up of the ordinary three-phase armature reaction with equal currents, and a single-phase armature reaction due to the excess current in the shorted phase.[2] This single-phase armature reaction produces a double-frequency pulsation of the field and a third harmonic of e.m.f. in all the phases. The reactances of an alternator are therefore different for a single-phase short circuit than for a three-phase short circuit. Table 28 gives the relations, single-phase to three-phase, for the various single-phase circuits through a Y-connected generator as shown in Fig. 201, *a*, *b*, and *c*.

[1] See Lightning Performance of 110–165 kv Transmission Lines, Report of Committee, *A.I.E.E. Trans.*, June, 1939.

[2] See Christie, C. V., "Electrical Engineering," McGraw-Hill Book Company, Inc.

TABLE 28.—RELATIONS OF GENERATOR REACTANCES, SINGLE-PHASE TO THREE-PHASE SHORT CIRCUITS[1]

a. For instantaneous short circuits

Generator connection	Current distribution	Number of times three-phase current	Multiplier for reactance
Y	1-0-0	1.5	0.666
Y	1-1-0	1.0	0.866
Y	1-1-2	1.155	0.866

b. For sustained short circuits

Generator connection	Current distribution	Number of times three-phase current		Multiplier for reactance	
		Salient pole	Distributed pole	Salient pole	Distributed pole
Y	1-0-0	2.0	2.5	0.5	0.4
Y	1-1-0	1.277	1.5	0.677	0.577
Y	1-1-2	1.47	1.73	0.677	0.577

[1] From LEWIS, W. W., "Transmission Line Engineering."

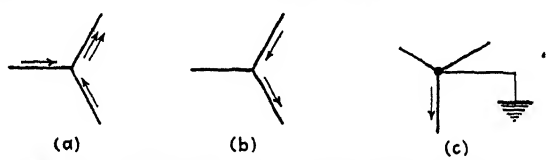


FIG. 201.—Diagram of single-phase short-circuit current flow in a Y-connected generator.

152. Solution of Single-phase Line-to-line Short Circuit.—Figure 202[1] shows a single-phase line-to-line short circuit on a transmission line supplied by a Y-connected generator through a delta-Y step-up transformer. If the resistance is neglected, the reactance is considered as normal three-phase ohms reactance based on the actual voltage of the circuit. The phase transformation is taken as 1:1 and later modified for the current ratio.

Let the high-tension short-circuit current flowing in the line be i, based on the voltage c-a, and the current distribution then be as shown. From the voltage relations in the circuit, we see that in line with c-a

[1] From LEWIS, W. W., "Transmission Line Engineering," McGraw-Hill Book Company, Inc.

$$\text{Volts } h - g - f = \frac{1}{2} \cdot \frac{E}{3} + \frac{E}{3} = \frac{E}{2} \tag{250}$$

$$\text{Volts } e - d = 0.866 \times 0.577E \tag{251}$$

$$\text{Volts } c - a = 2 \times 0.577E \times 0.866 = 2iX_3 + 2iX_4. \tag{252}$$

Now the generator voltage consumed in the short circuit is equal to the sum of the reactance drops and countervoltages round the

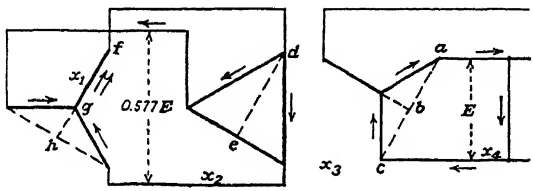


FIG. 202.—Diagram of a single-phase, line-to-line short circuit.

circuit. Summing up the voltages, therefore, from h round the circuit and return, we have

$$\frac{E}{2} = 3iX_1 + 3iX_2 + \text{voltage } e - d. \tag{253}$$

Substituting for voltage $e - d$ from Eq. (251), and its value in terms of $c - a$ from Eq. (252), then

$$\frac{E}{2} = 3iX_1 + 3iX_2 + iX_3 + iX_4 \tag{254}$$

or

$$E = 6iX_1 + 6iX_2 + 2iX_3 + 2iX_4 \tag{255}$$

and

$$i = \frac{E}{6X_1 + 6X_2 + 2X_3 + 2X_4}. \tag{256}$$

Assume the following values:

$$E = 110{,}000 \text{ volts.}$$

Generator voltage = 13,200 volts.

The turn ratio of the transformer $= \dfrac{63{,}500}{13{,}200} = 4.81.$

$X_1 = 0.808$ ohm, 13,200 volts.
$X_2 = 2.08$ ohms, 13,200 volts.
$X_3 = 39.1$ ohms, 110,000 volts.
$X_4 = 9.07$ ohms, 110,000 volts.

In order to express the low-tension reactance values in high-tension equivalents, the percentage reactance drop shall be the same in terms of either the high- or low-voltage winding, thus, for single-phase short circuit,

$$\frac{i_1 X_1 \text{ ohms}}{13{,}200}\ (\%\text{ low voltage}) = \frac{i_2 X_1'}{63{,}500}\ (\%\text{ high voltage}). \tag{257}$$

But

$$i_2 = i_1 \times \frac{13{,}200}{63{,}500} \tag{258}$$

then

$$\frac{i_1 X_1}{13{,}200} = i_1 X_1' \times \frac{13{,}200}{63{,}500^2} \tag{259}$$

and

$$X_1' = X_1 \left(\frac{63{,}500}{13{,}200}\right)^2 = X_1 R^2 \tag{260}$$

where R is the turn ratio of the transformation.

Then in high-tension equivalents the reactances are

$$\begin{aligned} X_1' &= 0.808 \times (4.81)^2 = 18.7 \text{ ohms.}\\ X_2' &= 2.08 \times (4.81)^2 = 48.1 \text{ ohms.}\\ X_3 &= \dots\dots\dots\dots\dots\dots 39.1 \text{ ohms.}\\ X_4 &= \dots\dots\dots\dots\dots\dots 9.07 \text{ ohms.} \end{aligned}$$

For the current flow shown in the generator, Table 28 gives the single-phase instantaneous generator current as 1.155 times the three-phase current; hence the ordinary instantaneous line-to-neutral reactance must be multiplied by the factor 0.866 to obtain the correct reactance for instantaneous single-phase short circuit. As corrected for single-phase, then, substituting in Eq. (256),

$$i = \frac{110{,}000}{(6 \times 18.7 \times 0.866) + (6 \times 48.1) + (2 \times 39.1) + (2 \times 9.07)} \tag{261}$$

$$= \frac{110{,}000}{482.1} = 228 \text{ amp.} \tag{262}$$

153. Solution of Single-phase Line-to-neutral Short Circuit.—Figure 203[1] shows a single-phase line-to-neutral short circuit on a

[1] From Lewis, W. W., "Transmission Line Engineering," McGraw-Hill Book Company, Inc.

system with a grounding transformer. Let i be the high-tension current flowing through the short-circuited transformer NA and the current distribution in the system be as shown. Since only the reactance of the circuit is considered, this current lags 90 deg. behind the transformer voltage NA, which is taken as a base for the solution. This primary current in NA produces an equal current, on a 1 : 1 ratio, in the secondary RT which must circulate

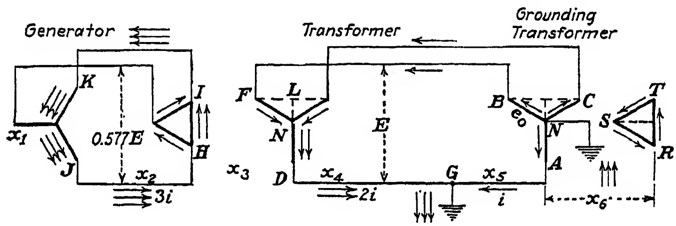


FIG. 203.—Diagram of a single-phase, line-to-neutral short circuit. The system neutral is established by a grounding transformer.

in the closed delta, therefore there are equal currents in NB and NC. Because of the current in NB, there is a drop in this leg, the component voltage in the direction AN being marked e_0. The applied terminal voltage e_0 minus the reactance drop iX_6 in the winding is equal to the induced voltage SR. But the vertical component of

$$SR = \frac{RT}{2} = \frac{1}{2}(iX_5 + iX_6), \tag{263}$$

since voltage RT is consumed in the short-circuited transformer leg.

Therefore,

$$e_0 = iX_6 + \frac{1}{2}(iX_5 + iX_6) = \frac{3}{2}iX_6 + \frac{1}{2}iX_5. \tag{264}$$

Writing the voltage equation of the circuit from the generator at K round the circuit and back, then

$$0.577E - 6iX_1 - 6iX_2 - 2iX_3 = \text{induced volts } IH \tag{265}$$

but

$$\text{Induced volts } IH = \tfrac{2}{3}LD \tag{266}$$

$$= \tfrac{2}{3}(3iX_4 + e_0 + iX_5). \tag{267}$$

Substituting for e_0 from Eq. (264),

$$\text{Induced volts } IH = \tfrac{2}{3}(3iX_4 + \tfrac{3}{2}iX_5 + \tfrac{3}{2}iX_6). \tag{268}$$

$$= 2iX_4 + iX_5 + iX_6. \tag{269}$$

Substituting this value for induced volts IH in Eq. (265), then

$$0.577E = 6iX_1 + 6iX_2 + 2iX_3 + 2iX_4 + iX_5 + iX_6 \quad (270)$$

and

$$i = \frac{0.577E}{6X_1 + 6X_2 + 2X_3 + 2X_4 + X_5 + X_6}. \quad (271)$$

Reactances X_1 and X_2 should be transferred, as in the previous example, to the high-voltage side of the transformer, and all values should be modified by the correct factor to transfer them from ordinary three-phase values of reactance to the special single-phase values. In particular, the reactance of a grounded conductor is larger than the normal three-phase reactance, depending upon the soil and local conditions. As the result of tests on five power systems, Lewis gives the following average values:[1]

Arrangement	Per Cent of Three-phase Reactance
Three-phase, one conductor to neutral no ground	100
Three conductors in multiple, ground return	117
Two conductors in multiple, ground return	136
One conductor, ground return	185

The zero phase-sequence reactance of transmission lines is three times the reactance of three conductors in multiple with ground return; *i.e.*, for a single-circuit line of average spacing $3 \times 1.17 = 3.5$ times the ordinary three-phase reactance. The three is introduced because the reactance is referred to the current in one phase only, which is one-third of the ground current.

The simplicity of the ohmic or percentage method of computing single-phase fault currents, illustrated above, recommends it in these cases for problems involving circuit-breaker capacity, the setting of the relays, and the stability of the system. However, for faults in complicated networks including polyphase rotating machines, the student is referred to the more systematic method of symmetrical components,[2] referred to in Sec. 113.

[1] "Transmission Line Engineering," p. 153, McGraw-Hill Book Company, Inc.

[2] See texts "Symmetrical Components," by Wagner and Evans, and "Applications of the Method of Symmetrical Components," by W. V. Lyon, McGraw-Hill Book Company, Inc.

CHAPTER X

PROTECTIVE RELAYS AND THEIR APPLICATIONS

154. Definition and Purpose.—In accordance with our plan to furnish continuous service of a high quality over a more or less widely spread power and lighting system, we must be prepared to protect the elements of such a system, its feeders, transformers, tie and transmission lines, and its generators, etc., from the hazards of storms, accidents, and abnormal conditions which arise in operation. To do this, it will frequently be necessary, upon the development of trouble, to take some part of the system out of service until the cause of the disturbance has been removed. It is extremely important that such a service interruption be minimized as much as possible, *i.e.*, that only the elements of the system absolutely involved be segregated surely and rapidly and be isolated by opening the controlling circuit breakers. High speed in the clearing of a fault is of the greatest importance in an interconnected system, since it will be most effective in maintaining the stability of the system, will prevent injurious heating in apparatus due to prolonged short-circuit currents, and will limit the duration of the drop of voltage in the adjoining healthy sections. These requirements have demanded in recent years the development of high-speed breakers which can clear line faults in 8, 5, or 3 cycles (60-cycle basis) and relays which are capable of operating in 1 cycle or less.

The selection of the protective system for any individual circuit cannot properly be made until the nature and characteristics of the entire system are considered, since the relays for such circuit must work in conjunction with the other protective apparatus installed. Naturally thrift enters prominently into the design, since the cost of various systems will spread over wide limits, the price increasing with the degree of elaboration and thoroughness of the protection. As counterbalancing the greater cost of an improved protection, we may consider the lesser damage to circuits and apparatus and the smaller invest-

ment in duplicate lines and equipment necessary to render the desired quality of service. An adequate system is certainly justified since, as H. R. Woodrow said, "Although relay protection constitutes only 1 per cent of the system investment, probably no other item has played a more important part in the development of reliability." The protective relay is entrusted with the task of selecting the necessary portion of the system which ceases operation for the moment, accomplishing this through its control of the associated circuit breakers. Therefore, we are vitally interested in the fundamental characteristics of some of the protective relays and the methods of their application to heavy power service. In this field, there are offered to the investigator fascinating problems in critical analysis of electrical conditions and wide opportunities for skill and ingenuity in design and application.

The protective relay is a combination electrical-mechanical device which, on the occurrence of the particular abnormal condition in the circuit or apparatus which it has been installed to detect, will operate after a proper discrimination and cause the circuit breaker to open in order to protect the circuit or equipment which it guards from the abnormality. It is expected automatically to isolate faulty lines and apparatus from the remainder of the system whenever the condition arises that it was installed to control. This again emphasizes the principle brought out in Chaps. VII and VIII that it is necessary to have an accurate knowledge of what current values may be expected for the various fault conditions occurring on the power system to be protected. One of the most useful types of relay protection, *viz.*, the principle of current selectivity, is absolutely dependent upon the correct analysis of trouble currents for the success of its application.

The relay consists essentially of a coil or coils connected to the circuit so as to be affected by what takes place in the circuit and a movable part which travels under control of the coil to close a contact device. This last element works in the tripping circuit of the breaker, as, for example, energizing the circuit to the trip coils.

155. Requirements of a Relay. 1. *Definite Operation, Accuracy.*—Until its individual operating condition occurs, a relay should remain in dormant position. When, however, the specific condition occurs for which it is installed, the relay must surely

operate and hold its operating contacts closed until the required movements outside the relay have been completed. For an instantaneous type, the travel must be very fast; for an inverse-time or definite-time type, the operation must accord with the type. All relays should give the same consistent performance whenever their operating conditions recur. They should operate properly in spite of slight variations in frequency and wave form or the presence of ordinary stray magnetic fields.

2. *Selective Operation.*—The same type of relay may be used to protect a number of parallel tie lines, or one relay may be installed for differential protection of two parallel tie lines. In such an application, if a fault occurs the relay must trip out the circuit breaker on the line carrying the greatest fault current. Or perhaps relays control both branch- and main-feeder circuits, in which case for a short circuit on the branch feeder only the branch-feeder circuit breaker should open. In the latter case and many similar ones, the selection will be made on the basis of the time grading of the system, the various relays being given appropriate time settings. In general, the relay may be required to select from among abnormal conditions those faults which are inherent in the apparatus or circuits of its own sphere of influence, as distinguished from faults that may reside in the remainder of the system.

3. *Sensitivity.*—Since the relay is generally called upon to perform in case of heavy overloads, grounds, short circuits, reversals, and such abnormal occurrences, it should be able to operate under low voltage, poor power factor, or, in some types, for currents less than the rated capacity of the circuit.

4. *Flexibility and Extension of System Operation.*—The individual relays and the plan of relay protection for the system must provide sufficient flexibility to meet all the ordinary methods of system operation and to permit extending the system without the necessity of radically revising the protective plan.

5. *Operating Experience.*—An analysis of practical operating experience with a modern high-speed relay system over a 5-year period leads to the following conclusions:[1]

1. Automatic recording devices are essential to a proper understanding of the performance of a high-speed relay system. It has been demon-

[1] See Experiences with a Modern Relay System, by G. W. Gerell, *A.I.E.E.*, October, 1936.

strated that they will determine: (*a*) type of fault and phases involved, (*b*) location of fault, (*c*) fault isolation time, (*d*) magnitude of current and voltage, (*e*) sequence of relay and switch operations, (*f*) causes of incorrect relay operation, and (*g*) empirical formulae to determine switching speeds necessary to preserve system stability. In addition, automatic recording devices are of great assistance in guiding the improvement and extension of the relay system.

2. The reduction of relay operating time, considered apart from oil-switch operating time, cannot be carried beyond a certain minimum. Experience shows that in certain specific applications relay operating times of the order of one cycle or less may result in unreliable performance. The application engineer can readily recognize these proposed installations wherein the minimum relay operating time should be carefully determined.

3. Most faults on grounded-neutral transmission systems are initiated between ground and one conductor (93%). Therefore greatest consideration should be given to the application of relay systems operating on zero-phase-sequence quantities. It appears probable that in the past, phase protection has been overstressed in importance. In some instances phase relays may be entirely eliminated.

4. By the application of high-speed relays and oil circuit breakers the stability of power systems can be materially improved, approaching a point where the transient stability attains equality with the static stability limits. It is highly important that proper consideration be given to the relaying of the secondary transmission system. In some instances on the Union system, faults of secondary origin have resulted in the trip-out of high-voltage transmission circuits, because of unstable conditions caused by the fault.

5. It has been forcibly shown that an extensive protective system cannot be installed once and for all and then expected to function perfectly under all circumstances. The protection engineer must constantly be alert, watching for incorrect or doubtful operations, and must be ready to remove obsolete equipment and install new equipment of approved design. Long experience invariably will demonstrate that the original installation is inadequate to comply with advanced conceptions of desirable relay performance and will point the way to necessary additions and improvements. In every year from 1931 to the present time, changes have been made on this company's protective system. Modern relays and circuit breakers have been installed in place of those of earlier designs, and numerous other additions and improvements have been effected with the result that instability and incorrect relay operations now are quite remote.

With reference to item 2 above, Gerell doubts that quick-acting balanced relays operating in less than one-half cycle can ride

through properly the transients that exist on a system during switching surges that occur while faults are in the process of being isolated. Such relays protecting parallel lines interconnecting two power sources furnish the most likely situation where incorrect relaying may occur.

156. Classification of Typical Types of Relays.—The protective relays may be classified according to their functions, to the source of their operation, whether current, potential, current and potential, or to time. It is to be noted, of course, that the time characteristics may be combined with other characteristics if it is desired. Such a group of classifications follows:[1]

Current	Potential	Current and potential	Time characteristics
Overcurrent.....	Overvoltage	Directional phase	Instantaneous
Thermal........	Undervoltage	Directional ground	Inverse time
Ratio differential	Single phase	Impedance or distance	Definite time
Current balance.	Phase reversal	Overpower	Automatic time
Phase sequence..	Frequency	Power directional	High speed
Directional ground.......	Voltage differential	Directional distance (Reactance)	

157. The Overcurrent Relay.—This device has its winding continuously in the circuit and is so designed that at normal current its electromagnetic strength does not close its contacts. A predetermined excess of current, the amount of which can be adjusted, will operate the contacts. The two general types for protection of apparatus and alternating-current circuits are as follows:

1. The plunger type, in which a soft-iron core moves in a solenoid, the movement of the core closing the relay contacts, as in Fig. 204.

2. The induction type, in which an aluminum disk rotates in the magnetic field of the relay windings, the disk closing the contacts by its travel, shown in Fig. 205. These relays possess permanent operating characteristics and have very accurate timing; hence they permit of reliable selection. Type IAC

[1] From HALMAN, T. R., and NORTH, J. R., paper on "Protective Relays," Detroit-Ann Arbor Section, A.I.E.E. See also A Condensation of the Theory of Relays, by A. R. van C. Warrington, *Gen. Elec. Rev.*, September, 1940.

provides an inverse or a very inverse time characteristic, *i.e.*, a decrease in the operating time as the current increases, with a greater rate of decrease in the very inverse case (see Fig. 218 for the time-current curves of this type).

Figure 206 shows an instantaneous overcurrent relay of the type PYC. It can be used in conjunction with existing induc-

FIG. 204.—General Electric instantaneous overcurrent relay, typical of type PBC-12-B.

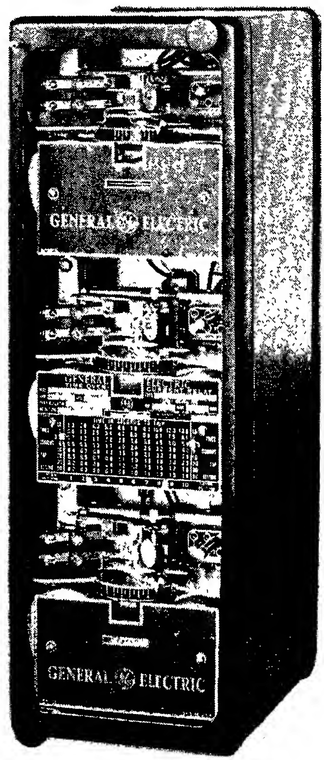


FIG. 205.—Time overcurrent relay, typical of type IAC-11-B.

tion-type overcurrent relays where instantaneous tripping is desired at high secondary currents. The pickup time varies from 0.005 to 0.05 sec. depending upon the time-current curve for which the relay is set.

If a directional element is added to an overcurrent relay, it may be used for directional overcurrent or directional ground

protection. Figure 207 illustrates a high-speed unit of this type. The high-speed directional element is identical with that used in the high-speed impedance relay described in Sec. 165.

158. The Current-balance Relay.—This relay is designed primarily for the short circuit and ground protection of parallel transmission lines. It compares the line currents, selecting the line carrying the heavier load; hence when applied the defective

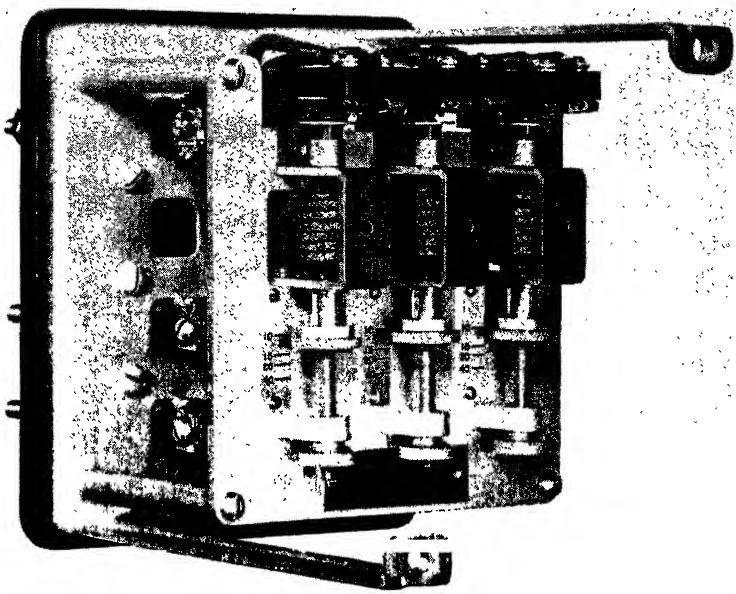


FIG. 206.—General Electric instantaneous overcurrent relay, model 12 PYC-11-A1.

line must be certain to carry the heavier current. "Through" faults then may not cause unbalance, yet a fault within its own section will operate the relay. Being dependent upon the balance in two lines, naturally this type of relay does not protect when one of the lines is out of service, or when both circuits of a double-circuit line are in trouble.

The type CD, shown in Fig. 208, works on the induction principle. Two overload elements act upon a common disk through a common magnetic circuit, each element being connected separately to its own current transformer in corresponding phases of the two balanced lines. The two elements are electrically opposed, and under balanced line loads the fluxes in the magnetic

circuit of the relay are equal and opposite, which results in zero torque on the relay disk. Under the proper conditions of current unbalance, the disk can rotate 80 deg. in either direction from zero and make contact on either side. This will trip the circuit breaker on the line carrying the heavier current.

If used for ground protection, the neutral currents of the two lines are balanced against each other and a ground on one line will show a predominance of current in the respective neutral circuit.

FIG. 207.—Type HR overcurrent, directional high-speed relay.

Figure 209 illustrates a high-speed relay, type HD, to clear faults on parallel lines. For phase-to-phase protection, six relays per pair of lines are required; and for ground protection, two additional relays per pair of lines are necessary.

Because of the extremely high speed involved, the mechanical construction is entirely different from that of the induction-type relay. A rectangular loop of aluminum is pivoted on the two ends and forms a short-circuited secondary of a small transformer, whose primary consists of two symmetrically tapped current windings. A current is induced in the loop which is proportional to the difference of the two line currents. This loop current interacts with the magnetic field set up in the secondary of the small saturating transformer. When the loop rotates counterclockwise, a silver plate bridges two stationary contacts.

159. The Ratio-differential Relay.—This relay is designed for the differential protection of alternating-current motors, generators, and transformers. It is a current-operated induction device, but unlike other differential relays, its operating current varies in proportion to the load. That is, if the instantaneous directions of currents in two sets of current transformers are the same, the current to operate the relay increases in proportion

to the line current rather than remaining constant for all load conditions. This special feature of the relay tends to prevent faulty tripping on through faults due to unbalanced current-transformer characteristics or loading, and allows the relay to be set for close protection at normal loads.

When used to protect generators or motors, the relay can be set to operate quickly on internal-trouble currents as low as 2.5 per

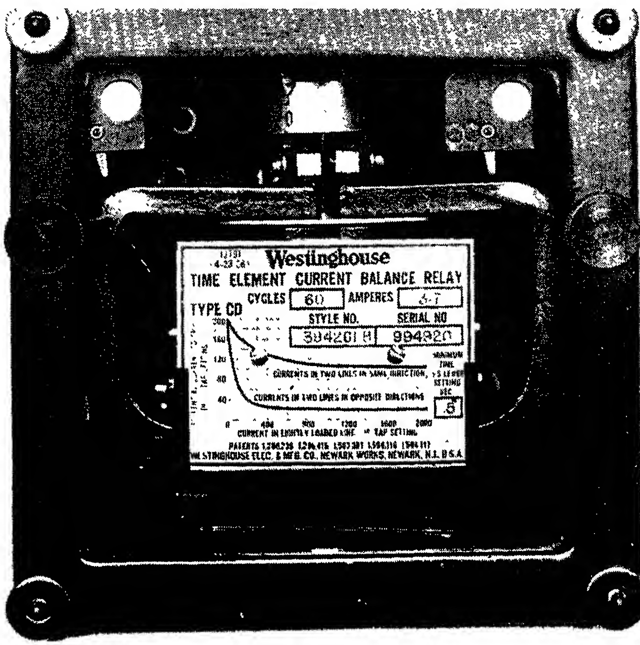


FIG. 208.—Westinghouse selective differential-current relay, type CD.

cent of the smaller secondary current without danger of tripping on heavy overloads or system troubles. The relay for generator protection is provided with taps so that it may be set to operate on an unbalance equal to 2.5, 5, 10, or 20 per cent of the smaller secondary current. For an application to generator, see Sec. 172.

For the protection of star-delta transformer banks, Scott-connected banks, or any case where the currents in the circuit to be balanced are unequal, the relay is provided with taps so that current-balancing autotransformers are unnecessary. For transformer protection, the relay operates on approximately 50 per

cent unbalance. Figure 210 shows the type CA of this relay, which is capable of operating in about 6 cycles. For an application to power transformers, see Sec. 173.

In addition to the older forms of the current-differential relay, new high-speed types have been developed. Figure 211 displays the RDD type for differential protection of alternating-current

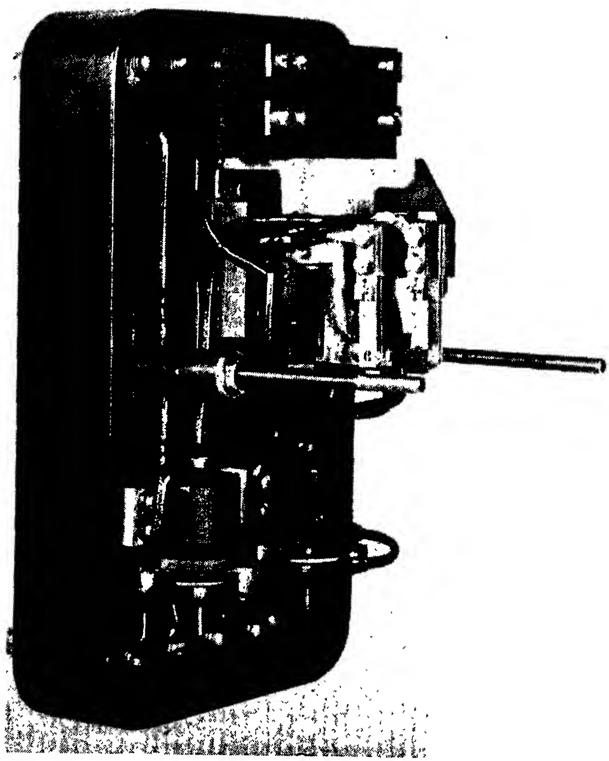


FIG. 209.—Westinghouse current-balance high-speed relay, type HD.

generators, frequency converters, synchronous condensers, and motors. Because of its balanced-beam construction, this relay can operate in one-half cycle. Under normal conditions, the equal secondary currents circulate through the current transformers and current restraining coils, connected in each phase, while none flows through the operating coil. When an internal fault occurs in the apparatus, however, additional current is supplied to the relay by one of the current transformers in the

faulty phase. This additional current passes through the operating coil and causes the relay to operate.

160. Overvoltage and Undervoltage Relays.—In the plunger type, these relays may be made like the overcurrent devices by supplying a suitable voltage coil to replace the current coil.

In the induction type, the elements are similar to those of the current relays but there are changes in operation. For the undervoltage relay, the springs tend to close the contacts but are

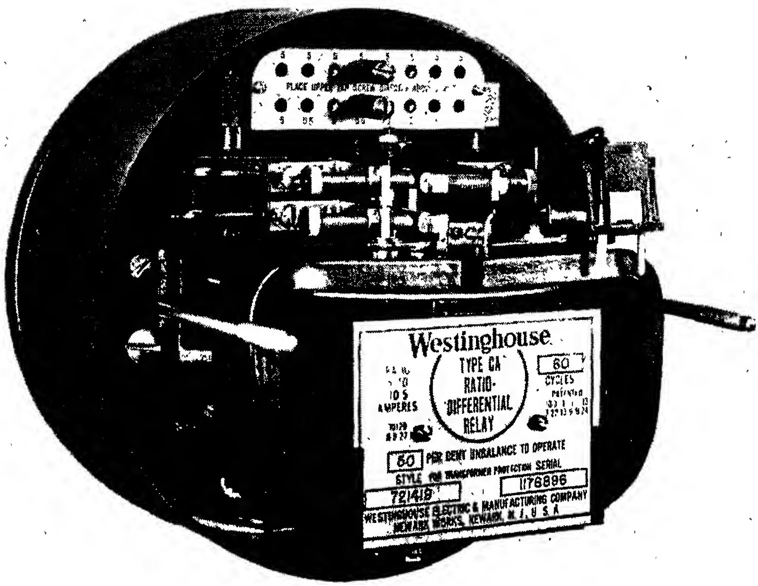


Fig. 210.—Westinghouse ratio-differential relay, type CA for transformer protection.

opposed by the electromagnetic action of the potential coils. For the overvoltage relay, the contacts are circuit closing on overvoltage when the torque of the potential winding has overcome that of the control spring. Many of the relays are adjustable as to their operating voltage, the speed of their operation increasing as the applied voltage varies from the relay setting.

161. Open-phase and Phase-rotation Relay.—This may be a plunger-induction instantaneous circuit-opening device for two- or three-phase circuits, in which an iron-clad solenoid has three potential windings connected across the various phases of the circuit. The relay operates on the rotating magnetic-field prin-

ciple. For sufficient potential and proper phase rotation, the plunger will hold the contacts closed. If the voltage is excessively low, the phase reversed or open-circuited, the plunger falls and opens the contacts.

The same protection may be had from an induction-disk type of relay for use on two- or three-phase circuits. Here the operating

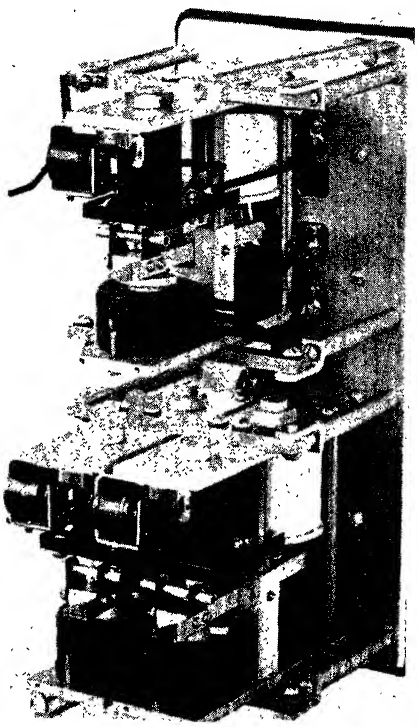


Fig. 211.—General Electric current-differential high-speed relay, typical of type RDD-13-A.

element is essentially the moving part of a single-phase induction watt-hour meter, except that the current coil is replaced by a potential coil. A restraining spring holds the disk in the de-energized position while torque is obtained from the operating element by action of the potential coil fluxes upon the aluminum disk. The interaction of the two fluxes produces a torque on the disk,

the direction and magnitude of which is dependent upon the phase relation of the two fluxes.

162. Alternating-current Power-directional Relays.—Power relays, in general, operate like induction watt-hour meters in that for each phase a current and potential coil act upon a metal disk

FIG. 212.—General Electric directional overcurrent relay, typical of type IBC-31-A.

mounted on a vertical shaft. The disk revolves and actuates the relay contacts. Unlike the watt-hour meter where current and potential elements are adjusted for maximum reaction at unity power factor of the load, the relay for reverse power protection is adjusted for maximum reaction with a lagging current. This is because a reverse current due to short circuit on a 60-cycle feeder may lag about 30 deg. behind its voltage. Figure 212 shows a type IBC directional overcurrent relay for single-phase protec-

tion on alternating-current circuits for short circuits and against ground faults. Each relay consists of a directional element and an overcurrent unit. The operating element of the latter is a current coil on a U-shaped magnet with wound shading coils which produce a split-phase field, thus developing torque on the operating disk. The shading coils are connected to an internal step-up transformer whose secondary is connected to the direc-

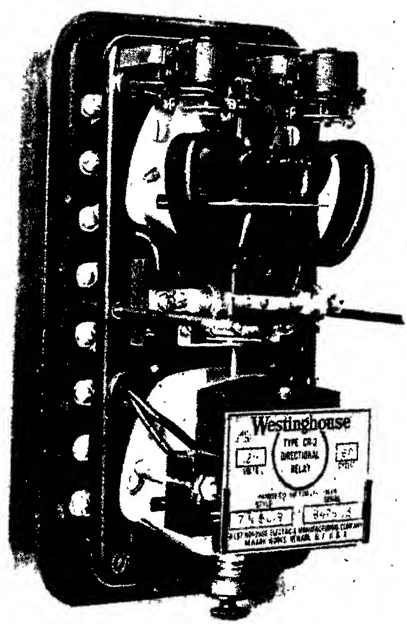


FIG. 213.—Type CR-3, polyphase-directional relay.

tional-unit contacts which control the operation of the overcurrent unit. When the transformer secondary is short-circuited, the overcurrent unit is operative, but when open-circuited it will not close its contacts. Therefore, the overcurrent unit is inoperative except when power flows in the proper direction for tripping.

For polyphase protection for transmission lines, three directional elements can be used mounted in a single case and acting on a common shaft. There are two disks on the shaft, two of the electromagnets operating on the lower disk, and the third

one on the upper disk. Figure 213 illustrates type CR-3. For the future, this will be superseded by a new type, CH-3.

163. Directional Ground Relay.—For speedy interruption of ground faults on transmission lines, type ICC relay, illustrated in Fig. 214, may be used where power-transformer neutrals are solidly grounded, or grounded through a low impedance. The construction is the same as the IBC type, previously described, with the exception of the overcurrent unit. A watt-hour-meter

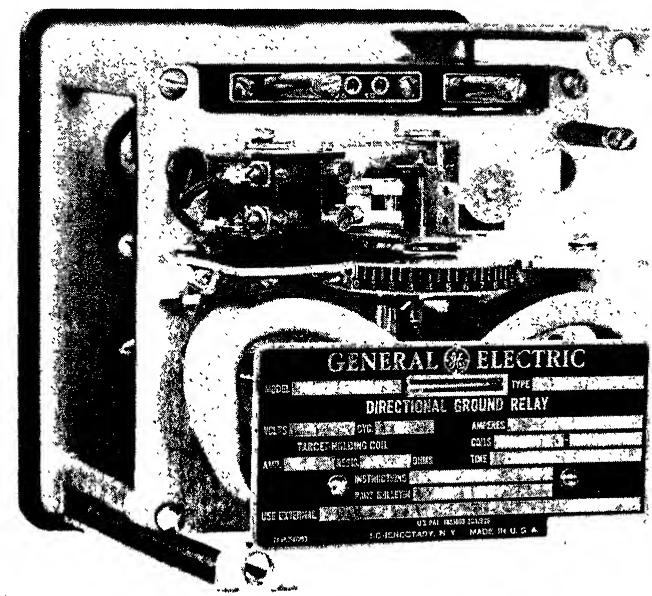


FIG. 214.—Directional ground relay, type ICC-11-A.

type of driving element with a spring return is used in the ICC relays instead of the U-magnet driving element with weight return. The upper half of the driving element has two windings. One of these is a tapped current winding, and the other is a coil that energizes two coils on the lower half of the driving magnet when the directional-unit contacts are closed. The directional feature is obtained by comparing the directions of the residual currents in the protected line and in the neutral of the power transformer. One current winding of the relay is connected in the residual circuit of the three line current transformers; the other is connected in the secondary of a current transformer in

the grounded neutral of the power transformer. The relay responds to the product of the residual current, neutral current, and a function of the phase angle between them. The disk will not rotate to close the contacts unless the residual current in the protected line flows in a predetermined direction. Normally there is no current in the relay coils.

164. Overpower and Underpower Relays.—These relays, generally polyphase, instantaneous, and circuit closing, have two sets of contacts. For normal power flow, the disk tends to rotate to close one set of the contacts. The rotation is opposed by spring tension so that at normal power flow the disk is held in neutral position. For an excess power flow, the disk will advance, or for less than normal flow the disk will be forced back and contact closed with the other operating circuit. The force of the spring may be adjusted to any desired power setting.

165. The Impedance, Reactance, or Distance Relay.—The distance relay is applicable to single and parallel transmission lines for rapid protection against three-phase and phase-to-phase faults. Two types are available, the impedance and the reactance. The impedance relay compares the magnitude of the voltage (proportional to the impedance between relay and fault) with that of the current, whereas the reactance relay compares the magnitude of that component of the voltage 90 deg. out of phase with the current (proportional to the line reactance) and the magnitude of the current. Some engineers believe that the arc resistance usually present in a fault would have an appreciable effect upon the indicated impedance of a faulty line section, particularly for short high-voltage lines, whereas a pure resistance would not affect the reactance to the fault.

Figure 215 shows a three-element high-speed impedance relay of the "balance-point" type, the three impedance elements operating with three impedance settings, which correspond to faults occurring within three distance zones from the relay. The time of operation is one or two cycles for faults in the first zone, a definite longer time for faults in the second zone, and a still longer time for faults in the third zone.

The first zone covers 80 per cent of the first transmission-line section, Fig. 216, and is protected by an instantaneous balanced-beam impedance element, operating in conjunction with a high-speed directional element. The second zone includes the remainder of the first section and extends

into the second section a distance of approximately 60 per cent of that line section. The third impedance element gives "back-up" protection and is usually set for a balance point about 25 per cent into the third

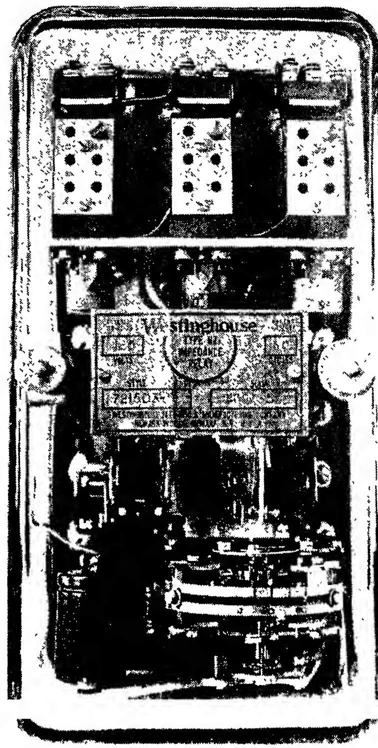


FIG. 215.—Type HZ, three-element impedance relay.

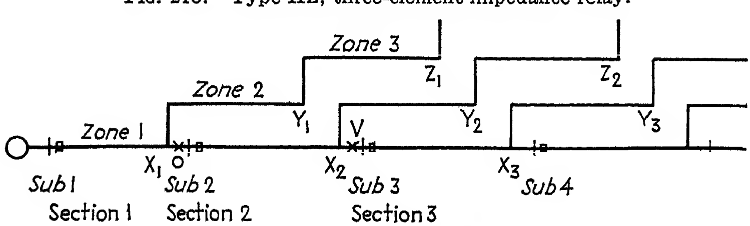


FIG. 216.—Time-distance characteristics of type HZ relay.

line section. This system of time-zone classification is necessary because the relaying indications are not an absolutely accurate measure of the distance to the fault, so that the relay is unable to determine whether the fault is just within or just beyond the end of the section.

The instantaneous element consists of a balanced beam of hipernik iron, which is pulled downward on the forward end by a current coil. The pull of the current coil is opposed by two voltage coils acting on the other end of the beam. The fluxes of these two potential coils are shifted 90 degrees out of phase with respect to each other, in order to produce a steady pull, so that a practically constant balance is obtained, regardless of the phase angle between the potential flux and the current flux.[1]

The contacts of the instantaneous element are in series with the contacts of the directional element, so that the breaker can be tripped only in case the power flow is in the proper direction. The second and third elements operate in conjunction with a synchronous timer. The synchronous timer includes two sets of adjustable contacts, the first in series with the second impedance element contacts and the second timer contact in series with the third impedance element contacts. By means of a core screw and taps on the current coil, the pull of this coil can be adjusted so that it just balances the pull of the voltage when a fault occurs at point X_1, Fig. 216. Under this balanced condition the contacts will not close. If a short-circuit occurs to the left of X, the current pull overcomes the voltage pull and closes the impedance element contacts. If the fault occurs at point O, the first impedance element cannot close contacts in that the ratio of E to I is too high. However, the second impedance element is adjusted to balance for a short-circuit at Y_1, and therefore operates when trouble occurs at O. The third element is adjusted to balance at Z_1.

For line-to-line fault conditions on an individual single-phase line, the impedance measured by the relay is equal to the actual impedance of the circuit from the relay to the fault, plus the effect of any fault impedance which is present. On polyphase systems the distance to the fault may not necessarily be correctly measured by the impedance relay and may be affected in various ways by the type of fault, the system load, and other conditions external to the section being protected, as well as by fault impedance.[2]

A later development of this distance relay (type HCZ)[3] provides extremely high-speed tripping for faults in the zone nearest the relay and a tripping time proportional to the distance for faults beyond the first zone. The relay has three main parts, an

[1] From Robinson and Monseth, Theory and Application of Relay Systems, *Elec. Jour.*, February, 1932, p. 84.

[2] See Lewis, W. A., and Tippett, L. S., Fundamental Basis of Distance Relaying on Three-phase Systems, Middle Eastern District Meeting, A.I.E.E., Mar. 11 to 13, 1931.

[3] See A High Speed Distance Relay, by L. N. Crichton, *Elec. Jour.*, December, 1935.

instantaneous impedance element, a distance impedance element, and a directional element. The first element is essentially the same as the corresponding part in the HZ relay.

The distance impedance element provides tripping for those faults beyond the balancing point of the instantaneous element, *i.e.*, zones 2 and 3. This element also consists of a pivoted beam, acted on by current and voltage forces. However, the current does not act on the beam directly but rotates an aluminum disk that is connected to the beam by a spring. The time required for the contacts to close, $t = K\frac{E}{I} = KZ$, and is therefore proportional to the distance to the fault.

The directional element ensures operation only when fault current is flowing away from the station, thus guarding against operation for faults on other circuits.

Type GCX directional distance relay (reactance) is illustrated in Fig. 217. It utilizes the modern induction-cylinder units, which give a steady high torque on a low-inertia element and provide high-speed operation.

In each phase, two units are used to determine the fault location by the relation between the current and voltage applied to the relay. One of these is the ohm unit which operates whenever the indicated reactance ohms are less than the value for which the unit is set. The other, the starting unit, is a directional fault detector which operates whenever fault current flows in a certain direction. Both units are adjustable. The starting unit can be adjusted in 10 per cent steps to operate for any fault on the protected line section plus whatever coverage of adjacent lines is desired. The ohm unit may be set within 1 per cent steps to definite ohmic values.

The starting unit controls operation of the complete relay. If a fault occurs within the ohm-unit setting, both the starting unit and the ohm unit operate immediately to close the tripping circuit. If only the starting unit operates, a timer is started which, after a definite time, operates an auxiliary transfer relay which changes the ohm-unit setting to a higher value.

These relays instantly isolate faults occurring within the protected section, operate in a short time on faults in the neighborhood of the next section, and in a longer time on faults in the other sections to which it is desired to give backup protection.

The minimum operating time of the relays is slightly less than one cycle, and the maximum time is adjustable up to 3 sec.

166. Instantaneous Relay.—"Instantaneous" as applied to a relay is defined as a qualifying term indicating that no delayed action is purposely introduced. The instantaneous type is

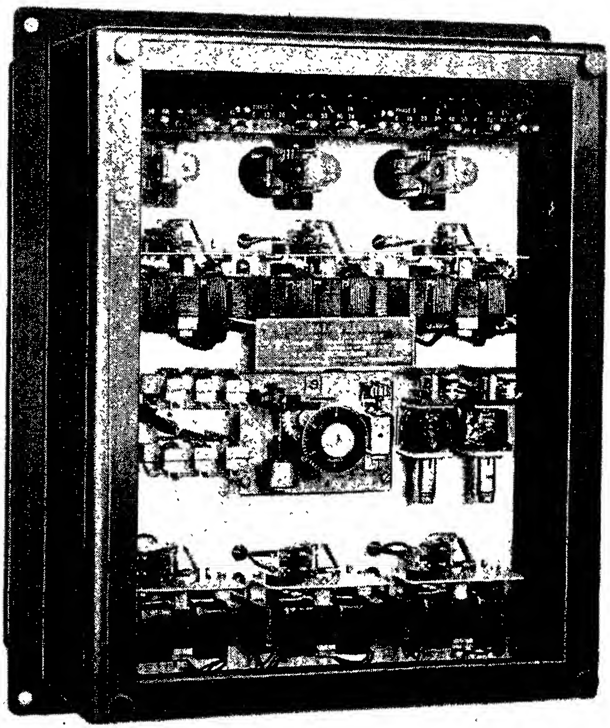


Fig. 217.—General Electric directional distance relay, three-phase, type GCX-12-B.

used principally for reducing the secondary burden imposed on current-transformer secondaries, for operating direct-current tripping circuits when the relays are energized by alternating-current sources, for use as an auxiliary relay operated by other relays (such as overcurrent or power-directional relays), for use as a locking relay to prevent the tripping of a circuit breaker in case the current flow at the time is above the interrupting capacity of

the breaker, and in fact for any case where it is desired to obtain a finer adjustment than can be secured by the use of the trip coils.

The induction overcurrent relays are available with an instantaneous attachment, consisting of an operating coil in series with the main coil of the relay and a set of circuit-closing contacts in multiple with the standard contacts of the relay. This will give instantaneous trip as rapidly as from 1 to 4 cycles in cases of heavy overcurrent.

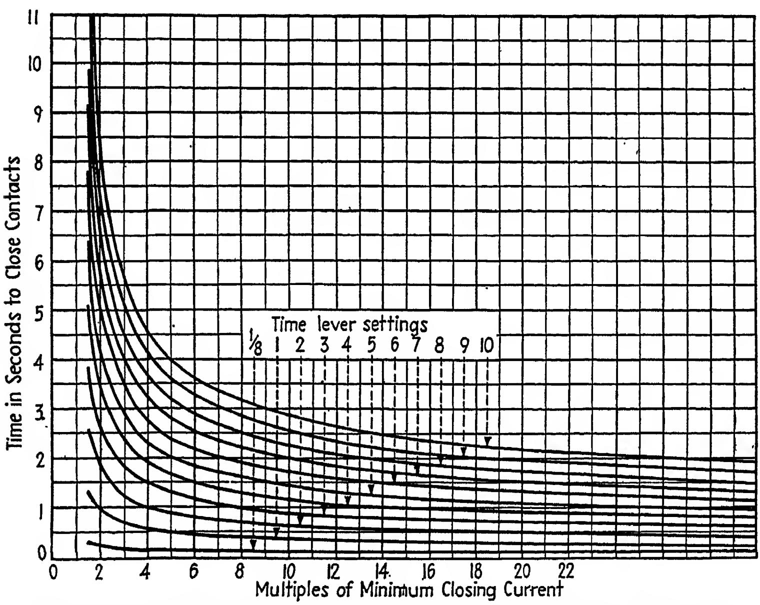


FIG. 218.—Inverse time-current curves of General Electric type IAC relay.

167. Inverse-time Relay.—The term "inverse time" is defined as indicating that there is purposely introduced a delayed action, which delay decreases as the operating force increases. The more important mechanical devices that have been developed to cause this time lag are the compressible leather bellows, the oil or air dashpot, and the rotating magnetic drag disk. The inverse-time overcurrent relay is used principally to prevent the interruption of circuits when the overcurrents are only momentary, to prevent false operation of the main relay upon transitory disturbances when used in conjunction with power-directional or with differential relays, and for selective action with a number of

stations in series. Figure 218 gives the typical characteristic curves of a General Electric IAC overcurrent relay at its various time-lever settings.

168. Definite-time Relay.—This term denotes that the action is purposely delayed, the periods of delay remaining substantially alike regardless of the magnitude of the operating forces. For forces slightly above the minimum operating value, the delay may be inverse. The definite-time overcurrent relay is used principally as an auxiliary to other relays where it will trip the

about 2 sec. as the maximum allowable, the increments depending upon the number of sections of the feeder. Different current settings as between branch ends and main lines may also be used to gain selectivity. The feeder relays will, in general, be set to operate at 200 to 400 per cent of full-load current. Figure 219 shows an elemental radial system equipped with the foregoing relays.

The overcurrent system will protect against phase-to-phase, single-phase, and polyphase short circuits, also against grounds if the ground current is large enough to operate the relay. The overcurrent relay does not protect for fault currents of full-load value or less. A ground relay may be used with the overcurrent relays for the small ground currents.

170. Typical Relay Application. Parallel Systems.—On parallel systems, two or more lines operate in parallel between two stations either or both of which may be generating stations or substations. When a short circuit occurs on such a line, power flows into the fault from both ends of the faulty line. The magnitude of the fault current flowing into each end of the line depends upon the system characteristics, the location of the fault, and the generating capacity available at that point. With two lines in parallel and a fault at the far end, the amperes flowing to the fault over the good line may be approximately equal to those flowing over the faulty line except for back feed from rotating machinery connected to the substation bus. Therefore, an overcurrent scheme of relaying cannot be relied upon for the proper discrimination. However, protection can be provided in two ways: (1) by means of the current-balance relays of Sec. 158, and (2) by means of the power-directional relays of Sec. 162. After only one of the parallel feeders is left in operation, it should automatically have overcurrent protection.

The selective-differential current relay may be applied at the generating end of any number of parallel feeders and will protect lines against unbalanced current in the similar phases, such as would be caused by a fault in one of the lines. At the receiving end of the feeders, it will be necessary to have at least three lines feeding the bus in order to secure sufficient differential in the currents. The connections are shown in Fig. 220. If one line of the balanced pair is open, the relay current necessary to trip-out the remaining line is about twice the differential current

setting. This is due to the fact that half the relay operating coil is no longer active, since it receives no current from the dead current transformer in the open line.

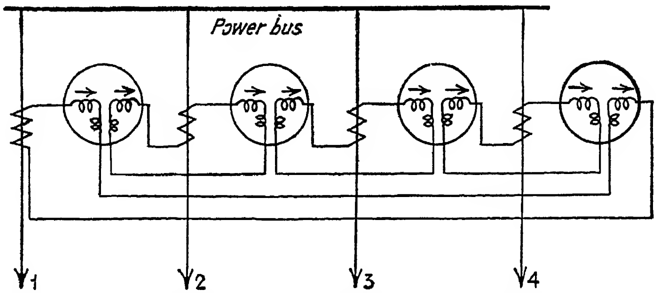


FIG. 220.—Type CD selective-differential current relays applied to parallel feeders. (*Westinghouse Bull.* 1666-*B*.)

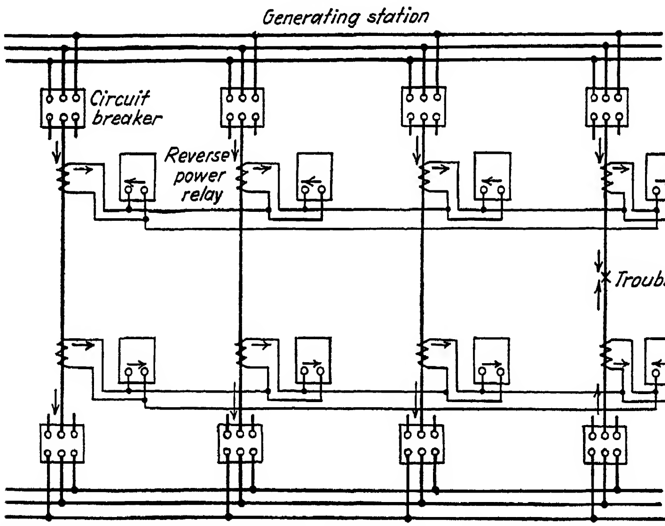


FIG. 221.—Cross-connected relay system (one phase shown, voltage and tripping circuits omitted) using directional-overcurrent type CR relays. Arrows show direction of current flow with short circuit on right-hand feeder. (*Westinghouse Bull.* 1666-*B*.)

The protection scheme shown in Fig. 221 may also be used with CR (directional-overcurrent) relays applied at both ends of the lines. The current transformers are connected in series, and each unidirectional relay is shunted across its own current transformer. Under balanced feeder loads, since the relays have a higher impedance than the current transformers, the current

from the latter will circulate through the transformers. However, for a fault on a line within the protected section the current in the defective feeder will exceed that of the others, and this excess must pass through the relays. The current in the relays will be in the correct direction to cause only the relay at each end of the defective feeder to act.

171. Typical Relay Application. Loop Systems.—Loop or ring feeders are those which consist of one or more lines starting

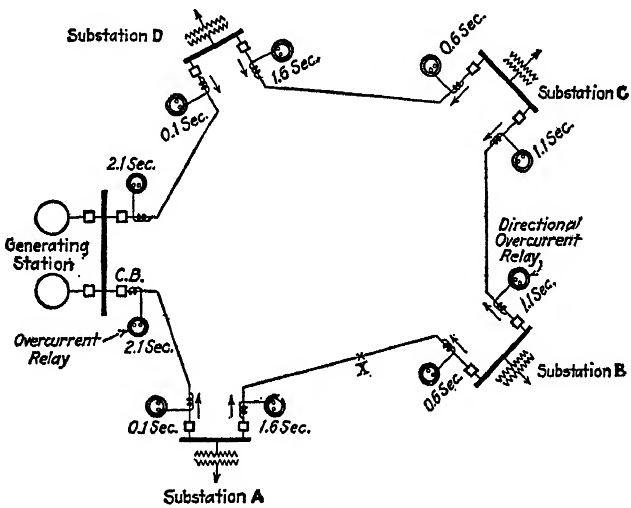


FIG. 222.—Loop feeder with overcurrent-time and directional-relay protection. (*Courtesy of Westinghouse Co.*)

out from a generating station, feeding a number of substations in series, and then returning to the original generating station or another one tied to it. Such a single-loop system is very commonly used with underground cable in supplying power to important customer substations which require two sources of supply.

1. *Overcurrent-directional Protection.*—Figure 222 shows such a system with the timings and direction settings to trip for the various relays. No directional elements are required at the generating station since there is no possibility of power flowing in to the station. The overcurrent relays here will take the maximum time which will then decrease in steps as we travel round the loop in order to provide the necessary selectivity with which to isolate a faulty section of feeder at the nearest points to the fault. Should a fault occur at X between Substations A and B, the

current flow from Substation B will be interrupted after 0.6 sec. and the flow from Substation A will be cut off after 1.6 sec. The troubled section will thus be isolated and the remainder of the system left intact. The overcurrent relays are set for operation at 100 to 300 per cent of normal current but cannot trip unless the directional elements find the current flow in the direction for which the relay is set to act.

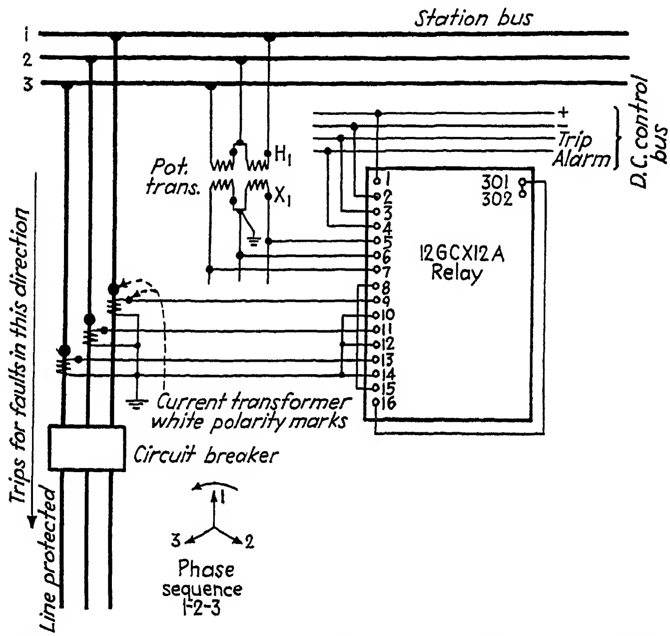


FIG. 223.—Connections for a General Electric type GCX-12, three-phase relay protecting a single line.

The plan of relaying shown in Fig. 222 protects also against substation bus short circuits and against grounds on a low-resistance system, but is limited for successful application to a maximum of four substations served. For a greater number of substations, the same plan is practical in conjunction with a pilot-wire system, or the distance relay can be used.

2. *Distance Protection.*—The directional type of impedance or reactance relay may also be applied to guard a loop system. However, because of the definite limits in its application to transmission circuits, two factors have to be assured for the impedance relay, *viz.*, with minimum short-circuit current flowing

there must be a voltage difference between the two ends of the circuit of at least 4.2 volts on a 110-volt base, and the minimum short-circuit current must not be less than 200 per cent of the current tap setting.

The reactance relay is subject to inaccuracy only when the arcing fault is fed from both ends of the line, and the currents from the two sources are out of phase owing to the normal phase displacement between the e.m.fs. of the two generating

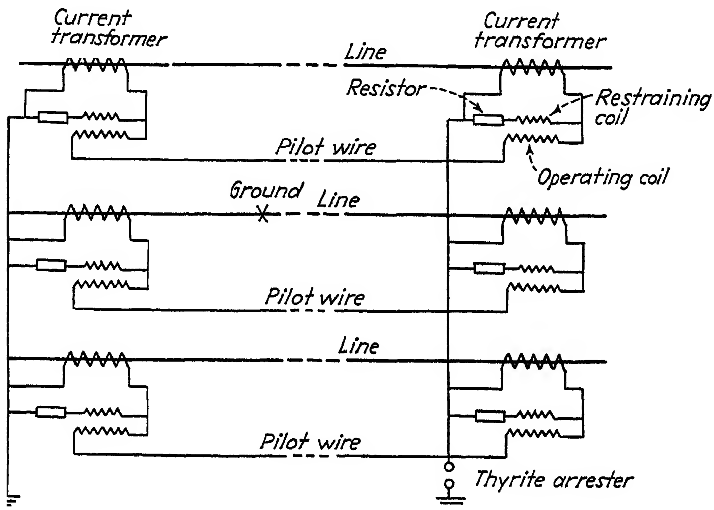


FIG. 224.—Connections for two General Electric type IJD pilot-wire relays protecting a three-phase line.

sources, or because of hunting between them. There is no error when the power is all supplied from one end. Figure 223 shows connections of type GCX relay to protect a single line.

3. *Pilot-wire Protection.*—The basic principle of current-balance pilot-wire protection is that equal currents will flow at both ends of a line if there is no fault on the line and that the currents will be unequal if a fault is present. The pilot-wire scheme is required to trip the breakers at each end of the faulty line which it guards but to pass any current due to trouble in another part of the system. Thus by balancing the currents at each end of a line over the pilot wire, practically instantaneous protection can be given that line. Furthermore, any number of lines in the loop may be so guarded since no change in time

setting is imposed on other overcurrent or power-directional relays. Figure 224 shows an application of pilot-wire protection for a single line. There are two parallel circuits across the secondary of each current transformer, a restraining coil in series with a resistor and the operating coil of each relay connected by a pilot wire and paralleled by the restraining coil and other transformer secondary. For corresponding currents at each relay, equal currents flow in the restraining coils and the potentials at the ends of the pilot wire are the same, so no current flows through the operating coils. If, however, the magnitudes or phases of the two end currents are not the same, then a difference of potential is impressed on the ends of the pilot wire and a proportional current flows. If it overcomes the restraining coil, then it trips the breaker at each station.

As against the complete and rapid protection given to any line section throughout its entire length, regardless of the nature of the fault and the number of lines between supply stations, the pilot-wire system incurs the cost of the signal wires and dependence upon their reliability. For short transmission sections, this will not be a great burden, and privately owned wires paralleling the transmission line may be utilized as the pilot channel. However, for long lines the cost and exposure are severe so that it may be preferable to arrange for leased wires or go to the use of carrier current. A number of companies secure good service from leased Bell System channels, and the cost may be less than for carrier equipment.

A type HCB relay has been developed[1] for protection against phase and ground faults which has only one moving part and uses a single relay current element. It derives a single-phase alternating-current discriminating function of limited magnitude from three asymmetrical currents and compares that function at each end of the protected line over a pair of pilot wires. Just two pilot wires are necessary and a communicating circuit can be used for the channel, since the signals are suited for use on telephone-company circuits. The relays will operate at 1 cycle maximum speed to 3 cycles at minimum trip currents.

4. *Carrier-current Relaying.*—When carrier current is employed, the conductors of the transmission line itself become the channel,

[1] See Ratio Differential Protection of Transmission Lines, by Smith and Bostwick, *A.I.E.E.*, August, 1939.

the carrier current being superimposed upon the power line by means of coupling capacitors. These same capacitors, by the addition of potential devices, may also furnish voltage .for the operation of directional relays. The essential function of the carrier is to replace the pilot wire and thus to lock out the relays at the two ends of a section whenever the fault is outside the section itself. Suitable carrier traps are generally added so that the capacitors may be used simultaneously for carrier telephony controlling the system operation. The traps at the end of each protected section confine the carrier signals within that section so that only one frequency, the best for the particular line constants, is required. There is no practical limitation to the length of the line over which the carrier current may be sent, and since these signals are transmitted only during external faults, the line is free from carrier signals during the long periods of normal operation.

As was indicated above, the carrier current operates to remove an important weakness of the protection by power-directional relays in that, even when equipped with voltage restraint, the latter cannot distinguish unfailingly between a fault occurring in the section itself and one occurring immediately beyond. Two GMB pilot relays, one at each end of a three-phase transmission line, provide fast selective protection for both phase and ground faults over the entire length of the line. This protection is unaffected by the settings of relays on adjoining lines or by changes in system setup. The GMB-11-B and GMB-12-B relays, shown in Fig. 225, are similar in construction except the latter have provision for out-of-step blocking and should be used for applications where severe swings or surges are likely to occur that may result in out-of-step conditions.

The GMB relay determines the location of a fault by comparing the direction of power flow at the line terminals, as found by the directional units at each station, and the two directions are compared by means of the pilot channel. During a fault, the directional units control the carrier-current transmitters, and the carrier-current receiver units control the tripping circuits. The carrier-current transmitting and receiving equipment functions only when a short circuit occurs, and then only to prevent tripping when the fault is not in the protected line. Failure of the pilot channel then, or its accessories, cannot jeopardize a

tripping function, but will leave a section unable to discriminate between internal and external faults.

Carrier-current transmission is started at a given end of a line whenever short-circuit current flows at that end regardless of the location of the fault. If the short-circuit flows into the line section, the transmission of carrier current is stopped at that end

FIG. 225.—General Electric carrier-current pilot relay, type GMB-12-B, for protection of three-phase transmission lines.

immediately. Fault current flowing out of a line section will not stop the transmission of carrier current because this condition indicates an external fault, and the breakers should not be tripped. This will be prevented by the carrier current flowing throughout the duration of short-circuit current flow. If the fault is in the protected line, the transmission of carrier current is stopped at both ends and the breakers are tripped if there is short-

circuit current sufficient to operate the fault detectors. The operating time will be one to three cycles.

For a comprehensive report on the performance of carrier equipment, the student should consult the paper "Application and Performance of Carrier Current Relaying," by Sporn and Muller, *A.I.E.E.*, January, 1938.

Two schemes have been developed for the many requirements of system protection, each being best adapted to certain operating conditions. Scheme 1 is to protect circuits where the phase-to-phase short-circuit current may be less than normal load. The

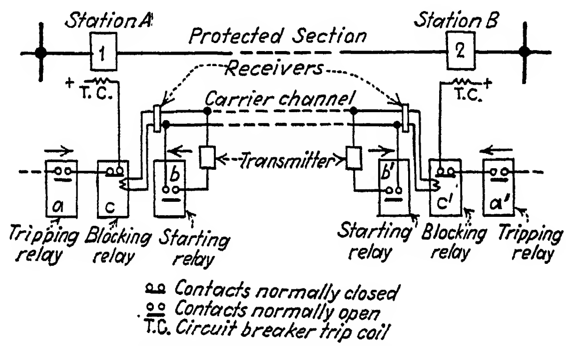


FIG. 226.—Simplified diagram of protection by means of polyphase directional relays. *a*, *a'*, *b*, *b'*, voltage-restrained power-directional relays with amplifying potential coils; *c*, *c'*, high-speed auxiliary relays. (*Pilot Protection by Power-directional Relays Using Carrier Current, Traver, Auchincloss and Bancker, Gen. Elec. Rev., November*, 1932.)

protection is equally good for ground faults, and the speed of clearing is the same for any fault location. There is a possibility, however, of incorrect tripping during oscillating conditions approaching instability. Scheme 2 may be used without directional ground relays provided the line-to-ground fault current will operate against the load torque of the polyphase power-directional relay when not subjected to voltage restraint. All fault currents must exceed normal load current, but the overcurrent setting need be only slightly above load. With directional ground relays, the sensitivity of both schemes is the same for ground faults.

Figure 226 shows the operation of scheme 1. For an external fault to the left of Station A, power flows from Station B toward Station A, which is in the proper direction for relay *a'* to close its contacts under the reduced voltage restraint accompanying

the fault. Unless the trip circuit is opened by the blocking relay c' (contacts normally closed), breaker 2 will be tripped. However, since the power flow out of this protected section at Station A is in the proper direction to close the contacts of relay b, this starting relay will initiate carrier-current transmission and cause blocking relays c and c' to open their contacts and prevent tripping of the breakers.

For an internal fault, power flows in at both Stations A and B; hence relays a and a' close their contacts but b and b' do not. Since b and b' do not initiate any carrier current, c and c' cannot open, and the breakers at both ends are tripped.

172. Typical Relay Application. Generators.—Continuity of service requires that working machines be taken off the bus only for internal faults. Hence generator circuit breakers are non-automatic for overcurrent or for short circuits in the outside system. Indicating instruments and alarm signals keep the operator informed as to the current and temperature conditions in the alternator so that troubles of this nature can be taken care of by manual control. For short circuits and grounds in the alternator itself, it is necessary to provide some protective relay system offering rapid and effective action. The most common generator fault is probably a ground from a phase winding to the frame. The relay action is complicated because it must not only trip the main oil circuit breaker but also kill the machine field, trip the emergency steam valve (or close the gates for a water turbine machine), and, perhaps, close the ventilating dampers to cut off the draft in case of fire.

The most common form of generator protection is the differential plan wherein a relay closes its contacts for unbalanced current in the two corresponding current transformers installed at both ends of each winding. If the ratio-differential relay, described in Sec. 159, is used for the scheme, then the current required for operation of the relay increases in proportion to the load current. This will prevent false tripping on outside faults due to unbalanced current-transformer characteristics or loading. Values of 2.5, 5, 10, and 20 per cent of full load for the differential current are commonly used. Figure 227 shows the wiring diagram for such an installation.

The simple differential plan alone will not protect against short-circuited turns, open circuits, or high-resistance grounds.

The simple scheme may, however, be readily combined with a split-conductor plan for large machines, to give full protection.

173. Typical Relay Application. Power Transformers.—The large power transformers in stations, like the alternators, are provided with indicating instruments and alarm signals for current loading and temperature and are therefore manually controlled for these conditions. Current differential protection is necessary, however, for transformers in parallel to open both

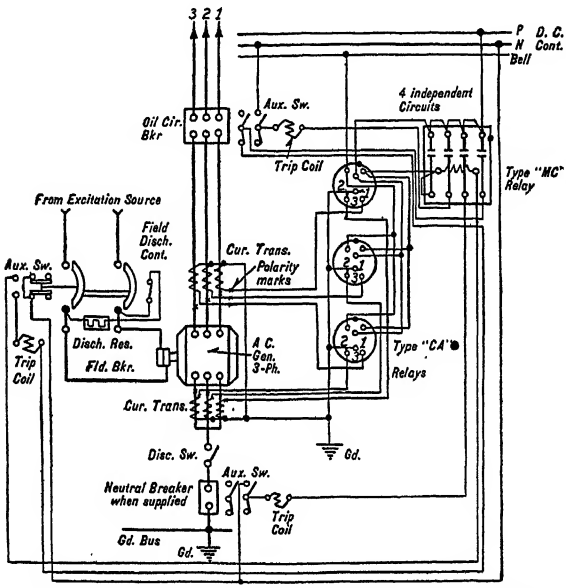


FIG. 227.—Wiring diagram illustrating application of Westinghouse type CA generator relays with one type MC relay for the protection of generators.

high- and low-voltage breakers when internal troubles occur. The breakers may be nonautomatic on through short circuit or overcurrent.

Figure 228 shows the application of IJD relays for current-differential protection of power transformers. Their operating characteristics are designed not to trip on large through fault currents because the difference current must exceed a fixed percentage of the through current. A system of current-balancing taps used with these relays permits balancing the secondary currents from the high- and low-tension current transformers.

The IJD relay will operate on a minimum current of 0.4 times tap rating. This value together with the short-time delay pro-

vided by the relay is usually sufficient to prevent incorrect operation on magnetizing inrush currents. In exceptional cases, desensitizing equipment may be used during the time period covering the maximum inrush transient, controlled by auxiliary relays operating on the transformer voltage.

The preceding methods have the disadvantage of requiring extra transformers and auxiliary relays and of seriously reducing

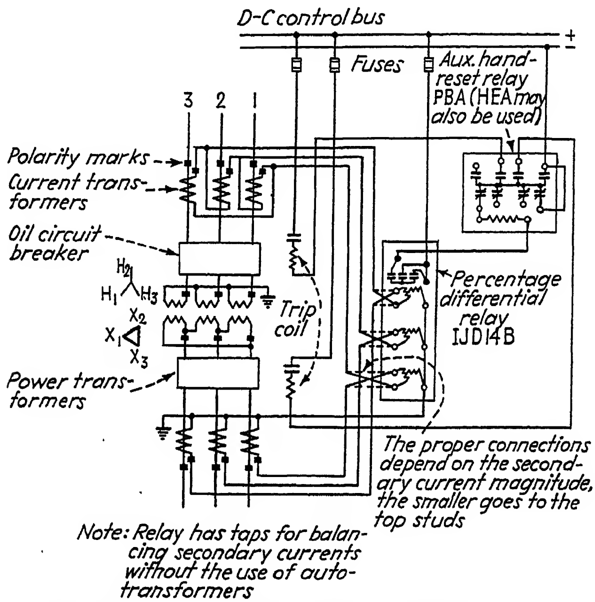


FIG. 228.—Diagram for General Electric type IJD relays for current-differential protection of power transformers.

the sensitivity of protection for faults present when the transformer is energized or which occur during the inrush period. A new relay has been developed[1] which is able to distinguish between the differential current due to an internal fault and that due to a magnetizing inrush by their difference in wave form. It will operate at high speed on the former but will be restrained from operating on the latter. Since it is entirely current operated, this relay does not require potential transformers.

174. Typical Relay Application. Buses.—Since the clearing of a bus section involves the loss of important generating or

[1] See Harmonic-Current-Restrained Relays for Transformer Differential Protection, by C. D. Hayward, *A.I.E.E.*, June, 1940.

transmitting capacity, it is necessary that relay operations be correct and that the isolation be rapid. If the fault remains on the system for an extended period, the insulation in the bus structure may be damaged, making it impossible to restore service without repairs.

1. *Fault Bus.*—For this plan of protection, every element of the switching structure is insul ted from ground and the sections are insulated also from each other. Each section is provided with a ground through a current relay. With such a design any fault must start as a line-to-ground fault, then if a fault occurs the current flows through the relay which opens the circuit breakers to isolate the defective section. Such faults can be cleared reliably and very rapidly. If, however, accidental grounds by-pass the relay, generally through reinforcing steel, it becomes inoperative. The practical difficulties of totally insulating an existing structure are so great as to practically eliminate the plan from consideration on a modernization program.

The Essex Switching Station of the Detroit Edison Company[1] employs a fault bus with isolated metal housings. As many instantaneous plunger-type overcurrent relays as there are breakers to be tripped to clear a fault are connected in series to the secondary of the current transformer in the ground connection. The metal housings are isolated from the building and grounded only through one connection to the ground bus. The ground bus is solidly connected to the building steel at frequent intervals to limit any voltage that might exist between them.

2. *Differential System.*—This plan, most commonly used, balances the incoming currents to a bus section against the outgoing currents from the same section so that the summation of the secondary currents of all the current transformers will be zero normally. The summation current flows through a relay, which in case of unbalanced flow inward, indicating a fault within the section, can trip all breakers connected to the section. The plan covers all types of faults since each phase is treated independently. The difficulty lies in obtaining exactly similar characteristics in the current transformers. For a given burden and primary current, duplicate transformers will have about the same current ratio, but at the time of a fault, when different currents

[1] See Switching at the Connors Creek Plant, by A. P. Fugill, *A.I.E.E.*, January, 1934.

flow, the ratios may vary. Accordingly, a net current may flow in the relay during a fault outside of the protected section, giving incorrect operation. This may be overcome by using a plain overcurrent relay with a high setting, by employing the percentage-differential relays or the harmonic-current restrained relay used for transformer and generator protection.

3. *Partial Differentiation with Impedance Relay.*—If current-limiting reactors are used in some of the circuits to a bus, the current transformers of those circuits may be omitted from the differential connections of the other current transformers and an impedance relay may be used in place of the overcurrent relay. Such a relay receives bus voltage and current from all the circuits without reactors. It will operate if the equivalent impedance to the fault is less than the minimum possible value to a fault outside of the protected section.

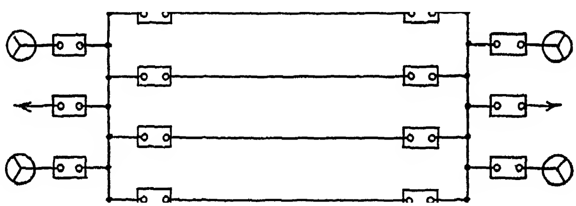


FIG. 229.—Parallel lines for Problem 3.

ACKNOWLEDGMENT: The typical examples of relays and their applications given in this chapter have been drawn from the "Switchgear Relay Handbook," of the General Electric Company; Westinghouse Electric and Manufacturing Company *Bull.* S.P.-1666-B.; Modernization of Switch-house Design, by Strang and Hanna, *A.I.E.E.*, January, 1939; High-speed-relaying Experience and Practice, by Relay Subcommittee of Protective Devices Committee, *A.I.E.E.*, November, 1939.

See also *A.I.E.E.* papers: Considerations in Applying Ratio Differential Relays for Bus Protection, by Smith, Sonneman, and Dodds, January, 1939; Ratio Differential Protection of Transmission Lines, by Smith and Bostwick, September, 1939; Out-of-step Blocking and Selective Tripping with Impedance Relays, by Vaughan and Sawyer, December, 1939; and A High-speed Differential Relay for Generator Protection, by W. K. Sonneman, June, 1940.

175. Problems.

1. Two large generating stations 5 miles apart are connected by a three-phase tie line of single-conductor 66-kv. cables. Suggest the relays to protect the tie line against internal faults.

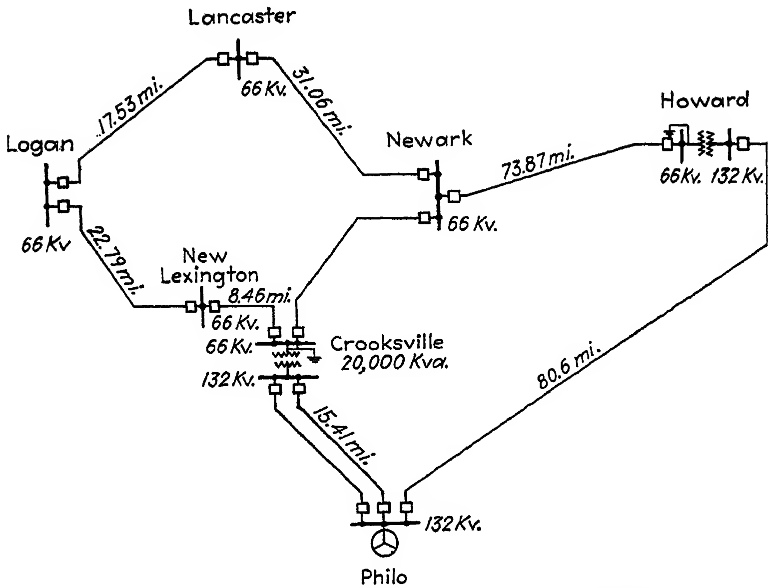


FIG. 230.—Doubly fed loop. Ohio Power Co., So. Div. (*Relaying*, *Sporn and Muller*, *Elec. World*, *Sept.* 10, 1932, *p.* 332.) Problem 4.

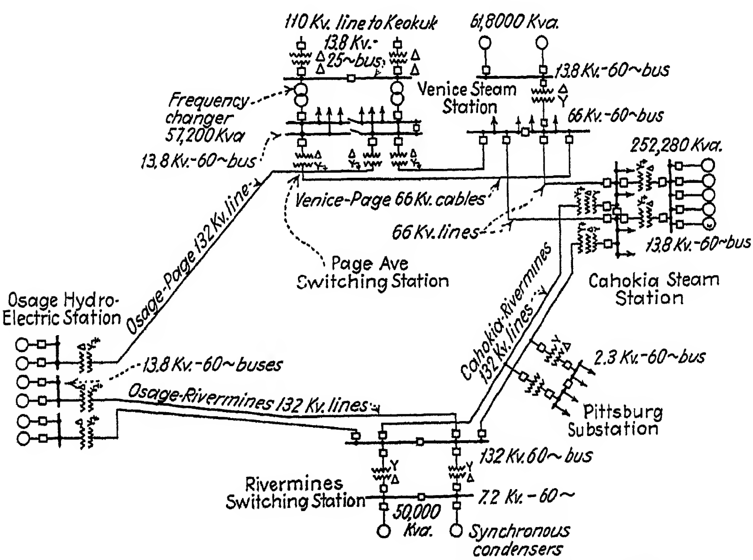


FIG. 231.—Transmission system of Union Electric Light and Power Co., So. Div. (*Application of High Speed Relays*, *G. W. Gerell*, *Elec. Eng.*, *June*, 1932.) Problem 5.

2. Suggest the relays for the loop system of Prob. 7, Chap. VIII. Illustrate by simple one-line diagram.

3. Suggest the relays for the protection of the parallel lines of Fig. 229. Illustrate by simple one-line diagram.

4. Suggest the relaying system for the doubly fed loop of Fig. 230. Illustrate by simple one-line diagram.

5. Suggest the relaying system for the transmission system of Fig. 231. Illustrate by simple one-line diagram.

CHAPTER XI

TRANSMISSION LINES WITH SYNCHRONOUS CONDENSERS

176. The Function of the Synchronous Condenser.—In Sec. 74 on the Ideal Conductor Section, it was stated that regulating devices should be considered where it is necessary to improve the line regulation before departing from the economic standards for conductor size. For direct-current service, there is the possibility of using a double bus system, one bus of higher and one of lower voltage, and connecting the various feeders to the appropriate bus. Again, a rotary converter or mercury-arc rectifier substation close to the direct-current load center may be employed. For alternating-current service, the induction regulator and the synchronous condenser are available in this field. In addition to the requirement of a satisfactory quality of voltage regulation, the great cost of heavy transmission lines makes it economically necessary that their full power-carrying capacity be used, and large-system operation and interconnection impose the maintenance of stability under surges and fault conditions. In the fulfillment of these demands, the modern synchronous condenser, because of its peculiar "V-curve" characteristics and its availability in large-sized units, is of the greatest assistance. By drawing a large leading current through the inductive reactance of the line and so raising the voltage at the load end, during the hours of the day when the industrial plants have a heavy inductive demand, the synchronous phase modifier maintains the substation voltage. At times of light load on the line, as for example, on Saturday afternoons and Sundays, when it may be chiefly the charging current of the conductors, the synchronous-condenser excitation is reduced and the voltage drop due to lagging current through the reactance prevents the rise of voltage at the substation. By thus controlling the power factor of the transmission-line load, the synchronous condenser may be arranged to maintain automatically a given delivery voltage.

In order to economize on the capacity of synchronous-condenser apparatus, to do this for a wide range of loading, the primary delivery voltage of the line is allowed to rise somewhat at light load and to fall at heavy load, but the delivery secondary voltage from the step-down transformers is kept constant by the use of tap changers on the transformers.

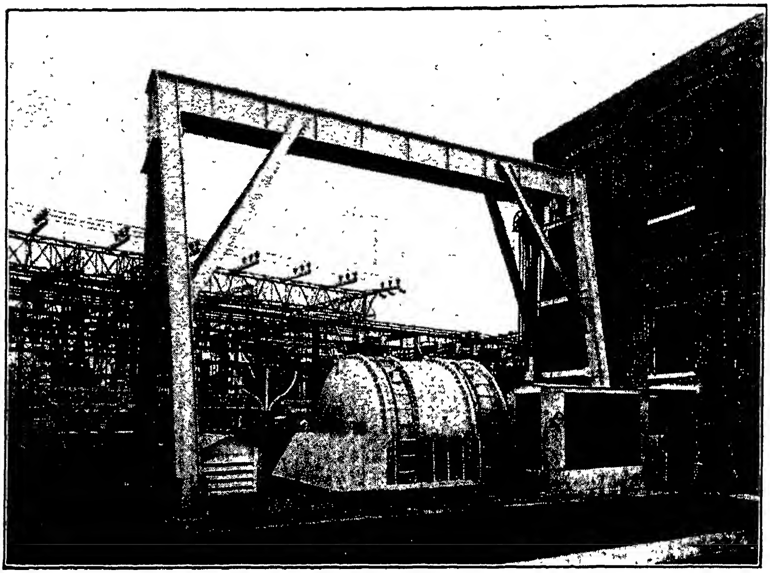


FIG. 232.—General Electric type ATI-8-20,000 kva.-900, 11,500-volt outdoor hydrogen-cooled synchronous condenser, Appalachian Electric Power Co., Charleston, W. Va.

In recent installations, three-winding transformers have been used in which the tertiary winding supplies the synchronous condenser.[1] The voltage on the secondary load bus is maintained constant by a voltage regulator controlling the condenser excitation, the actuating potential transformer of the regulator being connected across the load bus, even though the condenser is connected to the tertiary winding. In designing the three-winding transformers for use with synchronous condensers, the reactances can be varied so that the condenser can be closely coupled to the primary or to the secondary, whichever is better for maintaining stability.

[1] See BERGVALL, R. C., Use of Synchronous Condensers with Three-winding Transformers, *Elec. Jour.*, May, 1924.

177. Baum's Principle of Constant-voltage Transmission.—In order to remove the limit on the distance to which power could be transmitted, Frank G. Baum[1] presented a notable plan for economical long-distance transmission of energy. In this scheme, the effect of the restricting reactance of the line was to be canceled by adding synchronous condensers to the line, thus securing high power factor, large kilovolt-ampere capacity, high efficiency,

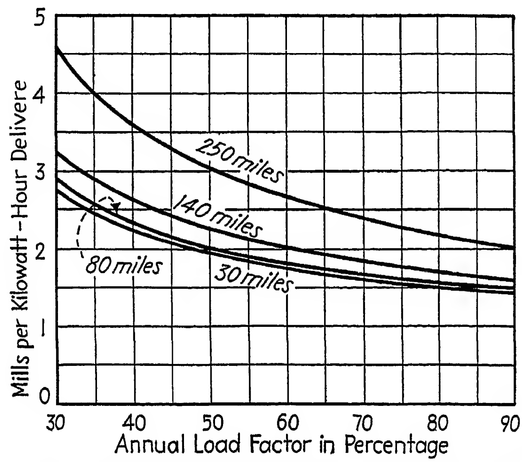


FIG. 233.—Cost of electrical transmission of energy (fixed charges on transmission facilities 12.25 per cent, 10 per cent losses included). (*From Cost of Generation of Electric Energy by Philip Sporn, Trans. A.S.C.E.* 1939.)

and good voltage regulation. The transmission line was to be so loaded with the synchronous condensers at certain points, perhaps every 100 miles, that it would be supplied at each place with the correct reactive kilovolt-amperes required to keep the current and voltage very closely in phase. Such a long transmission line would have practically a uniform voltage at all points of the line, and the transmission distance could be extended by simply adding constant-voltage units of line so long as it paid to transmit the power.

Since these synchronous-phase modifiers were essential elements in the transmission of power over the entire line, they were part of the transmission system and must remain connected to it except in extreme emergencies. The cost of the large synchronous reactive capacities required on heavy transmissions is, of

[1] See Voltage Regulation and Insulation for Large Power Long-distance Transmission Systems, *A.I.E.E. Trans.*, 1921.

course, an increased burden on the system. The synchronous-condenser power losses, about 4 per cent of their full-load kilovolt-ampere rating, lessen by that amount the power savings they effect in the transmission-line losses by correcting the line to nearly unity power factor.

Figure 233 shows transmission costs in mills per kilowatt-hour delivered for lengths of 250, 140, 80, and 30 miles plotted against various values of annual load factor. The fixed charges on the transmission facilities have been taken at 12.25 per cent, and 10 per cent losses have been included.

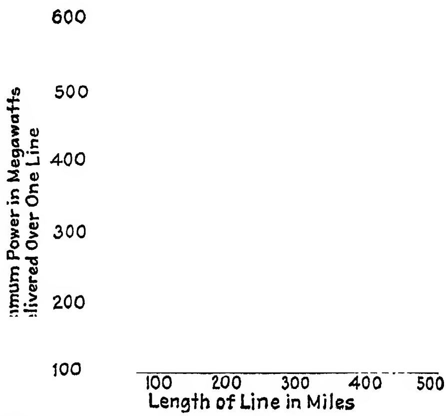


FIG. 234.—Maximum amount of power that can be transmitted over a three-phase, 60-cycle, 220-kv. straightaway line, sending and receiving voltages 220 kv.

The maximum amounts of power that can be delivered for straightaway transmission over varying distances at three-phase, 60 cycles, with 220 kv. at sending and receiving ends, are given by Charles L. Fortescue[1] in Fig. 234. Under these conditions, the maximum power is the same as the static stability limit of the system. The limit for 400 miles of line, *viz.*, 150,000 kw., is approximately the amount of power that would have to be delivered in order to justify the line economically.

Fortescue has also presented an illustrative example[1] of the application of Baum's principle based on a transmission line 400 miles long over which it is desired to transmit 150,000 kw. with a load factor of 80 per cent. The sending, receiving, and inter-

[1] The Baum Principle of Transmission, *Elec. World*, May 7, 1927.

mediate condenser-station voltages are all held constant, and the phase angle between the voltages at adjacent stations is held to 22 deg. for stable operation.. Under these conditions, the amount of power that can be transmitted over the 400-mile straightaway line is 53,000 kw. with a line efficiency, exclusive of condenser losses, of 92.3 per cent. At no load, the synchronous condensers at the receiving end would have to supply 50,000 lagging kilovolt-amperes to maintain the voltage at 220 kv.

With the introduction of one intermediate condenser station, the system changes to two 200-mile units. For the stability angle chosen, the delivered power is then 99,000 kw. and the line efficiency is 89.2 per cent. Similarly, with two intermediate condenser stations, *i.e.*, three 133⅓-mile sections, the power at the receiver end is 150,000 kw. and the line efficiency is 85.5 per cent. For the delivery of 150,000 kw., the condenser kilo-volt-amperes would be: at receiver end, 31,000 kva.; at intermediate condenser Station A, 22,000 kva.; at intermediate Station B, 28,000 kva.; all leading power delivered to the line. At no load, each intermediate station would have to deliver 45,000 kva.

For a practical operating system, at least two transmission lines must be considered on account of maintaining continuity of service. Therefore, two 400-mile straightaway lines would transmit 106,000 kw. and require, including one spare unit, 120,000 kva. of synchronous condensers. The generators would operate at approximately 77 per cent leading power factor, which is not conducive to stable operation. For the two-intermediate-station system, the two lines would transmit 300,000 kw. (the economic minimum for two 400-mile transmissions) and the synchronous condensers would be 90,000 kva. per intermediate station and 60,000 kva. at the receiving end, a total, including one 30,000-kva. spare unit at each station, of 330,000 kva. The generators would have a slightly leading power factor of approximately 99.5 per cent, which is a very favorable operating condition for stability. This remarkable increase from 106,000 to 300,000 kw. in the power that can be transmitted, with the same degree of stability in the two cases, is obtained with an increase of only 13 per cent in cost.

178. The Circle Diagram of a Constant-voltage Transmission Line.—In any particular application of the constant-voltage principle to a transmission problem, the determination of the

operating result of the line losses, power factor, efficiency, rating of synchronous condensers required at the different loads, etc., is best made by use of the labor-saving circle diagram, as developed by H. B. Dwight.[1] The process is based upon a direct

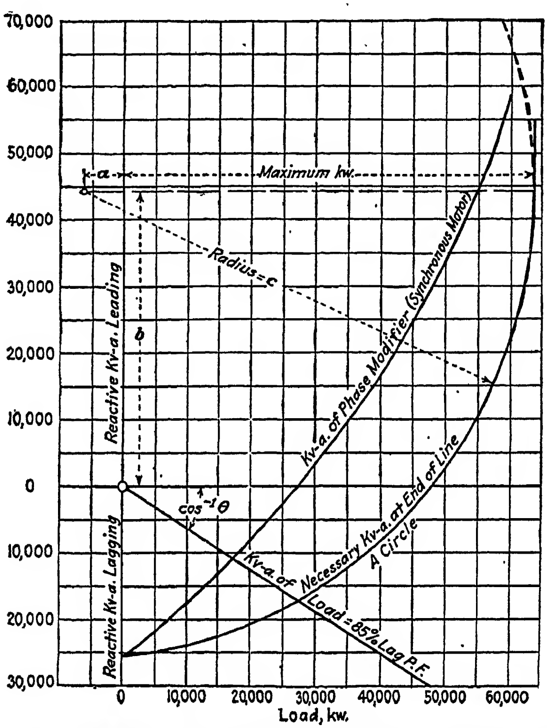


FIG. 235.—Circle diagram for three-phase, 200-mile, constant-voltage transmission line.

solution according to the hyperbolic theory, and if the circle diagram is drawn on fine cross-section paper very accurate results may be obtained.

Figure 235 represents a long transmission line with distributed constants delivering load only at the end of the line.

Let E and I be the voltage and current at the load end.

$I = (P + jQ)$ where P is the in-phase component and $+ jQ$ the leading quadrature component of current, in the line, all in equivalent single-phase or "total" amperes.

[1] The Calculation of Constant Voltage Transmission Lines, *Elec. Jour.*, September, 1914.

Let E_s and I_s be the voltage and current at the supply end.

E_l = voltage of line, l miles from the load end.
I_l = current in line, l miles from the load end.
r = resistance per conductor per mile, ohms.
x = reactance per conductor per mile, ohms.
$z = (r + jx)$, impedance per conductor per mile, ohms.
g = leakage conductance from conductor per mile, mhos.
b = capacity susceptance per conductor per mile, mhos.
$y = (g + jb)$, admittance of conductor per mile, mhos.
L = total length of line, miles.
Z = total line impedance per conductor.
Y = total line admittance per conductor.

(The formulas using the foregoing constants are for three-phase lines. For single-phase lines, use $2R$ and $2X$ in place of R and X and $0.5Y$ in place of Y.)

Then the fundamental formulas for the conditions at the supply end of the line, as developed in standard works on transmission lines,[1] are

$$E_s = E \cosh \sqrt{YZ} + \frac{1}{\sqrt{YZ}} IZ \sinh \sqrt{YZ} \tag{272}$$

and

$$I_s = I \cosh \sqrt{YZ} + \frac{1}{\sqrt{YZ}} EY \sinh \sqrt{YZ}. \tag{273}$$

To convert these forms to those using convergent series, substitute for $\cosh \sqrt{YZ}$ and $\sinh \sqrt{YZ}$, by definition, then

$$E_s = E\left(\frac{\epsilon^{\sqrt{YZ}} + \epsilon^{-\sqrt{YZ}}}{2}\right) + \frac{IZ}{\sqrt{YZ}}\left(\frac{\epsilon^{\sqrt{YZ}} - \epsilon^{-\sqrt{YZ}}}{2}\right) \tag{274}$$

and

$$I_s = I\left(\frac{\epsilon^{\sqrt{YZ}} + \epsilon^{-\sqrt{YZ}}}{2}\right) + \frac{EY}{\sqrt{YZ}}\left(\frac{\epsilon^{\sqrt{YZ}} - \epsilon^{-\sqrt{YZ}}}{2}\right). \tag{275}$$

Now expand the binomials, $(\epsilon^{\sqrt{YZ}} + \epsilon^{-\sqrt{YZ}})$ by Maclaurin's series, then

$$\epsilon^{\sqrt{YZ}} + \epsilon^{-\sqrt{YZ}} = 1 + \sqrt{YZ} + \frac{(\sqrt{YZ})^2}{2} + \cdots + 1 - \sqrt{YZ} + \frac{(\sqrt{YZ})^2}{2} + \cdots \tag{276}$$

[1] See STILL, "Electric Power Transmission," McGraw-Hill Book Company, Inc.

$$= 2\left(1 + \frac{YZ}{2} + \frac{(YZ)^2}{4!} + \cdots\right). \tag{277}$$

Similarly,

$$\epsilon^{\sqrt{YZ}} - \epsilon^{-\sqrt{YZ}} = 2\sqrt{YZ}\left(1 + \frac{YZ}{3!} + \frac{(YZ)^2}{5!} + \cdots\right). \tag{278}$$

Substituting these values back in Eqs. (274) and (275), we have

$$E_s = E\left(1 + \frac{YZ}{2} + \frac{(YZ)^2}{4!} + \cdots\right) + IZ\left(1 + \frac{YZ}{3!} + \frac{(YZ)^2}{5!} + \cdots\right) \tag{279}$$

and

$$I_s = I\left(1 + \frac{YZ}{2} + \frac{(YZ)^2}{4!} + \cdots\right) + EY\left(1 + \frac{YZ}{3!} + \frac{(YZ)^2}{5!} + \cdots\right). \tag{280}$$

In order to work Eq. (279) into the circle form, let all components in phase with E be collected and likewise all components in quadrature with E, then for the first bracket,

$$E\left(1 + \frac{YZ}{2} + \frac{(YZ)^2}{4!} + \cdots\right) = E' + jE'' \tag{281}$$

and writing for Z its equivalent $(R + jX)$, for the second bracket, let

$$(R + jX)\left(1 + \frac{YZ}{3!} + \frac{(YZ)^2}{5!} + \cdots\right) = R' + jX'. \tag{282}$$

Substitute these values from Eqs. (281) and (282) in Eq. (279), which may then be written

$$E_s = E' + jE'' + (P + jQ)(R' + jX') \tag{283}$$

where P is total amperes in-phase current at load end and Q is total leading reactive amperes at load end. Gathering in-phase and quadrature components together, Eq. (283) may be written

$$E_s^2 = (E' + PR' - QX')^2 + (E'' + PX' + QR')^2 \tag{284}$$

$$\begin{aligned} &= E'^2 + (PR')^2 + (QX')^2 + 2E'PR' - 2E'QX' \\ &\qquad 2PQR'X' \\ &\quad + E''^2 + (PX')^2 + (QR')^2 + 2E''QR' + 2PQR'X' \\ &\qquad + 2E''PX'. \end{aligned} \tag{285}$$

Rearranging,

$$E_s^2 - E'^2 - E''^2 = P^2(R'^2 + X'^2) + 2P(E'R' + E''X') + Q^2(R'^2 + X'^2) + 2Q(E''R' - E'X') \tag{286}$$

or

$$\frac{E_s^2 - E'^2 - E''^2}{R'^2 + X'^2} = P^2 + 2P\left(\frac{E'R' + E''X'}{R'^2 + X'^2}\right) + Q^2 + 2Q\left(\frac{E''R' - E'X'}{R'^2 + X'^2}\right). \tag{287}$$

By adding terms to both sides of the equation, the squares can be completed for P and Q, then

$$\frac{E_s^2 - E'^2 - E''^2}{R'^2 + X'^2} + \left(\frac{E'R' + E''X'}{R'^2 + X'^2}\right)^2 + \left(\frac{E''R' - E'X'}{R'^2 + X'^2}\right)^2 = \left(P + \frac{E'R' + E''X'}{R'^2 + X'^2}\right)^2 + \left(Q + \frac{E''R' - E'X'}{R'^2 + X'^2}\right)^2, \tag{288}$$

which simplifies to

$$\frac{E_s^2}{R'^2 + X'^2} = \left(P + \frac{E'R' + E''X'}{R'^2 + X'^2}\right)^2 + \left(Q + \frac{E''R' - E'X'}{R'^2 + X'^2}\right)^2. \tag{289}$$

This is the equation of a circle in the form

$$\text{Radius}^2 = (x - a)^2 + (y - b)^2 \tag{290}$$

having its center at a, b.

Therefore, Eq. (289) specifies a circle having the current components P and Q as axes. If we multiply these current variables by $E/1{,}000$ (kilovolts), then the axes may represent real power $\left(\frac{E}{1{,}000} \times P\right)$ and quadrature kva. $\left(\frac{E}{1{,}000} \times Q\right)$ at the load end. Thus, using these axes,

$$a = -\frac{E'R' + E''X'}{R'^2 + X'^2} \cdot \frac{E}{1{,}000}. \tag{291}$$

$$b = +\frac{E'X' - E''R'}{R'^2 + X'^2} \cdot \frac{E}{1{,}000}. \tag{292}$$

$$\text{Radius} = +\frac{E_s}{\sqrt{R'^2 + X'^2}} \cdot \frac{E}{1{,}000}. \tag{293}$$

For a constant supply voltage then, E_s, and a constant voltage at the load, E, as prescribed in constant-voltage transmission, the kilowatts and kilovolt-amperes at the end of the line bear such a relation to each other that their locus describes a circle. Since it is necessary to the maintenance of the fixed voltage E at the load end that the reactive kilovolt-amperes be that shown by the circle diagram, if the actual reactive kilovolt-ampere demand of the load itself does not equal that of the circle diagram, then the kilovolt-ampere difference must be supplied by the synchronous condenser connected in parallel with the load.

In Fig. 235, the abscissas represent various values of kilowatt power delivered to the load; the ordinates represent reactive kilovolt-amperes. The coordinates of the center of the circle a, b and the radius of the circle C are shown. In order to represent the reactive kilovolt-amperes of the load, draw the kilovolt-ampere load line from the origin, making the proper lagging angle with the base line to accord with the power factor $\cos\theta$ of the load. For any given abscissa, the ordinate of the circle giving the reactive kilovolt-amperes necessary at the end of the line, less the ordinate of the load line giving the reactive kilovolt-amperes present in the load, gives the ordinate for the reactive kilovolt-amperes required from the synchronous condenser at that particular power load. Thus, point by point, the ellipse of the synchronous-condenser kilovolt-amperes can be plotted. Certain significant points will be readily seen; *viz.*, at no load the synchronous condenser must furnish all the required reactive kilovolt-amperes; at the intersection of the circle and the load line, the synchronous-condenser kilovolt-amperes required are zero and the condenser changes from supplying lagging kilovolt-amperes to furnishing leading kilovolt-amperes.

The following data may now be obtained from the diagram:

1. The theoretical limit of load at the given voltage. This is the maximum abscissa for the circle and, as shown,

$$\text{Max. kw.} = \text{radius} - a. \tag{294}$$

2. The reactive kilovolt-amperes in the line at the load end. This is the ordinate to the circle at any abscissa. It may also be computed as follows.

Arrange Eq. (286) in the form of the quadratic equation

$$a_1Q^2 - 2b_1Q + c_1 = 0,$$

where

$$a_1 = R'^2 + X'^2,$$
$$b_1 = E'X' - E''R'$$

and

$$c_1 = P^2(R'^2 + X'^2) + 2P(E'R' + E''X') + E'^2 + E''^2 - E_s^2.$$

Then

$$Q = \frac{b_1 \pm \sqrt{b_1^2 - a_1c_1}}{a_1}. \tag{295}$$

Use only the negative value of the radical, multpilied by $E/1{,}000$, which gives the reactive kilovolt-amperes ($EQ/1{,}000$) for a given kilowatt ($PE/1{,}000$).

3. Line power factor at load end. The power factor of the combined load and synchronous phase modifiers is

$$\frac{100P}{\sqrt{P^2 + Q^2}} \text{ per cent} \tag{296}$$

where P is determined from the load in kilowatts and Q is either read from the circle diagram or calculated by means of Eq. (295). This will be leading for a positive Q and lagging for a negative Q.

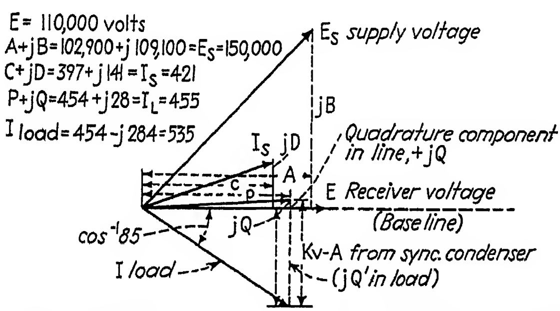


FIG. 236.—Vector diagram to accompany circle diagram for constant-voltage transmission line.

4. Reactive kilovolt-amperes of synchronous motors. This may be read from the circle diagram, or may be found by using the calculated value of Q in the expression

$$\frac{EQ + EP \tan \theta}{1{,}000} = \text{reactive kva.} \tag{297}$$

For when the load takes $P - jQ'$, but the line at the load end has $P + jQ$, the reactive kilovolt-amperes taken by the load, as

shown in Fig. 236,

$$EQ' = \sin\theta\left(\frac{P}{\cos\theta}\right)E,$$

since

$$\frac{P}{I\ (\text{load})} = \cos\theta.$$

Then the reactive kilovolt-amperes taken by the synchronous condensers at the receiver end

$$\begin{aligned}
&= \frac{E}{1{,}000}\left[Q \pm \sin\theta\left(\frac{P}{\cos\theta}\right)\right] \\
&= \frac{E}{1{,}000}\left[Q \pm \frac{\sin\theta}{\cos\theta}\cdot P\right] \\
&= \frac{E}{1{,}000}\left[Q \pm \tan\theta\, P\right].
\end{aligned}$$

5. Line losses in kilowatts. Let Eq. (284) be written as

$$E_s = A + jB,$$

where

$$\begin{aligned}
A &= E' + PR' - QX' \\
B &= E'' + PX' + QR'
\end{aligned}$$

and similarly from Eq. (280),

$$I_s = C + jD = (P + jQ)\left(1 + \frac{YZ}{2} + \frac{(YZ)^2}{4!}\right) + EY\left(1 + \frac{YZ}{3!} + \frac{(YZ)^2}{5!}\right).$$

Then

$$\text{Line losses} = \frac{1}{1{,}000}(AC + BD - EP)\ \text{kw.} \tag{298}$$

The vector relations will then be as shown in Fig. 236, the diagram being built on E, the line-to-line receiver voltage.

6. Kw. at generators $= \dfrac{1}{1{,}000}(AC + BD)$. (299)

7. Kva. at generators $= \dfrac{1}{1{,}000}E_s\sqrt{C^2 + D^2}$. (300)

8. Power factor at generators $= \dfrac{100(AC + BD)}{E_s\sqrt{C^2 + D^2}}$ per cent. (301)

9. Reactive kva. at generators $= \dfrac{1}{1,000}(AD - BC)$. (302)

10. Efficiency of the transmission line $= \dfrac{100EP}{AC + BD}$ per cent. (303)

Figure 235[1] shows the circle diagram for the following 200-mile constant-voltage line:

Constant supply voltage, E_s	150,000 volts
Constant load voltage, E	110,000 volts
Full delivered load	50,000 kw.
Power factor of load, lagging	0.85
Frequency	60 cycles
Conductor, copper cable	400,000 cir. mils.
Effective spacing	20 ft.
Resistance per conductor plus transformers, etc., R	31.8 ohms
Reactance per conductor plus transformers, etc., X	244.4 ohms
Conductance, leakage neglected, G	0
Capacity susceptance per conductor, Y	$+j0.001038$ mhos

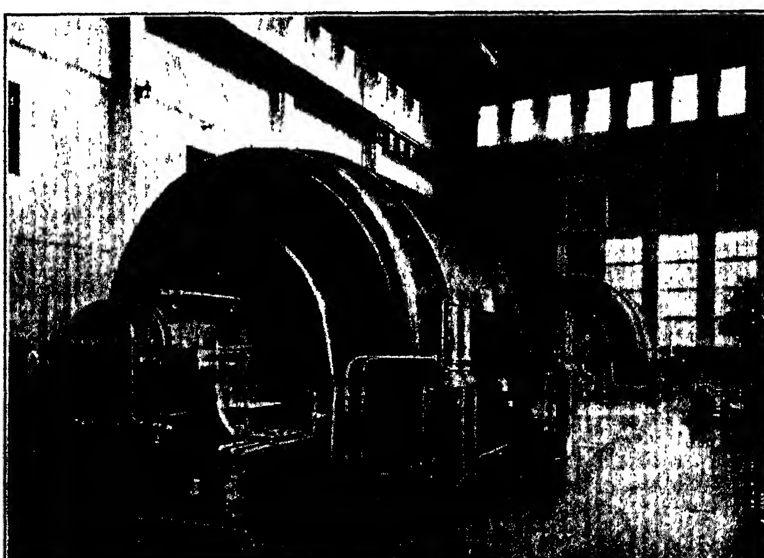


FIG. 237.—Two General Electric type ATI, 50,000-kva. synchronous condensers, Southern California Edison Co.

[1] From DWIGHT, H. B., The Calculation of Constant-voltage Transmission Lines, *Elec. Jour.*, September, 1914.

It will be noted by examination of Eqs. (291) and (292), for a and b, respectively, that since $E = E' + jE''$, for a different fixed voltage at the load end of the transmission line, the supply voltage being constant, another circle diagram would result with the new a and b center coordinates, directly proportional to the square root of the delivery voltage. For another voltage E, the radius C from Eq. (293) would vary directly with E. Hence, for various values of E, a family of circles could be drawn showing the relation between the kilowatts and the reactive kilovolt-amperes for each value of E at the load end of the line.

179. Problems.

1. A three-phase transmission line delivers 600 kw. at 2,200 volts with 0.8 power factor lagging to induction motors. RI drop in each wire is 2 per cent of star voltage, XI drop 6 per cent. A synchronous motor is substituted for one-third load. Find the kilovolt-amperes and power factor of synchronous motor to secure same voltage at load and generator ends of line. How much could the leading quadrature amperes of the synchronous motor be reduced and still hold the 2,200 volts at the receiver end? How much line reactance would be needed?

a. Solve by vector diagram, let $PR = QX$ approximately.

b. Solve Q by computation, as for circle diagram.

2. The Conowingo-Philadelphia 220-kv. transmission lines are each three phase, single circuit, with 795,000-cir. mil. A.C.S.R. conductors on 25 ft. 6 in. flat spacing for a length of 58 miles.

Conductor resistance per mile =	0.119 ohm.
60-cycle reactance per mile of single conductor =	0.81 ohm.
Capacitance to neutral per mile of single conductor =	0.0136 mf.
($jb = 2\pi fC \times 10^{-6}$ mho) per mile.	

With a line delivering 100,000 kw. at 220 kv. with 0.8 power factor lagging and its regulating synchronous condenser carrying 30,000 leading kva., what is the voltage at the generator end of line?

3. The following 225-mile transmission line is proposed:

Supply voltage	220,000
Receiver voltage	95 % of E_s
Resistance per mile	0.0782 ohm
Reactance per mile	0.8 ohm
Susceptance per mile	0.00000542 mho

750,000-cir. mil coppers, spaced 21 ft., 60 cycles.

What is the theoretical steady-state limit of the load at the given voltages?

4. Winnipeg is supplied by a double-circuit three-phase line on one set of steel towers, 77 miles long.

Cable 287,000	cir. mils. aluminum
R per cable	22.8 ohms
X per cable, 60 cycles	53.8 ohms
jB per cable	0.000462 mho.
Supply volts	72 kv.
Receiver volts	66 kv.

Load power factor is 0.78 lagging. What is the theoretical steady-state maximum capacity of each circuit?

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